On revising offer status in argument-based negotiations

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Abstract: Negotiation is a form of interaction in which agents with conflicting preferences try to reach an agreement on an issue by exchanging offers. Since early nineties, the benefits of exchanging arguments, in addition to offers, has been advocated in the literature. The idea is that an offer supported by an argument has a better chance to be accepted by another agent. Unfortunately, a little has been done on showing how a new argument may change the status of an offer for an agent. In other words, how an acceptable (resp. rejected) offer becomes rejected (resp. acceptable) for an agent when he receives a new argument from another agent.

In this paper, we assume that each negotiating agent is equipped with a particular argument-based decision system. This system assigns a status to each offer on the basis of the acceptability of their supporting arguments. We will study under which conditions an offer may change its status when a new argument is received and under which conditions this new argument is useless. This amounts to study how the acceptability of arguments evolves when the decision system is extended by new arguments.

Contents

1 Introduction

In their seminal book [12], Walton and Krabbe define negotiation as a form of interaction in which autonomous agents having different interests or goals try to find a compromise on an issue, called *negotiation object*. Examples of negotiation objects are: the price of a given product, an allocation of resources, etc. During a negotiation, agents exchange *offers*. An offer represents a possible value of the negotiation object. Since agents may have conflicting interests, they may have also conflicting preferences on the set of all possible offers. It may be the case that the most preferred offer for one agent is the worst one for another agent. Consequently, agents need to make *concessions*, i.e. to accept less preferred offers.

At early nineties, Sycara has emphasized the importance of using argumentation techniques in negotiation dialogues [10]. The basic idea is to allow agents not only to exchange offers but also reasons that support these offers in order to mutually influence their preferences on the set of offers, and consequently the outcome of the dialogue. Since there, several works on argumentation-based negotiation have been done including work by Parsons and Jennings [8], Reed [9], Kraus et al. [7], Tohmé [11], Amgoud et al. [3], and Kakas and Moraitis [6]. Most of these works have focused on relating arguments to protocols, and on giving definitions of arguments in favor of an offer. In [1], an abstract model for argument-based negotiation has been proposed. That model shows clearly how arguments are related to the preference relation between offers. The idea is that each agent is equipped with a decision model that computes a status for each offer, and a preference relation among all the offers. The whole process is based on arguments supporting or attacking offers. While this work is very important for understanding the role of argumentation in negotiation, it is not clear how and under which conditions the status of an offer may change when a given argument is received from another agent without having to compute the new extensions of arguments. This issue is very important for studying dialog strategies, where an agent should guess what is the best argument to utter at each step of a dialog. For instance, if we know that an argument will not change the status of an offer which is currently rejected, then it is useless to utter it, and the agent has to look for a more relevant one.

Grounded on a recent argument-based decision model [2], this paper studies deeply the revision of offer status in light of a new argument. This amounts to study how the acceptability of arguments evolves when the decision system is extended by new arguments. Note that proofs of all properties and theorems are in the appendix of this document.

This paper is organized as follows: Section 2 recalls briefly the decision model proposed in [2]. Section 3 studies the revision of offer status when a new argument is received. In section 4 we study the revision of offer status under some assumptions on the decision model. The last section concludes.

2 An argumentation framework for decision making

This section recalls briefly the argument-based framework for decision making that has been proposed in [2]. In that work, a decision problem amounts to defining a preordering, usually a complete one, on a set of possible *choices* on the basis of the different consequences of each decision.

2.1 The decision system

Let $\mathcal{L}$ denote a logical language. From $\mathcal{L}$, a finite set $\mathcal{O}$ of n distinct *options* is identified. Two kinds of arguments are distinguished in that framework: arguments supporting options, called *practical arguments* and arguments supporting beliefs, called *epistemic arguments*. Arguments supporting options are collected in a set $\mathcal{A}_o$ and arguments supporting beliefs are collected in a set $\mathcal{A}_b$ such that $\mathcal{A}_o \cap A_b = \emptyset$ and $\mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_o$. Note that the structure of arguments is assumed not known. Moreover, arguments in $\mathcal{A}_o$ highlight positive features of their conclusions, i.e., they are *in favor* of their conclusions. Practical arguments are linked to the options they support by a function $\mathcal{H}$ defined as follows:

$$\mathcal{H}\colon \mathcal{O} \to 2^{\mathcal{A}_o} \text{ such that } \forall i, j \text{ if } i \neq j \text{ then } \mathcal{H}(o_i) \cap \mathcal{H}(o_j) = \emptyset \text{ and}$$
$$\mathcal{A}_o = \bigcup_{i=1}^{n} \mathcal{H}(o_i) \text{ with } \mathcal{O} = \{o_1, \ldots, o_n\}.$$

Each practical argument a supports only one offer o. We say also that o is the conclusion of the practical argument a, and we write $\texttt{Conc}(a) = o$. Note that there may exist offers that are not supported by arguments (i.e. $\mathcal{H}(o) = \emptyset$).

Example 1 *Let us assume a set* $\mathcal{O} = \{o_1, o_2, o_3\}$ *of three options, a set* $\mathcal{A}_b = \{b_1, b_2, b_3\}$ *of three epistemic arguments, and finally a set* $\mathcal{A}_o = \{a_1, a_2, a_3\}$ *of three practical arguments. The arguments supporting the different offers are summarized in table below.*

$\mathcal{H}(o_1)$	$= \{a_1\}$
$\mathcal{H}(o_2)$	$= \{a_2, a_3\}$
$\mathcal{H}(o_3)$	$= \emptyset$

Three binary relations between arguments have been defined. They express the fact that arguments may not have the same strength. The first preference relation, denoted by $\geq_b$, is a partial preorder[1] on the set $\mathcal{A}_b$. The second relation, denoted by $\geq_o$, is a partial preorder on the set $\mathcal{A}_o$. Finally, a third preorder, denoted by $\geq_m$ (m for *mixed* relation), captures the idea that any epistemic argument is stronger that any practical argument. The role of epistemic arguments in a decision problem is to validate or to undermine the beliefs on which practical arguments are built. Indeed, decisions should be made under certain information. Thus, $(\forall a \in \mathcal{A}_b)(\forall a' \in \mathcal{A}_o)\ (a, a') \in \geq_m \wedge\ (a', a) \notin \geq_m$. Note that $(a, a') \in \geq_x$ with $x \in \{b, o, m\}$ means that a is *at least as good as*

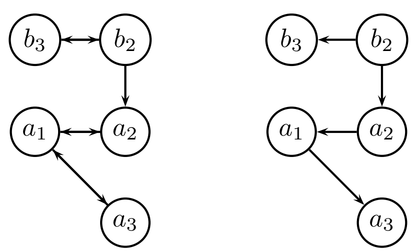
[1]Recall that a relation is a preorder iff it is *reflexive* and *transitive*.

a'. In what follows, $>_x$ denotes the strict relation associated with $\geq_x$. It is defined as follows: $(a, a') \in >_x$ iff $(a, a') \in \geq_x$ and $(a', a) \notin \geq_x$. We will sometimes write $(a, a') \in \odot$ to refer to one particular of the four possible situations: $(a, a') \in \geq_x \wedge (a', a) \in \geq_x$, meaning that the two arguments a and a' are *indifferent* for the decision maker, $(a, a') \in >_x$, meaning that a is *strictly preferred* to a', $(a', a) \in >_x$, meaning that a' is strictly preferred to a, $(a, a') \notin \geq_x \wedge (a', a) \notin \geq_x$, meaning that the two arguments are *incomparable*.
Generally arguments may be conflicting. These conflicts are captured by a binary relation on the set of arguments. Three such relations are distinguished. The first one, denoted by $\mathcal{R}_b$ captures the different conflicts between epistemic arguments. The second relation, denoted $\mathcal{R}_o$ captures the conflicts among practical arguments. Two practical arguments are conflicting if they support different options. Formally, $(\forall a, b \in \mathcal{A}_o)$ $(a, b) \in \mathcal{R}_o$ iff $\texttt{Conc}(a) \neq \texttt{Conc}(b)$. Finally, practical arguments may be attacked by epistemic ones. The idea is that an epistemic argument may undermine the belief part of a practical argument. However, practical arguments are not allowed to attack epistemic ones. This avoids wishful thinking, i.e., avoids making decisions according to what might be pleasing to imagine instead of by appealing to evidence or rationality. This relation, denoted by $\mathcal{R}_m$, contains pairs (a, a') where $a \in \mathcal{A}_b$ and $a' \in \mathcal{A}_o$. Before introducing the framework, we need first to combine each preference relation $\geq_x$ (with $x \in \{b, o, m\}$) with the conflict relation $\mathcal{R}_x$ into a unique relation between arguments, denoted $\texttt{Def}_x$, and called *defeat* relation.

