Hypergraph Partitioning for Compiling Pseudo-Boolean Formulae

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ROADEF'21, Session *Partitionnement des Graphes* – April 29, 2021

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The problem is often to check whether such a formula is satisfiable, i.e., has a solution

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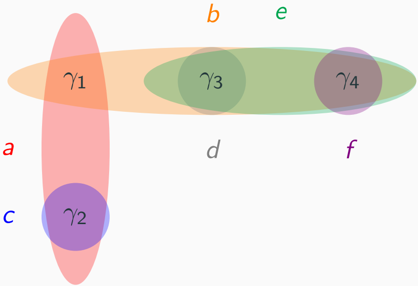

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In such cases, it may be interesting to rely on knowledge compilation

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Compiling a formula is translating it (offline) into another language to obtain an equivalent formula on which performing the wanted (online) operations is easier

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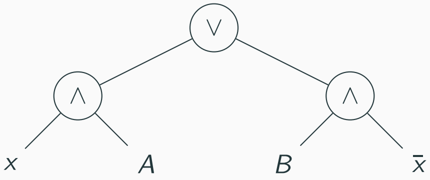
These two properties allow the efficient computation of different queries

Ensuring Determinism

To ensure determinism, each disjunction node in the circuit will be a decision node



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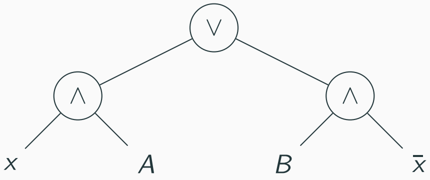

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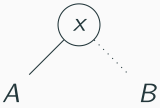

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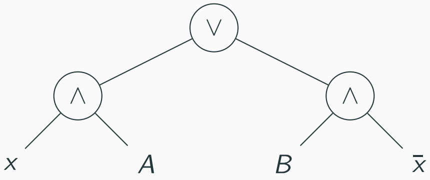

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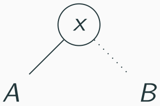

The d-DNNFs we obtain in this case are called Decision-DNNF

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The connected components can then be compiled independently, before adding their conjunction to the build d-DNNF

Compiling our CNF Formula

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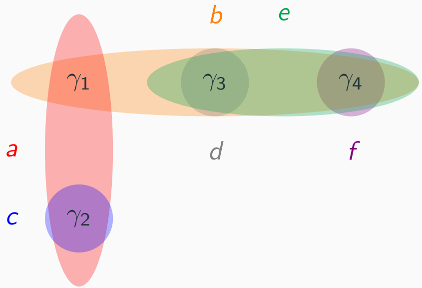

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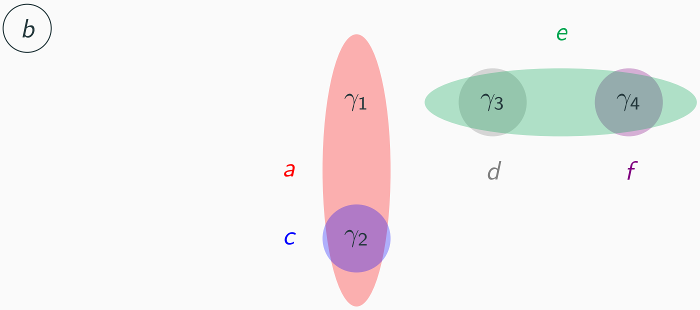

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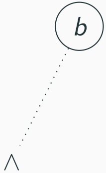

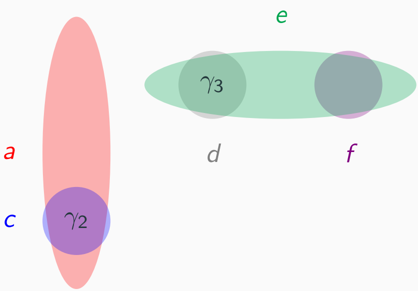

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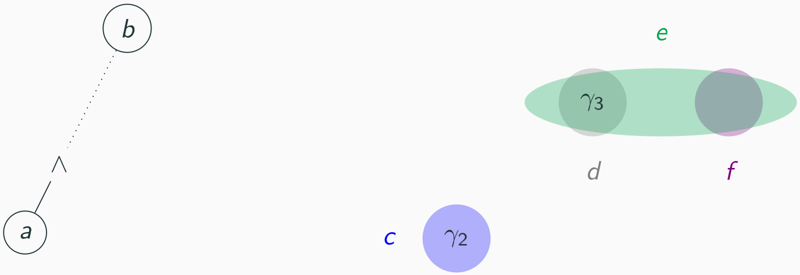

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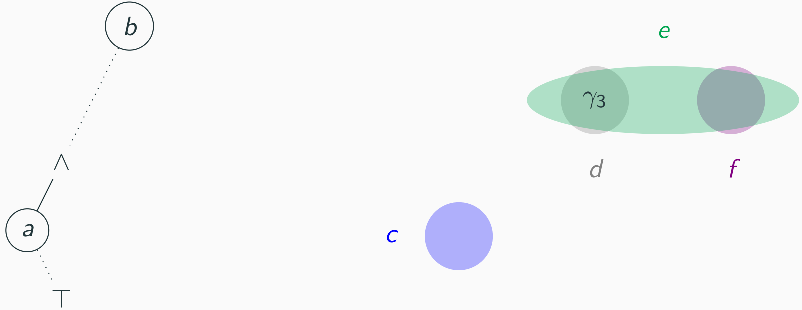

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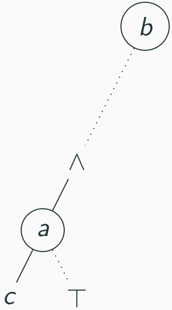

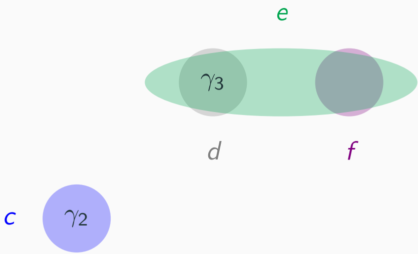

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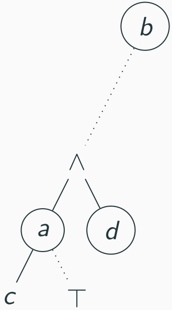

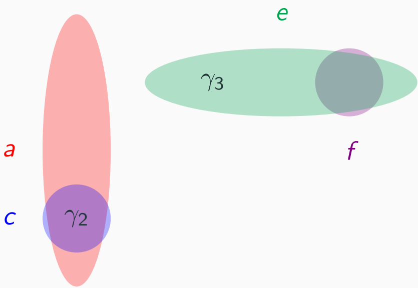

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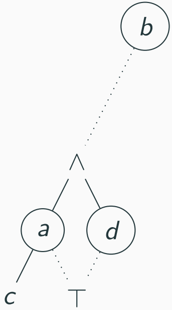

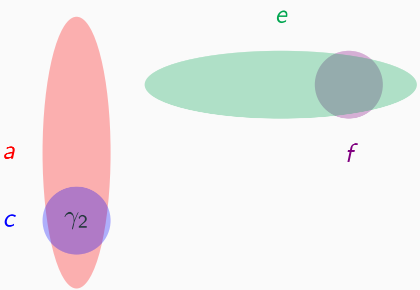

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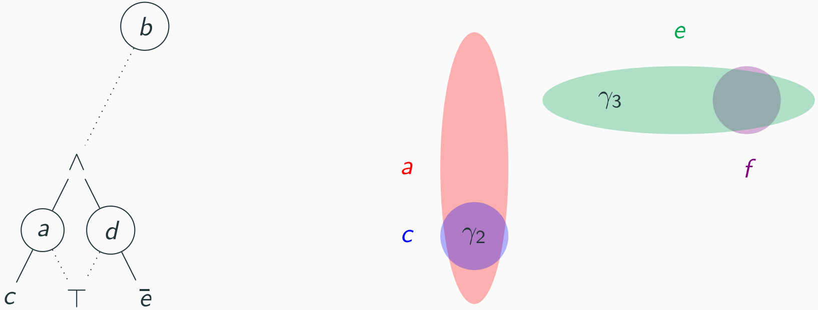