Definition 1 *(Defeat relation) Let $a, b \in \mathcal{A}$. $(a, b) \in \texttt{Def}_x$ iff $(a, b) \in \mathcal{R}_x$ and $(b, a) \notin \geq_x$.*

Let $\texttt{Def}_b$, $\texttt{Def}_o$ and $\texttt{Def}_m$ denote the three defeat relations corresponding to three attack relations. Since arguments in favor of beliefs are always preferred (in the sense of $\geq_m$) to arguments in favor of offers, it holds that $\mathcal{R}_m = \texttt{Def}_m$.

Example 2 *(Example 1 cont.) The graph on the left depicts different attacks among arguments. Let us assume the following preferences: $(b_2, b_3) \in \geq_b$, $(a_2, a_1) \in \geq_o$ and $(a_1, a_3) \in \geq_o$. The graph of defeat is depicted on the right of figure below.*



The different arguments of $\mathcal{A}_b \cup \mathcal{A}_o$ are evaluated within the system $\mathcal{AF} = \langle \mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_o, \texttt{Def} = \texttt{Def}_b \cup \texttt{Def}_o \cup \texttt{Def}_m \rangle$ using any Dung's acceptability semantics. In order to define these semantics, we introduce first the notion of conflict-free set and that of defense.

Definition 2 *(Conflict-free, Defense) Let* $\langle \mathcal{A}, \texttt{Def} \rangle$ *be an argumentation system,* $\mathcal{B} \subseteq \mathcal{A}$*, and* $a \in \mathcal{A}$*.*

- $\mathcal{B}$ *is* conflict-free *iff* $\nexists\, a,\, b \in \mathcal{B}$ *s.t.* $(a, b) \in \texttt{Def}$*.*
- $\mathcal{B}$ defends a *iff* $\forall\, b \in \mathcal{A}$*, if* $(b, a) \in \texttt{Def}$*, then* $\exists\, c \in \mathcal{B}$ *s.t.* $(c, b) \in \texttt{Def}$*.*

The main semantics introduced by Dung are recalled in the following definition.

Definition 3 *(Acceptability semantics) Let* $\mathcal{AF} = \langle \mathcal{A}, \texttt{Def} \rangle$ *be an argumentation system, and* $\mathcal{B}$ *be a conflict-free set of arguments.*

- $\mathcal{B}$ *is a* grounded extension*, denoted* $\texttt{GE}$*, iff* $\mathcal{B}$ *is the least fixpoint of function* $\mathcal{F}$ *where* $\mathcal{F}(S) = \{a \in \mathcal{A} \mid S \text{ defends } a\}$*, for* $S \subseteq \mathcal{A}$*.*
- $\mathcal{B}$ *is an* admissible extension *iff* $\mathcal{B} \subseteq \mathcal{F}(\mathcal{B})$*.*
- $\mathcal{B}$ *is a* preferred extension *iff* $\mathcal{B}$ *is a maximal (w.r.t set* $\subseteq$*) admissible extension.*
- $\mathcal{B}$ *is a* stable extension *iff* $\mathcal{B}$ *attacks any argument in* $\mathcal{A} \setminus \mathcal{B}$*.*
- $\mathcal{B}$ *is a* complete extension *iff* $\mathcal{B} = \mathcal{F}(\mathcal{B})$*;*

Using these acceptability semantics, the status of each argument can be defined as follows.

Definition 4 *(Argument status) Let* $\mathcal{AF} = \langle \mathcal{A}, \texttt{Def} \rangle$ *be an argumentation system, and* $\mathcal{E}_1, \ldots, \mathcal{E}_x$ *its extensions under a given semantics. Let* $a \in \mathcal{A}$*.*

- a *is* skeptically accepted *iff* $a \in \mathcal{E}_i$*,* $\forall \mathcal{E}_{i=1,\ldots,x}$*,* $\mathcal{E}_i \neq \emptyset$*.*
- a *is* credulously accepted *iff* $\exists \mathcal{E}_i$ *s.t.* $a \in \mathcal{E}_i$ *and* $\exists \mathcal{E}_j$ *s.t.* $a \notin \mathcal{E}_j$*.*
- a *is* rejected *iff* $\nexists \mathcal{E}_i$ *s.t.* $a \in \mathcal{E}_i$*.*

A direct consequence of Definition 4 is that an argument is skeptically accepted iff it belongs to the intersection of all extensions, and that it is rejected iff it does not belong to the union of all extensions. Formally:

Property 1 *Let* $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation framework, and* $\mathcal{E}_1, \ldots, \mathcal{E}_n$ *its extensions under a given semantics. Let* $a \in \mathcal{A}$*.*

1. *a is skeptically accepted iff* $a \in \bigcap_{i=1}^{n} \mathcal{E}_i$*.*
2. *a is rejected iff* $a \notin \bigcup_{i=1}^{n} \mathcal{E}_i$*.*

Example 3 *(Example 1 cont.) There is one preferred extension, which is also the grounded one,* $\{a_1, b_1\}$*. It is clear that* a_1 *and* b_1 *are skeptically accepted while other arguments are rejected.*

Let $\mathcal{AF} = \langle \mathcal{A}, \mathtt{Def} \rangle$ be an argumentation system. $\mathtt{Sc}(\mathcal{AF})$, $\mathtt{Cr}(\mathcal{AF})$ and $\mathtt{Rej}(\mathcal{AF})$ denote respectively the sets of skeptically accepted arguments, credulously accepted arguments and rejected arguments. It can be shown that these three sets are disjoint. Moreover, their union is the set $\mathcal{A}$ of arguments.

Property 2 *Let $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation system and $\mathtt{Sc}(\mathcal{AF})$, $\mathtt{Cr}(\mathcal{AF})$, $\mathtt{Rej}(\mathcal{AF})$, its sets of arguments.*

1. $\mathtt{Sc}(\mathcal{AF}) \cap \mathtt{Cr}(\mathcal{AF}) = \emptyset$, $\mathtt{Sc}(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF}) = \emptyset$, $\mathtt{Cr}(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF}) = \emptyset$
2. $\mathtt{Sc}(\mathcal{AF}) \cup \mathtt{Cr}(\mathcal{AF}) \cup \mathtt{Rej}(\mathcal{AF}) = \mathcal{A}$.

Status of an option is defined from the status of its arguments.

Definition 5 *(Option status) Let $o \in \mathcal{O}$.*

- *o is* acceptable *iff $\exists a \in \mathcal{H}(o)$ such that $a \in \mathtt{Sc}(\mathcal{AF})$.*
- *o is* rejected *iff $\mathcal{H}(o) \neq \emptyset$ and $\forall a \in \mathcal{H}(o)$, $a \in \mathtt{Rej}(\mathcal{AF})$.*
- *o is* negotiable *for the negotiating agent iff $(\nexists a \in \mathcal{H}(o))$ $(a \in \mathtt{Sc}(\mathcal{AF})) \wedge$ $(\exists a' \in \mathcal{H}(o))$ $(a' \in \mathtt{Cr}(\mathcal{AF}))$.*
- *o is* non-supported *iff it is neither acceptable, nor rejected nor negotiable.*

Let $\mathcal{O}_a$ (resp. $\mathcal{O}_n$, $\mathcal{O}_{ns}$, $\mathcal{O}_r$) be the set of acceptable (resp. negotiable, non-supported, rejected) options.

The following simple property can be shown.

Property 3 *An option $o \in \mathcal{O}$ is non-supported iff $\mathcal{H}(o) = \emptyset$.*

Example 4 *(Example 1 cont.) Option o_1 is acceptable, o_2 is rejected and o_3 is non-supported.*

It can be checked that an option has only one status. This status may change in light on new argument as we will show in next sections.

Property 4 *Let $o \in \mathcal{O}$. o has exactly one status.*

2.2 Impact of a semantics on the status of options

The aim of this section is to study the impact of the different acceptability semantics on the status of options, consequently on the relation $\succeq$ on the set $\mathcal{O}$ of options. Before starting the study, let us first introduce a useful property that will be used for showing our results.

Property 5 *Let $k, n \in \mathcal{N}$, $1 \leq k \leq n$. Let $A_1, \ldots, A_n$ be arbitrary sets. Then:*

- $\bigcap_{i=1}^{n} A_i \subseteq \bigcap_{i=1}^{k} A_i$

- $\bigcup_{i=1}^{k} A_i \subseteq \bigcup_{i=1}^{n} A_i$

In the particular case where $n \geq 1$ and $k = 1$, we have:

- $\bigcap_{i=1}^{n} A_i \subseteq A_1$
- $A_1 \subseteq \bigcup_{i=1}^{n} A_i$

The choice of a semantics has an impact on the acceptability of arguments and, consequently, on the status of options. We have studied the impact of several semantics on the status of options. Note that the results presented in this section hold not only in the case of the particular system defined in the previous section, but also in the general case (i.e. for any attack relation $\mathcal{R}$).