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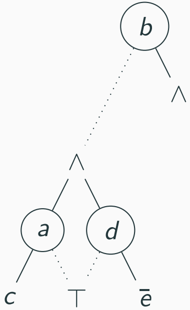

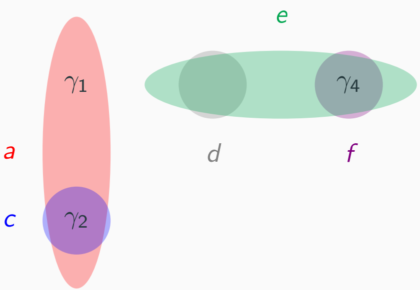

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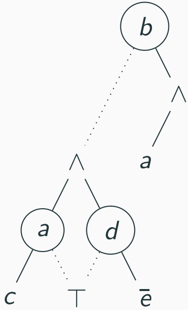

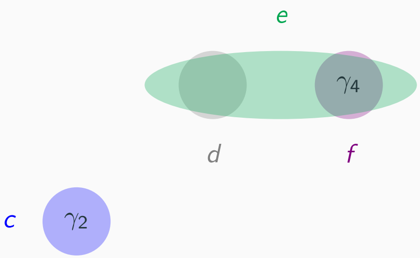

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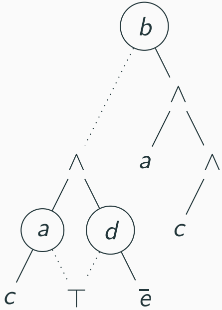

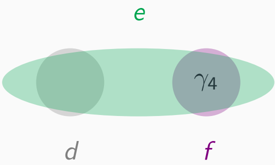

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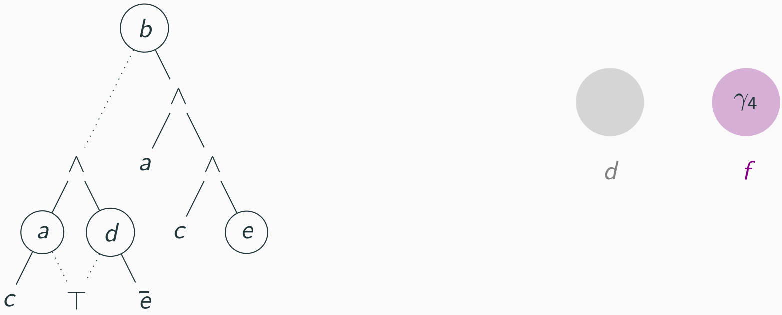

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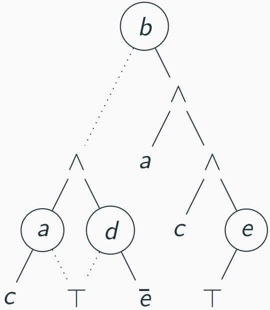

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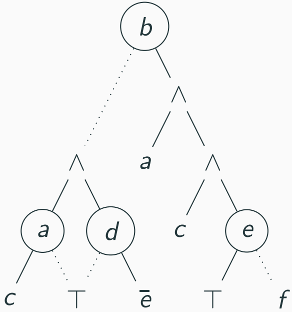

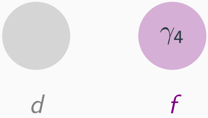

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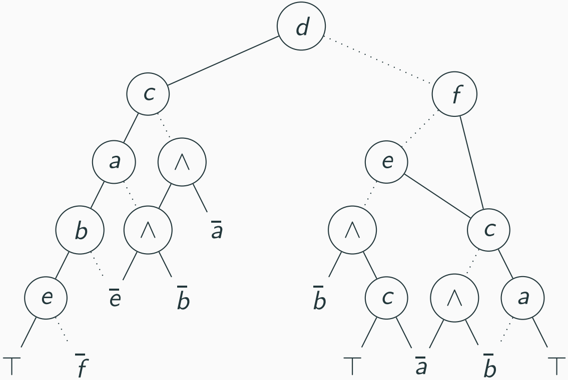

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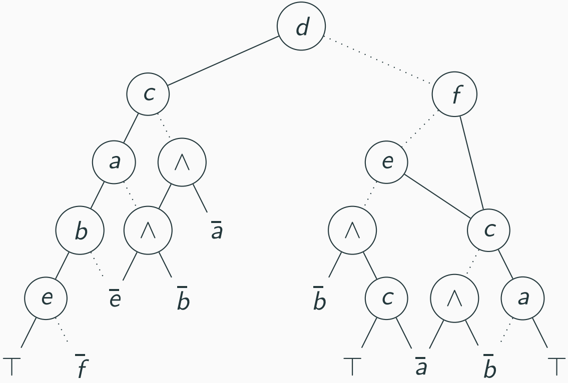

Ideally, we need small cutsets and balanced partitions

Outline of *D4* (Lagniez and Marquis, 2017)

1. Invoke a SAT Solver on the input
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3. If all variables are assigned, then the compiled form is $\top$
4. For each connected component φ of the formula:
   a. Choose a variable v based on a cutset of φ computed with PaToH (Çatalyürek and Aykanat, 2011)
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D4 is available at `https://github.com/crillab/d4`

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On such instances, pseudo-Boolean reasoning can offer better performance

Pseudo-Boolean (PB) Constraints

PB solvers are generalizations of SAT solvers that allow to consider

- normalized PB constraints $\sum_{i=1}^{n} \alpha_i \ell_i \geq \delta$
- cardinality constraints $\sum_{i=1}^{n} \ell_i \geq \delta$
- clauses $\sum_{i=1}^{n} \ell_i \geq 1$

in which

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PB constraints allow in general more succinct encodings than CNF, and are often more natural to use

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In general, PB representations may be exponentially smaller than CNF representations

Compiling PB Formulae

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To support PB compilation, one basically needs to replace by a PB solver the SAT solver used in the compilation procedure

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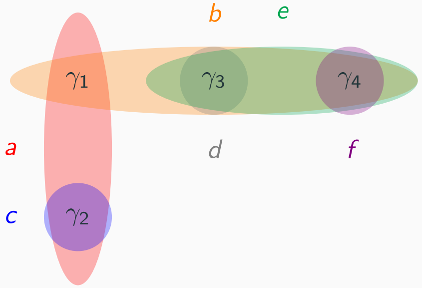

Outline of *PBD4*

1. Invoke a PB Solver on the input
2. If the formula is `UNSAT`, then the compiled form is $\bot$
3. If all variables are assigned, then the compiled form is $\top$
4. For each connected component φ of the formula:
   a. Choose a variable v based on a cutset of φ computed with KaHyPar (Schlag, 2020)
   b. Compile $\varphi|v$ as φ_v
   c. Compile $\varphi|\bar{v}$ as $\varphi_{\bar{v}}$
   d. The compiled form of φ is $\text{ite}(v, \varphi_v, \varphi_{\bar{v}})$
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PBD4 is available at `https://github.com/crillab/pbd4`

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- Hypergraph partitioning provides a heuristic to decide in which order to assign variables when building the compiled form
- Modern and efficient SAT solvers are used as oracles to determine whether it is worth compiling subformulae
- For compiling certain problems, using PB solvers instead may be more efficient

Perspectives

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- Take advantage of native PB compilation for considering new applications of knowledge compilation (e.g., for explaining (binarized) neural networks)
- Use speculation techniques to speed up compilation:
    - by predicting satisfiability before invoking the SAT/PB solver as an oracle
    - by predicting cutsets before computing a partition of the hypergraph

Hypergraph Partitioning for Compiling Pseudo-Boolean Formulae

Romain Wallon

ROADEF'21, Session *Partitionnement des Graphes* – April 29, 2021

Laboratoire d'Informatique de l'X (LIX), École Polytechnique, X-Uber Chair

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