Let $\mathcal{O}_y^x$ denote the set of options having status y under semantics x, with $x \in \{ad, p, g, s, c\}$, where ad stands for admissible, p for preferred, g for grounded, s for stable and c for complete. For example, $\mathcal{O}_a^p$ denotes the set of all accepted options under preferred semantics. In [2], the status of options makes it possible to compare these options, thus to define a preference relation $\succeq$ on the set O. The basic idea is the following: acceptable options are preferred to negotiable ones. Negotiable options are themselves preferred to non-supported options, which in turn are better than rejected options.

Let us start with admissible semantics. It is worth noticing that under this semantics, there are no skeptically accepted arguments, thus there are no acceptable options. This is due to the fact that the empty set is an admissible extension of the argumentation framework $\mathcal{AF} = \langle \mathcal{A}, \mathtt{Def} \rangle$. Formally:

Property 6 *Let $\mathcal{O}$ be a set of options. It holds that $\mathcal{O}_a^{ad} = \emptyset$.*

It can be shown that acceptable options are the same under grounded and complete semantics. Formally:

Property 7 *Let $\mathcal{O}$ be a set of options. The equality $\mathcal{O}_a^g = \mathcal{O}_a^c$ holds.*

The following result shows that acceptable options under grounded semantics are a subset of acceptable options under preferred semantics.

Property 8 *Let $\mathcal{O}$ be a set of options. The inclusion $\mathcal{O}_a^g \subseteq \mathcal{O}_a^p$ holds.*

Let us now focus on the link between accepted options under preferred and stable semantics. There are two situations here: the case where the argumentation system has stable extensions and the case where there is no stable extension. The following result shows that the direction of the inclusion differs from one case to another.

Property 9 *Let $\mathcal{O}$ be the set of options, and let $\mathcal{AF} = \langle \mathcal{A}, \mathtt{Def} \rangle$ be the argumentation system for rank-ordering elements of $\mathcal{O}$.*

1. *If $\mathcal{AF}$ has no stable extensions, then $\mathcal{O}_a^s = \emptyset$ and $\mathcal{O}_a^s \subseteq \mathcal{O}_a^p$.*
2. *If $\mathcal{AF}$ has stable extensions, then $\mathcal{O}_a^p \subseteq \mathcal{O}_a^s$.*

All these results and summarized below.

Let $\mathcal{O}$ be the set of options, and let $\mathcal{AF}$ be the argumentation system.

- *Case 1:* If $\mathcal{AF}$ has no stable extension, then
$\mathcal{O}_a^s = \mathcal{O}_a^{ad} = \emptyset \subseteq \mathcal{O}_a^g = \mathcal{O}_a^c \subseteq \mathcal{O}_a^p$.

- *Case 2:* If $\mathcal{AF}$ has at least one stable extension, then
$\mathcal{O}_a^{ad} = \emptyset \subseteq \mathcal{O}_a^g = \mathcal{O}_a^c \subseteq \mathcal{O}_a^p \subseteq \mathcal{O}_a^s$.

The above result shows that admissible semantics does not provide very rich framework for decision making since there are no acceptable options at all. Grounded semantics accepts very few arguments as expected, because it is very cautious. When stable extensions exist, this semantics accepts more acceptable options than any other semantics.

We will now explore the links between the sets of rejected offers under different semantics.

Property 10 *Let $\mathcal{O}$ be a set of options. It holds that:*

1. $\mathcal{O}_r^{ad} = \mathcal{O}_r^c = \mathcal{O}_r^p$
2. $\mathcal{O}_r^p \subseteq \mathcal{O}_r^g$.

Regarding stable semantics, there are here also two cases depending on whether there are stable extensions or not.

Property 11 *Let $\mathcal{O}$ be the set of options, and let $\mathcal{AF}$ be the argumentation system.*

1. *If $\mathcal{AF}$ has no stable extensions, then $\mathcal{O}_r^s = \mathcal{O}$, i.e. all the options are rejected.*
2. *If $\mathcal{AF}$ has stable extensions, then $\mathcal{O}_r^p \subseteq \mathcal{O}_r^s \subseteq \mathcal{O}_r^g$.*

Let us now summarize all these results.

Let $\mathcal{O}$ be the set of options, and let $\mathcal{AF}$ be the argumentation system.

- *Case 1:* If $\mathcal{AF}$ has no stable extension, then
$\mathcal{O}_r^{ad} = \mathcal{O}_r^c = \mathcal{O}_r^p \subseteq \mathcal{O}_r^g \subseteq \mathcal{O}_r^s = \mathcal{O}$.
- *Case 2:* If $\mathcal{AF}$ has at least one stable extension, then
$\mathcal{O}_r^{ad} = \mathcal{O}_r^c = \mathcal{O}_r^p \subseteq \mathcal{O}_r^s \subseteq \mathcal{O}_r^g$.

In case there is no stable extension, all the options are rejected under stable semantics. However, when the system has at least one stable extension, the grounded semantics rejects the most arguments, and consequently, the most options. We can also check that admissible, complete and preferred semantics reject exactly the same set of arguments. As expected, the number of rejected options is very high under grounded semantics. This result is not surprising since grounded semantics is very cautious and accepts very few arguments.

As mentioned before, an option o is non-supported iff there are no arguments in its favor, i.e. $\mathcal{H}(o) = \emptyset$. It is clear that this is independent of the acceptability semantics. So, we have the following property:

Property 12 *Let $\mathcal{O}$ be the set of options. It holds that:*
$\mathcal{O}^{ad}_{ns} = \mathcal{O}^{c}_{ns} = \mathcal{O}^{p}_{ns} = \mathcal{O}^{g}_{ns} = \mathcal{O}^{s}_{ns}$.

Now, recall that an option o is negotiable if there are no skeptically accepted arguments in its favor, but it is supported by at least one credulously accepted argument. An agent will always prefer an accepted option to a negotiable one, while it prefers a negotiable option to a non-supported one.

Generally, an argumentation system has more admissible extensions than any other type of extensions, thus this semantics will return the greatest number of credulously accepted arguments, and consequently, the greatest number of negotiable options. On the other hand, since there is always exactly one grounded extension, there can never exist a negotiable option under this semantics. Since every stable extension is a preferred one, an argumentation system may have more preferred extensions than stable ones. Consequently, it can be shown that there are more credulously accepted arguments under preferred semantics than under stable one. Thus, there are more negotiable offers.

Property 13 $\mathcal{O}^{g}_{n} = \emptyset \subseteq \mathcal{O}^{s}_{n} \subseteq \mathcal{O}^{p}_{n} \subseteq \mathcal{O}^{c}_{n} \subseteq \mathcal{O}^{ad}_{n}$.

Note that in case there is no stable extension the set $\mathcal{O}^{s}_{n}$ is empty, while this is not necessarily true in the general case.

3 Revising offer status

In the remaining of this paper, we assume that a negotiation takes place between two or more agents on a negotiation object. In what follows, we assume that the different values of this object are gathered in a "finite" set $\mathcal{O}$. Those values, calles offers, will be exchanged by agents in a negotiation. Each negotiating agent is assumed to know this set, and is equipped with a decision system like the one introduced in the previous section that compute a status for each offer in $\mathcal{O}$. Let $\mathcal{AF} = \langle \mathcal{A}, \mathtt{Def} \rangle$ be that system where $\mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_o$, $\mathtt{Def} = \mathtt{Def}_b \cup \mathtt{Def}_o \cup \mathtt{Def}_m$, with $\mathtt{Def}_x$ ($x \in \{b, o, m\}$) is built from an attack relation $\mathcal{R}_x$ and a preference relation $\geq_x$ between arguments. Let $\mathcal{H}$ be a function that relates the offers to the arguments of the agent. Agents may have different sets of arguments and different preferences between them. However, each agent is able to recognize a received argument, i.e. to interpret it correctly. Since agents may have different sets of arguments, the decision systems of these agents may return different status for the same offer. In this section, we assume that "grounded semantics" is used for defining the acceptability of arguments. Consequently, an argument can either be skeptically accepted or rejected. Similarly, an offer may be either acceptable, or rejected or non-supported.

During a negotiation, an agent may receive an argument e from another agent. This argument is either epistemic or practical. In this paper, we assume that e is practical. The decision system of this agent will be extended by this new argument. Let $\mathcal{AF} \oplus e = \langle \mathcal{A}', \mathtt{Def}' \rangle$ denote the new system. It is clear that when $e \in \mathcal{A}$ (i.e. the received argument is already owned by the agent), then $\mathcal{A}' = \mathcal{A}$ and $\mathtt{Def}' = \mathtt{Def}$. Consequently, all the arguments and all the offers will keep their original status provided by $\mathcal{AF}$. Things are different when $e \notin \mathcal{A}$. In this case: $\mathcal{A}' = \mathcal{A} \cup \{e\}$ and $\mathtt{Def}' = \mathtt{Def} \cup \{(x,e) \mid x \in \mathcal{A}_b$ and $(x,e) \in \mathcal{R}_b^{\mathcal{L}}$[2]$\} \cup \{(e,y) \mid y \in \mathcal{A}_o$ and $\mathtt{Conc}(y) \neq \mathtt{Conc}(e)$ and $(y,e) \notin \geq_o\} \cup \{(y,e) \mid y \in \mathcal{A}_o$ and $\mathtt{Conc}(y) \neq \mathtt{Conc}(e)$ and $(e,y) \notin \geq_o\}$. Throughout the paper, we assume that the argument e is not already in $\mathcal{A}_o$.

We will denote by $\mathcal{O}_x(\mathcal{AF})$, with $x \in \{a, r, ns\}$, the set of acceptable, respectively, rejected and non-supported offers of the original system $\mathcal{AF}$ and $\mathcal{O}_x(\mathcal{AF} \oplus e)$ the corresponding sets of the new system. For example, $\mathcal{O}_r(\mathcal{AF} \oplus e)$ is the set of rejected offers in light of the new argument.

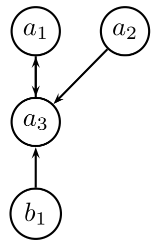
Let us summarize below the different assumptions made in this section.

1. The set $\mathcal{O}$ of offers is finite.
2. Grounded semantics is used for computing the status of arguments.
3. The new argument e is a practical one and $e \notin \mathcal{A}_o$.

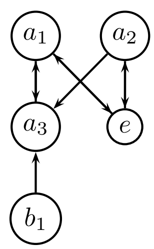
In this section, we will study the properties of an argument that can change the status of an offer. We will show when an accepted argument in system $\mathcal{AF}$ remains accepted (resp. becomes rejected) in $\mathcal{AF} \oplus e$. Similarly, we will show under which conditions an offer in $\mathcal{O}_x(\mathcal{AF})$ will move to $\mathcal{O}_y(\mathcal{AF} \oplus e)$ with $x \neq y$. Let us first consider the following illustrative example.

Example 5 *Let $\mathcal{O} = \{o_1, o_2\}$ be the set of offers, $\mathcal{A}_b = \{b_1\}$ be a set of epistemic arguments and $\mathcal{A}_o = \{a_1, a_2, a_3\}$ be a set of practical arguments. Let the following attacks hold between the arguments: $\mathcal{R}_b = \emptyset$, $\mathcal{R}_m = \{(b_1, a_3)\}$. Recall that $\mathcal{R}_o = \{(x,y) \mid x, y \in \mathcal{A}_o \wedge (\mathtt{Conc}(x) \neq \mathtt{Conc}(y))\}$. Let us also suppose that the argument a_2 is preferred to argument a_3. Thus, $\geq_b = \{(b_1, b_1)\}$ and $\geq_o = \{(a_1,a_1), (a_2,a_2), (a_2,a_3), (a_3,a_3)\}$. Recall that we suppose that all the epistemic arguments are preferred to practical ones, i.e. $\geq_m = \{(x,y) \mid x \in \mathcal{A}_b \wedge y \in \mathcal{A}_o\}$. Let $\mathcal{H}(o_1) = \{a_1, a_2\}$ and $\mathcal{H}(o_2) = \{a_3\}$. Now, we can compute the defeat relations, and obtain: $\mathtt{Def}_b = \emptyset$, $\mathtt{Def}_m = \{(b_1, a_3)\}$, $\mathtt{Def}_o = \{(a_1,a_3), (a_2,a_3), (a_3,a_1)\}$. The picture below depicts the different attacks in the sense of* $\mathtt{Def}$.

[2] $\mathcal{R}_b^{\mathcal{L}}$ is a set that contains all the conflicts between all the epistemic arguments that may be built from a logical language $\mathcal{L}$.



The grounded extension of this argumentation framework is $\texttt{GE} = \{a_1, a_2, b_1\}$*, so* $\texttt{Sc}(\mathcal{AF}) = \{a_1, a_2, b_1\}$ *and* $\texttt{Rej}(\mathcal{AF}) = \{a_3\}$*. Consequently, the offer* o_1 *is acceptable and* o_2 *is rejected. Let us now suppose that the owner of the system receives a new practical argument* e *in favor of option* o_2*. Assume also that this argument is incomparable to the other arguments of this agent, thus* $e \in \mathcal{H}(o_2)$*,* $\mathcal{R}'_b = \mathcal{R}_b$[3]*,* $\mathcal{R}'_m = \mathcal{R}_m$*,* $\mathcal{R}'_o = \mathcal{R}_o \cup \{(a_1, e), (e, a_1), (a_2, e), (e, a_2)\}$*,* $\texttt{Def}'_b = \texttt{Def}_b$*,* $\texttt{Def}'_m = \texttt{Def}_m$*,* $\texttt{Def}'_o = \texttt{Def}_o \cup \{(a_1, e), (e, a_1), (a_2, e), (e, a_2)\}$*. The attacks (in the sense of* $\texttt{Def}$*) between the arguments of this argumentation framework are depicted in picture below.*



The new grounded extension is $\texttt{GE} = \{b_1\}$*. Note that this new argument has an impact on the acceptability of the previous arguments. Namely,* $\texttt{Sc}(\mathcal{AF} \oplus e) = \{b_1\}$ *and* $\texttt{Rej}(\mathcal{AF}\oplus) = \{a_1, a_2, a_3, e\}$*. Consequently,* $\mathcal{O} = \mathcal{O}_r = \{o_1, o_2\}$*.*

The previous example shows that the arrival of a new argument may influence the status of existing arguments. For instance, the two arguments a_1 and a_2 were accepted in $\mathcal{AF}$ and became rejected when e is received. However, the status of b_1 has not changed. The following result shows that new practical argument will never influence the status of epistemic ones. The reason is that practical arguments are not allowed to attack epistemic ones. Formally:

Property 14 *Let* e *be a new practical argument. It holds that* $\texttt{Sc}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b = \texttt{Sc}(\mathcal{AF}) \cap \mathcal{A}_b$*.*

As can be checked in Example 5, this result is not true for the practical arguments of the set $\mathcal{A}_o$. However, this result holds in one particular case: when the new argument is defeated by a skeptically accepted epistemic argument. In this case, the argument e is clearly useless.

[3] $\mathcal{R}'_b, \mathcal{R}'_m$ and $\mathcal{R}'_o$ denote the new attack relations after the new argument has arrived.

Property 15 *Let e be a new practical argument. If $(\exists a \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF}))$ such that $(a, e) \in \mathtt{Def}$ then $\mathtt{Sc}(\mathcal{A} \oplus e) \cap \mathcal{A}_o = \mathtt{Sc}(\mathcal{AF}) \cap \mathcal{A}_o$.*

From the two above properties, the following trivial result holds:

Property 16 *Let e be a new practical argument. If $(\exists a \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF}))$ such that $(a, e) \in \mathtt{Def}$ then $\mathtt{Sc}(\mathcal{A} \oplus e) = \mathtt{Sc}(\mathcal{AF})$.*

It can be shown that each skeptically accepted practical argument can be defended either by an epistemic argument or by another practical argument that supports the same offer. Before presenting formally this result, let us first introduce a notation. Recall that $\mathtt{Sc}(\mathcal{AF}) = \bigcup_{i=1}^{\infty} \mathcal{F}^{(i)}(\emptyset)$. Let $\mathtt{Sc}^1(\mathcal{AF}) = \mathcal{F}(\emptyset)$ and let $(\forall i \in \{2, 3, \ldots\})$ $\mathtt{Sc}^i(\mathcal{AF})$ denote $\mathcal{F}^{(i)}(\emptyset) \setminus \mathcal{F}^{(i-1)}(\emptyset)$, i.e., the arguments reinstated at step i.

Property 17 *Let $o \in \mathcal{O}$, $a_i \in \mathcal{H}(o)$, $a_i \in \mathtt{Sc}^i(\mathcal{AF})$ and $x \in A$ such that $(x, a_i) \in \mathtt{Def}$.*

1. *If $x \in \mathcal{A}_b$ then $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i) \wedge (\exists a_j \in \mathcal{A}_b \cap \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j, x) \in \mathtt{Def}$,*
2. *If $x \in \mathcal{A}_o$ then $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i) \wedge (\exists a_j \in (\mathcal{A}_b \cup \mathcal{H}(o)) \cap \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j, x) \in \mathtt{Def}$.*

The following result states that a new practical argument will never influence the accepted arguments supporting the same offer as the new argument e.

Theorem 1 *Let e be a new argument such that $\mathtt{Conc}(e) = o$. Then, $(\forall a \in \mathcal{H}(o))$ $a \in \mathtt{Sc}(\mathcal{AF}) \Rightarrow a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$.*

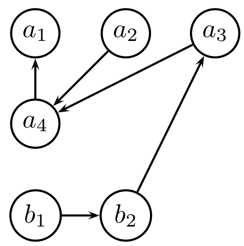
We can also show that if the new practical argument e induces a change in the status of a given practical argument from rejection to acceptance, then this argument supports the same offer as e. This means that a new practical argument can improve the status of arguments supporting its own conclusion, thus it can improve the status of the offer it supports. However, it can never improve the status of other offers.

Theorem 2 *Let $o \in \mathcal{O}$, and $a \in \mathcal{H}(o)$. If $a \in \mathtt{Rej}(\mathcal{AF})$ and $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$, then $e \in \mathcal{H}(o)$.*

Before continuing with the results on the revision of the status of offers, let us define the set of arguments defended by epistemic arguments in $\mathcal{AF}$.

Definition 6 *(Defense by epistemic arguments) Let $\mathcal{AF} = \langle \mathcal{A}, \mathtt{Def} \rangle$ be an argumentation system and $a \in \mathcal{A}$. We say that a is defended by epistemic arguments in $\mathcal{AF}$ and we write $a \in \mathtt{Dbe}(\mathcal{AF})$ iff $(\forall x \in \mathcal{A})$ $(x, a) \in \mathtt{Def} \Rightarrow (\exists \alpha \in \mathtt{Sc}(\mathcal{AF}) \cap \mathcal{A}_b)$ $(\alpha, x) \in \mathtt{Def}$.*

Example 6 *Let* $\mathcal{O} = \{o_1, o_2\}$ *be the set of offers,* $\mathcal{A}_b = \{b_1, b_2\}$ *set of epistemic arguments and* $\mathcal{A}_o = \{a_1, a_2, a_3, a_4\}$ *a set of practical arguments. Let the following attacks hold between the arguments:* $\mathcal{R}_b = \{(b_1, b_2)\}$, $\mathcal{R}_m = \{(b_2, a_3)\}$, $\mathcal{R}_o = \{(x, y) \mid x, y \in \mathcal{A}_o \wedge \texttt{Conc}(x) \neq \texttt{Conc}(y)\}$. *Let us also suppose that we have the following preference relations:* $\geq_b = \{(b_1, b_1), (b_2, b_2)\}$, $\geq_o$ $\{(a_1, a_1), (a_2, a_2), (a_2, a_3), (a_3, a_3), (a_3, a_4), (a_3, a_1), (a_2, a_4), (a_2, a_1), (a_4, a_1))\}$. *Recall that we suppose that all the epistemic arguments are preferred to practical ones, i.e.* $\geq_m = \{(x, y) \mid x \in \mathcal{A}_b \wedge y \in \mathcal{A}_o\}$. *Let* $\mathcal{H}(o_1) = \{a_1, a_2, a_3\}$ *and* $\mathcal{H}(o_2) = \{a_4\}$. *Now, we can compute the defeat relations, and obtain:* $\texttt{Def}_b = \{(b_1, b_2)\}$, $\texttt{Def}_m = \{(b_2, a_3)\}$, $\texttt{Def}_o = \{(a_2, a_4), (a_3, a_4), (a_4, a_1)\}$. *The attacks (in the sense of* $\texttt{Def}$*) between the arguments of this argumentation framework are depicted in picture below.*



It can be checked that $\texttt{Sc}(\mathcal{AF}) = \{a_1, a_2, a_3, b_1\}$. *Note that* $\texttt{Dbe}(\mathcal{AF}) = \{b_1, a_2, a_3\}$.

Note that, since elements of $\texttt{Sc}^1(\mathcal{AF})$ are not attacked at all, they are also defended by epistemic arguments, i.e. $\texttt{Sc}^1(\mathcal{AF}) \subseteq \texttt{Dbe}(\mathcal{AF})$. As a consequence, the set of arguments defended by epistemic arguments is skeptically accepted.

Property 18 *It holds that* $\texttt{Dbe}(\mathcal{AF}) \subseteq \texttt{Sc}(\mathcal{AF})$.

The following example shows that the converse is not true.

Example 7 *(Example 6 cont.) We have already seen that* $\texttt{Dbe}(\mathcal{AF}) = \{b_1, a_2, a_3\}$. *Note that the argument* a_1 *is skeptically accepted, but is not defended by epistemic arguments.*

Given an offer which is accepted in the system $\mathcal{AF}$, it becomes rejected in the new system $\mathcal{AF} \oplus e$ if three conditions are satisfied: e is not in favor of the offer o, there is no skeptically accepted epistemic argument that defeats e, and e defeats all the arguments in favor of offer o that are defended by epistemic arguments.

Theorem 3 *Let* $o \in \mathcal{O}_a(\mathcal{AF})$ *and let* e *be a new practical argument.* $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$ *iff* $e \notin \mathcal{H}(o) \wedge (\nexists x \in \mathcal{A}_b \cap \texttt{Sc}(\mathcal{AF}))\ (x, e) \in \texttt{Def} \wedge (\forall a \in \texttt{Dbe}(\mathcal{AF}) \cap \mathcal{H}(o))\ (e, a) \in \texttt{Def}$.

This result is important in a negotiation. It shows the properties of a good argument that may kill an offer that is not desirable for an agent.

Similarly, we can show that it is possible for an offer to move from a rejection to an acceptance. The idea is to send a practical argument that supports this offer and that is accepted in the new system. Formally:

Theorem 4 *Let $o \in \mathcal{O}_r(\mathcal{AF})$ and let e be a new practical argument. $o \in \mathcal{O}_a(\mathcal{AF} \oplus e)$ iff $e \in \mathcal{H}(o) \wedge e \in \mathtt{Sc}(\mathcal{AF} \oplus e)$.*

4 Revising offer status in complete decision systems

In [1], another variant of decision system has been used in a negotiation context. The main difference with the system presented in the previous section is that not only arguments in favor of different options are conflicting, but also those in favor of the same offer. The idea behind this assumption is that the decision system should return the "best" offer as well as the "best" argument in its favor. This class of decision systems is called "complete" system. Formally, the attack relation $\mathcal{R}_o$ is defined as follows:

$$(\forall a, a' \in \mathcal{R}_o) \quad (a, a') \in \mathcal{R}_o$$

For the sake of simplicity, in the rest of this section we assume that the set $\mathcal{A}_b$ of epistemic arguments is empty. The decision system that will be used is thus $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$. The general case (i.e., $\mathcal{A}_b \neq \emptyset$) will be the subject of future work.

In this section, we assume that the status of arguments is evaluated using preferred semantics.

The status of each argument in this system can be characterized as follows:

Property 19 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be a complete argumentation framework for decision making, and a be an arbitrary argument. Then:*

1. *a is skeptically accepted iff $(\forall x \in \mathcal{A}_o)$ $(a, x) \in \geq_o$.*

2. *a is rejected iff $(\exists x \in \mathcal{A})$ $(x, a) \in >_o$.*

3. *a is credulously accepted iff* $((\exists x' \in \mathcal{A})\ (a, x') \notin \geq_o) \wedge ((\forall x \in \mathcal{A})\ ((a, x) \notin \geq_o) \Rightarrow (x, a) \notin \geq_o))$.

The next property highlights the link between argument status and option status.

Property 20 *The following equivalences hold.*

1. *There is at least one skeptically accepted argument iff there is at least one acceptable offer.*

2. *There is at least one credulously accepted argument iff there is at least one negotiable offer.*

If an argument a is rejected, then there is some argument x such that x defeats a and a does not defend itself. The next property shows that arguments that defeat a cannot be all rejected.

Property 21 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be a complete argumentation framework for decision making and $a \in \mathcal{A}_o$. If $a \in \mathtt{Rej}(\mathcal{AF}_o)$ then $(\exists x' \in \mathcal{A}_o)$ such that $x' \notin \mathtt{Rej}(\mathcal{AF}_o) \wedge (x', a) \in >_o$.*

The next property proves that if there is exactly one non-rejected argument, then it is skeptically accepted. This result is important because it guaranties that it cannot be the case that all the options are rejected.

Property 22 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be a complete argumentation framework for decision making and $a \in \mathcal{A}_o$. If $\mathcal{A}_o \setminus \{a\} \subseteq \mathtt{Rej}(\mathcal{AF}_o)$ then $a \in \mathtt{Sc}(\mathcal{AF}_o)$.*

It can be checked that all skeptically accepted arguments in this system are equally preferred.

Property 23 *Let $a, b \in \mathtt{Sc}(\mathcal{AF}_o)$. Then $(a, b) \in \geq_o$ and $(b, a) \in \geq_o$.*

We will now prove that in this particular system, there are two possible cases: the case where there exists at least one skeptically accepted argument but there are no credulously accepted arguments, and the case where there are no skeptically accepted arguments but there is "at least" one credulously accepted argument. This means that one cannot have a state with both skeptically accepted and credulously accepted arguments. Moreover, it cannot be the case that all the arguments are rejected. Formally:

Theorem 5 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system. The following implications hold:*

1. *If $\mathtt{Sc}(\mathcal{AF}_o) \neq \emptyset$ then $\mathtt{Cr}(\mathcal{AF}_o) = \emptyset$.*
2. *If $\mathtt{Cr}(\mathcal{AF}_o) = \emptyset$ then $\mathtt{Sc}(\mathcal{AF}_o) \neq \emptyset$.*

We will now show that an arbitrary argument x is in the same relation with all accepted arguments. Recall that we use the notation $(x, a) \in \odot$ to refer to one particular relation between the arguments x and a.

Property 24 *Let x be an arbitrary argument.*
If $(\exists a \in \mathtt{Sc}(\mathcal{AF}_o))$ such that $(a, x) \in \odot$ then $(\forall a' \in \mathtt{Sc}(\mathcal{AF}_o))$ $(a', x) \in \odot$.

Let us now have a look at credulously accepted arguments. While all the skeptically accepted arguments are in the same class with respect to the preference relation $\geq_o$, this is not always the case with credulously accepted arguments. The next property shows that credulously accepted arguments are either incomparable or indifferent with respect to $\geq_o$.

Property 25 *$\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system and $\mathtt{Cr}(\mathcal{AF}_o)$ its set of credulously accepted arguments. Then $(\forall a, b \in \mathtt{Cr}(\mathcal{AF}_o)$ it holds that*

$$((a, b) \in \geq_o \wedge (b, a) \in \geq_o) \vee ((a, b) \notin \geq_o \wedge (b, a) \notin \geq_o).$$

The next property shows that if a' is credulously accepted then there exists another credulously accepted argument a'' such that they are incomparable in the sense of preference relation.

Property 26 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system for decision making, and $\mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. Then it holds that: $(\forall a' \in \mathtt{Cr}(\mathcal{AF}_o))$ $(\exists a'' \in \mathtt{Cr}(\mathcal{AF}_o))$ $(a', a'') \notin \geq_o \wedge (a'', a') \notin \geq_o$.*

The next property will make some reasoning easier, because it shows that, in this particular framework, the definition of negotiable offers can be simplified.

Property 27 *Let $o \in \mathcal{O}$. The offer o is negotiable iff there is at least one credulously accepted argument in its favor.*

As a consequence of the above properties, the following result shows that negotiable offers and acceptable ones cannot exist at the same time.

Theorem 6 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be a complete argumentation framework for decision making. The following holds: $\mathcal{O}_a \neq \emptyset \Leftrightarrow \mathcal{O}_n = \emptyset$.*

4.1 Revising the status of an argument

Like in the previous section, we assume that an agent receives a new practical argument e. The question is, how the status of an argument given by the system $\mathcal{AF}_o$ may change in the system $\mathcal{AF} \oplus e$ without having to compute the preferred extensions of $\mathcal{AF}_o \oplus e$.

The first result states that rejected arguments in $\mathcal{AF}_o$ remain rejected in the new system $\mathcal{AF}_o \oplus e$. This means that rejected arguments cannot be "saved".

Property 28 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system. If $a \in \mathtt{Rej}(\mathcal{AF}_o)$, then $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$.*

We can also show that an argument that was credulously accepted in $\mathcal{AF}_o$ can never become skeptically accepted in $\mathcal{AF}_o \oplus e$. It can either remain credulously accepted, or become rejected.

Property 29 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system. If $a \in \mathtt{Cr}(\mathcal{AF}_o)$, then $a \notin \mathtt{Sc}(\mathcal{AF}_o \oplus e)$.*

The next property is simple but will be very useful later in this section.

Property 30 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system for decision making.*

1. *If $a \in \mathtt{Sc}(\mathcal{AF}_o)$ then $a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ iff $(a, e) \in \geq_o$.*
2. *If $a \notin \mathtt{Rej}(\mathcal{AF}_o)$ then $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$ iff $(e, a) \in >_o$.*

The next property shows that all the skeptically accepted arguments will have the "same destiny" when a new argument is received.

Property 31 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system and $a, b \in \mathtt{Sc}(\mathcal{AF}_o)$. Let $e \notin \mathcal{A}_o$.*

1. *If $a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ then $b \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$.*
2. *If $a \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$ then $b \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$.*
3. *If $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$ then $b \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$.*

The next theorem analyzes the status of all skeptically accepted arguments after a new argument has arrived.

Theorem 7 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be a complete argumentation framework for decision making, $a \in \mathtt{Sc}(\mathcal{AF}_o)$ and $e \notin \mathcal{A}_o$. The following holds:*

1. *$a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e) \wedge e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ iff $((a, e) \in \geq_o) \wedge ((e, a) \in \geq_o)$*
2. *$a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e) \wedge e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ iff $(e, a) \in >_o$*
3. *$a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e) \wedge e \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$ iff $(a, e) \in >_o$*
4. *$a \in \mathtt{Cr}(\mathcal{AF}_o \oplus e) \wedge e \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$ iff $((a, e) \notin \geq_o) \wedge ((a, e) \notin \geq_o)$*

Note that, according to Property 24, all skeptically accepted arguments are in the same relation with e as a is. Formally, if a and e are in a particular relation i.e. $(a, e) \in \odot$, then $(\forall b \in \mathcal{A}_o)\ ((b \in \mathtt{Sc}(\mathcal{AF}_o)) \Rightarrow (b, e) \in \odot)$. Hence, the condition "let $a \in \mathtt{Sc}(\mathcal{AF}_o)$ and $(a, e) \in \odot$" in the previous theorem is equivalent to the condition $(\forall a \in \mathcal{A}_o)\ ((a \in \mathtt{Sc}(\mathcal{AF}_o)) \Rightarrow (a, e) \in \odot)$.

Theorem 7 stands as a basic tool for reasoning about the status of new arguments as well as about the changes in the status of other arguments. Once the argument status is known, it is much easier to determine the status of offers.

We will now analyze the relation between credulously accepted arguments and new arguments. The next result shows that if there are credulously accepted arguments in $\mathcal{AF}_o$ and the new argument e is preferred to all of them, then it is strictly preferred to all of them.

Property 32 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system such that $\mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. The following result holds: $((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in >_o)$ iff $((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in \geq_o)$.*

Property 33 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be and argumentation system s.t. $\mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. The following holds: $((\forall a \in \mathtt{Cr}(A_o))\ a \in \mathtt{Rej}(A_o \oplus e))$ iff $((\forall a \in \mathtt{Cr}(A_o))\ (e, a) \in >_o)$.*

The next theorem analyzes the case when there are no skeptically accepted arguments in $\mathcal{AF}_o$.

Theorem 8 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation framework such that $\mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. Then, the following holds:*

1. $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in >_o$ *iff* $e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e) \wedge \mathcal{A}_o = \mathtt{Rej}(\mathcal{AF}_o \oplus e)$.

2. $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \notin >_o \wedge (\nexists a' \in \mathtt{Cr}(\mathcal{AF}_o))$
$(a', e) \in >_o$ *iff* $e \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$

3. $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (a, e) \in >_o$ *iff* $e \in \mathtt{Rej}(\mathcal{AF}_o \oplus e) \wedge \mathcal{A}_o = \mathtt{Cr}(\mathcal{AF}_o \oplus e)$.

Recall that, according to Property 32, the condition $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in >_o$ in the previous theorem is equivalent to the condition $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in \geq_o$. While all the skeptically accepted arguments have the "same destiny" after a new argument arrives, this is not the case with credulously accepted arguments. Some of them may remain credulously accepted while the others may become rejected.

4.2 Revising the status of an offer

We will now show under which conditions an offer can change its status. We start by studying acceptable offers.

Theorem 9 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system and $o \in \mathcal{O}_a(\mathcal{AF}_o)$. Suppose that $a \in \mathtt{Sc}(\mathcal{AF}_o)$ is an arbitrary skeptically accepted argument. Then:*

1. $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ *iff*
$((a, e) \in \geq_o) \vee (e \in \mathcal{H}(o)) \wedge ((e, a) \in >_o)$

2. $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ *iff*
$((a, e) \notin \geq_o) \wedge ((e, a) \notin \geq_o))$

3. $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ *iff*
$(e \notin \mathcal{H}(o)) \wedge (e, a) \in >_o)$

Recall that, according to Property 24, all skeptically accepted arguments are in the same relation with an arbitrary argument. Hence, the condition $(\exists a \in \mathtt{Sc}(\mathcal{AF}_o))\ (a, e) \in \odot)$ in the previous theorem is equivalent to the condition $(\forall a \in \mathtt{Sc}(\mathcal{AF}_o))\ (a, e) \in \odot)$.

A similar characterization is given bellow for negotiable offers.

Theorem 10 *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system and $o \in \mathcal{O}_n\mathcal{AF}$. Then:*

1. $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ *iff*
$(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathtt{Cr}(\mathcal{A}_o))\ (e, a) \in >)$

2. $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ *iff*
$((e \in \mathcal{H}(o)) \wedge (\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a') \notin >_o \wedge$
$(\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'', e) \in >_o)$
$\vee$
$((\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a' \in \mathcal{H}(o) \wedge (e, a') \notin >_o))$

3. $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ *iff* $((e \notin \mathcal{H}(o)) \wedge$
   $((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (a \in \mathcal{H}(o)) \Rightarrow (e, a) \in >_o))$.

Note that, according to Property 32, the condition $(\forall a \in \mathtt{Cr}(\mathcal{A}_o))\ (e, a) \in >$ in the previous theorem is equivalent to condition $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in \geq_o$.

Let us now analyze when a rejected offer in $\mathcal{AF}_o$ may change its status in $\mathcal{AF} \oplus e$.

Theorem 11 *Let* $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ *be an argumentation system and* $o \in \mathcal{O}_r(\mathcal{AF})$. *Then:*

1. $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ *iff*
   $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)\ (e, a) \in \geq_o)$

2. $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ *iff*
   $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)\ (a, e) \notin >_o) \wedge ((\exists a \in \mathcal{A}_o)\ (e, a) \notin >_o)$

3. $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ *iff*
   $(e \notin \mathcal{H}(o)) \vee ((e \in \mathcal{H}(o)) \wedge (\exists a \in \mathcal{A}_o)(a, e) \in >)$

5 Conclusion

This paper has tackled the problem of revising an offer status in argument-based negotiations. As already said in the introduction, during a negotiation an agent may receive, in addition to offers, arguments. These arguments may have a great impact on the preferences of this agent. By preferences here we mean its ordering of the set of all possible offers.

To the best of our knowledge, in this paper we have proposed the first investigation on the impact of a new argument on the preferences of an agent. The basic idea is to check when the status of an offer may shift when a new argument is received without having to compute the whole new ordering on offers. For that purpose, we have considered a decision model that has recently been proposed in the literature. This model computes a status for each offer on the basis of the status of their supporting arguments. We have studied two cases: the case where an offer may be supported by several arguments and the case where an offer is supported by only one argument. In both cases, we assumed that the new argument is practical, i.e. it supports an offer. We have provided a full characterization of acceptable offers that become rejected, negotiable or remain accepted. Similarly, we have characterized any shift from one status to another. These results are based on a characterization of a shift of the status of arguments themselves.

These results may be used to determine strategies of a negotiating agent, since at a given step of a dialog an agent has to choose an argument to send to another agent in order to change the status of an offer. Moreover, they may help to understand which arguments are useful and which ones are useless in a given situation, which allows us to understand the role of argumentation in a negotiation.

Note that a recent work has been done on revision in argumentation systems in [4]. That paper addresses the problem of revising the set of extensions of an abstract argumentation system. It studies how the extensions of an argumentation system may evolve when a new argument is received. Nothing is said on the revision of a particular argument. In our paper, we are more interested by the evolution of the status of a given argument without having to compute the extensions of the new argumentation system. We have also studied how the status of offers change when a new argument is received. Another main difference with this work is that in [4] only the case of adding an argument having only one interaction with an argument of the initial argumentation system is studied. In our paper we have studied the more general case, i.e. the new argument may attack and be attacked by an arbitrary number of arguments of the initial argumentation system.

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Appendix

Property 1 Let $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, and $\mathcal{E}_1, \ldots, \mathcal{E}_n$ its extensions under a given semantics. Let $a \in \mathcal{A}$.

1. a is skeptically accepted iff $a \in \bigcap_{i=1}^{n} \mathcal{E}_i$.
2. a is rejected iff $a \notin \bigcup_{i=1}^{n} \mathcal{E}_i$.

Proof *Proof follows directly from Definition 4.* ■

Property 2 Let $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $\texttt{Sc}(\mathcal{AF})$, $\texttt{Cr}(\mathcal{AF})$, $\texttt{Rej}(\mathcal{AF})$, its sets of arguments.

1. $\texttt{Sc}(\mathcal{AF}) \cap \texttt{Cr}(\mathcal{AF}) = \emptyset$, $\texttt{Sc}(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF}) = \emptyset$, $\texttt{Cr}(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF}) = \emptyset$
2. $\texttt{Sc}(\mathcal{AF}) \cup \texttt{Cr}(\mathcal{AF}) \cup \texttt{Rej}(\mathcal{AF}) = \mathcal{A}$.

Proof *Let $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework*

1. *Let us prove that three sets mentioned above are pairwise disjoint.*

   (a) *Assume that $\texttt{Sc}(\mathcal{AF}) \cap \texttt{Cr}(\mathcal{AF}) \neq \emptyset$. So, there exists an argument a such that $a \in \texttt{Sc}(\mathcal{AF})$ and $a \in \texttt{Cr}(\mathcal{AF})$. Since $a \in \texttt{Cr}(\mathcal{AF})$ then there exists an extension $\mathcal{E}_i$ such that $a \in \mathcal{E}_i$ and there exists an extension $\mathcal{E}_j$ such that $a \notin \mathcal{E}_j$. Since $a \in \texttt{Sc}(\mathcal{AF})$, then a is in all extensions. In particular, $a \in \mathcal{E}_j$. Contradiction.*

   (b) *Assume that $\texttt{Sc}(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF}) \neq \emptyset$. So, there exists an argument a such that $a \in \texttt{Sc}(\mathcal{AF})$ and $a \in \texttt{Rej}(\mathcal{AF})$. Since $a \in \texttt{Rej}(\mathcal{AF})$ then for all extensions $\mathcal{E}_i$, $a \notin \mathcal{E}_i$. Since $a \in \texttt{Sc}(\mathcal{AF})$ then there exists at least one non-empty extension and a is in all extensions. Contradiction.*

(c) *Assume that* $\texttt{Cr}(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF}) \neq \emptyset$*. So, there exists an argument* a *such that* $a \in \texttt{Cr}(\mathcal{AF})$ *and* $a \in \texttt{Rej}(\mathcal{AF})$*. Since* $a \in \texttt{Rej}(\mathcal{AF})$ *then for all extensions* $\mathcal{E}_i$*,* $a \notin \mathcal{E}_i$*. Since* $a \in \texttt{Cr}(\mathcal{AF})$ *then there exists an extension* $\mathcal{E}_i$ *such that* $a \in \mathcal{E}_i$*. Contradiction.*

2. *The inclusion* $\texttt{Sc}(\mathcal{AF}) \cup \texttt{Cr}(\mathcal{AF}) \cup \texttt{Rej}(\mathcal{AF}) \subseteq \mathcal{A}$ *is trivial. Let us now assume that* $a \in \mathcal{A}$*. If the argumentation system* $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ *has no extensions then* a *is rejected i.e.* $a \in \texttt{Rej}(\mathcal{AF})$*.*
   Let us now assume that there are exactly n *extensions* $\mathcal{E}_1, \ldots, \mathcal{E}_n$*, with* $n \geq 1$*. There are three possible cases.*

   (a) $a \in \bigcap_{i=1}^{n} \mathcal{E}_i$*. This means that* $a \in \texttt{Sc}(\mathcal{AF})$*.*

   (b) *There is at least one extension* $\mathcal{E}_i$ *such that* $a \in \mathcal{E}_i$ *and there is at least one extension* $\mathcal{E}_j$ *such that* $a \notin \mathcal{E}_j$*. In this case,* a *is credulously accepted, i.e.* $a \in \texttt{Cr}(\mathcal{AF})$*.*

   (c) $a \notin \bigcup_{i=1}^{n} \mathcal{E}_i$*. This means that* $a \in \texttt{Rej}(\mathcal{AF})$*.*

■

Property 3 An offer $o \in \mathcal{O}$ is non-supported iff $\mathcal{H}(o) = \emptyset$.

Proof *Let* $o \in \mathcal{O}$*. Let us assume that* $\mathcal{H}(o) \neq \emptyset$*. This means that there are two possibilities:*

1. *all the arguments are rejected, consequently the offer is rejected*

2. *there exists at least one argument, say* a*, which is not rejected. Since* a *is not rejected, then:*

   (a) a *is skeptically accepted. This means that* o *is accepted.*

   (b) a *is credulously accepted. This means that the offer* o *is negotiable.*

■

Property 5 Let $k, n \in \mathcal{N}$, $1 \leq k \leq n$. Let A_1, …, A_n be arbitrary sets. Then:

- $\bigcap_{i=1}^{n} A_i \subseteq \bigcap_{i=1}^{k} A_i$
- $\bigcup_{i=1}^{k} A_i \subseteq \bigcup_{i=1}^{n} A_i$

In the particular case where $n \geq 1$ and $k = 1$, we have:

- $\bigcap_{i=1}^{n} A_i \subseteq A_1$
- $A_1 \subseteq \bigcup_{i=1}^{n} A_i$

Proof *Let* n *and* k *be arbitrary but fixed integers which satisfy the condition* $1 \leq k \leq n$*.*

- *Let us prove the inclusion* $\bigcap_{i=1}^{n} A_i \subseteq \bigcap_{i=1}^{k} A_i$. *Suppose* $x \in \bigcap_{i=1}^{n} A_i$. *This means that* $(x \in A_1) \wedge \ldots \wedge (x \in A_k) \wedge \ldots \wedge (x \in \mathcal{A}_n)$. *So,* $(x \in A_1) \wedge \ldots \wedge (x \in A_k)$, *and, consequently,* $x \in \bigcap_{i=1}^{k} A_i$.

- *Let us prove the inclusion* $\bigcup_{i=1}^{k} A_i \subseteq \bigcup_{i=1}^{n} A_i$. *Suppose* $x \in \bigcup_{i=1}^{k} A_i$. *This means that* $(x \in A_1) \vee \ldots \vee (x \in A_k)$. *So,* $(x \in A_1) \vee \ldots \vee (x \in A_k) \vee \ldots \vee (x \in A_n)$, *and, consequently,* $x \in \bigcup_{i=1}^{n} A_i$.

■

Property 6 Let $\mathcal{O}$ be a set of options. $\mathcal{O}_a^{ad} = \emptyset$.

Proof *Let* $\mathcal{E}_1, \ldots, \mathcal{E}_n$ *be the admissible extensions of* $\langle \mathcal{A}, \mathtt{Def} \rangle$. *Let us assume that* $\mathcal{O}_a^{ad} \neq \emptyset$. *So,* $\exists o \in \mathcal{O}$ *such that* $o \in O_a^{ad}$. *This means that* $\exists a \in \mathcal{H}(o)$ *such that* a *is skeptically accepted. According to Property 1, we have* $a \in \bigcap_{i=1}^{n} \mathcal{E}_i$. *However, one can easily see that the empty set is always an admissible extension. This means that* $\exists \mathcal{E}_i = \emptyset$, *with* $1 \leq i \leq n$. *Since* $\mathcal{E}_i = \emptyset$, *we have* $\bigcap_{i=1}^{n} \mathcal{E}_i = \emptyset$. *Contradiction with the fact that* $a \in \bigcap_{i=1}^{n} \mathcal{E}_i$. ■

Property 7 Let $\mathcal{O}$ be a set of options. The equality $\mathcal{O}_a^{g} = O_a^{c}$ holds.

Proof *Let* $\langle \mathcal{A}, \mathtt{Def} \rangle$ *be an argumentation system. Let* $\mathtt{GE}$ *be its grounded extension and* $\mathcal{E}_1, \ldots, \mathcal{E}_n$ *its complete extensions.*

We will show that $\mathcal{O}_a^{g} \subseteq O_a^{c}$. *Let* $o \in \mathcal{O}$. *Let us assume that* $o \in \mathcal{O}_a^{g}$ *and* $o \notin O_a^{c}$. *Since* $o \in \mathcal{O}_a^{g}$, *then* $\exists a \in \mathcal{H}(o)$ *such that* a *is skeptically accepted under grounded semantics. Thus,* $a \in \mathtt{GE}$. *Since* $o \notin O_a^{c}$ *then* $\forall a' \in \mathcal{H}(o)$ a' *is not skeptically accepted with respect to complete semantics. According to Property 1,* $(\forall a' \in \mathcal{H}(o))$ $a' \notin \bigcap_{i=1}^{n} \mathcal{E}_i$. *In [5], it has been shown that the grounded extension is exactly the intersection of all complete extensions. So, it holds that* $\bigcap_{i=1}^{n} \mathcal{E}_i = \mathtt{GE}$. *Thus,* $(\forall a' \in \mathcal{H}(o))$ $a' \notin \mathtt{GE}$, *hence* $a \notin \mathtt{GE}$. *Contradiction.*

We will now show that $\mathcal{O}_a^{c} \subseteq O_a^{g}$. *Let* $o \in \mathcal{O}$. *Let us assume that* $o \in \mathcal{O}_a^{c}$ *and* $o \notin O_a^{g}$. *Since* $o \in \mathcal{O}_a^{g}$, *then* $\exists a \in \mathcal{H}(o)$ *such that* a *is skeptically accepted under complete semantics. Thus,* $a \in \bigcap_{i=1}^{n} \mathcal{E}_i$. *Since* $o \notin O_a^{g}$ *then* $(\forall a' \in \mathcal{H}(o))$ a' *is not skeptically accepted with respect to grounded semantics. So,* $(\forall a' \in \mathcal{H}(o))$ $a \notin \mathtt{GE}$. *In [5], it has been shown that the grounded extension is exactly the intersection of all complete extensions, i.e.,* $\bigcap_{i=1}^{n} \mathcal{E}_i = \mathtt{GE}$. *Thus,* $(\forall a' \in \mathcal{H}(o))$ $a' \notin \bigcap_{i=1}^{n} \mathcal{E}_i$, *hence* $a \notin \bigcap_{i=1}^{n} \mathcal{E}_i$. *Contradiction.*

Since $\mathcal{O}_a^{g} \subseteq O_a^{c}$ *and* $\mathcal{O}_a^{c} \subseteq O_a^{g}$, *we have* $\mathcal{O}_a^{g} = O_a^{c}$. ■

Property 8 Let $\mathcal{O}$ be a set of options. The inclusion $\mathcal{O}_a^{g} \subseteq O_a^{p}$ holds.

Proof *Let* $\langle \mathcal{A}, \mathtt{Def} \rangle$ *be an argumentation system. Let* $\mathtt{GE}$ *be its grounded extension and* $\mathcal{E}_1, \ldots, \mathcal{E}_n$ *its preferred extensions. Let* o *be an option and* $\mathcal{H}(o)$ *its set of arguments. Since* o *is accepted under grounded semantics then there exists an argument* $a \in \mathcal{H}(o)$ *such that* a *is in the grounded extension. In [5], it has been shown that the grounded extension is the subset of the intersection of all preferred extensions. Since,* $a \in \mathtt{GE}$ *then* $a \in \bigcap_{i=1}^{n} \mathcal{E}_i$. *So there is at least one skeptically accepted argument in favor of the option* o *under preferred semantics, which means that* o *is accepted under preferred semantics.* ■

Property 9 Let $\mathcal{O}$ be the set of options, and let $\mathcal{AF} = \langle \mathcal{A}, \texttt{Def} \rangle$ be the argumentation system for rank-ordering elements of $\mathcal{O}$.

1. If $\mathcal{AF}$ has no stable extensions, then $\mathcal{O}_a^s = \emptyset$ and $\mathcal{O}_a^s \subseteq O_a^p$.
2. If $\mathcal{AF}$ has stable extensions, then $\mathcal{O}_a^p \subseteq \mathcal{O}_a^s$.

Proof

1. *If $\mathcal{AF}$ has no stable extensions then there are no skeptically accepted arguments under stable semantics, thus there are no accepted options under this semantics.*
2. *Let us now assume that $\mathcal{E}_1, \ldots, \mathcal{E}_n$ are stable extensions of $\mathcal{AF}$ and $\mathcal{E}_{n+1}, \ldots, \mathcal{E}_{n+k}$ are preferred extensions that are not stable. In [5], it has been shown that every stable extension is preferred. Since $\mathcal{E}_1, \ldots, \mathcal{E}_n$ are stable, these are also preferred. According to Property 5, $\bigcap_{i=1}^{n+k} A_i \subseteq \bigcap_{i=1}^{n} A_i$, thus the set of skeptically accepted arguments under preferred semantics are a subset of the set of skeptically accepted arguments under the stable one. Let us now assume that $\exists o \in \mathcal{O}$ such that $o \in \mathcal{O}_a^p$ and $o \notin \mathcal{O}_a^s$. Since $o \in \mathcal{O}_a^p$ this means that $(\exists a \in \mathcal{H}(o))$ such that $a \in \bigcap_{i=1}^{n+k} A_i$. But, since $\bigcap_{i=1}^{n+k} A_i \subseteq \bigcap_{i=1}^{n} A_i$, then $a \in \bigcap_{i=1}^{n} A_i$. Thus, a is skeptically accepted under the stable semantics and $o \in \mathcal{O}_a^s$.*

■

Property 10 Let $\mathcal{O}$ be a set of options. It holds that:

1. $\mathcal{O}_r^{ad} = \mathcal{O}_r^c = \mathcal{O}_r^p$
2. $\mathcal{O}_r^p \subseteq \mathcal{O}_r^g$.

Proof

1. *$\mathcal{O}_{ad}^r = \mathcal{O}_c^r$. Let $\mathcal{E}_1, \ldots, \mathcal{E}_n$ be the complete extensions. One can easily see that every complete extension is admissible, so $\mathcal{E}_1, \ldots, \mathcal{E}_n$ are also admissible extensions. However, there can exist one or more admissible extensions which are not complete. So, let $\mathcal{E}_1, \ldots, \mathcal{E}_n, \ldots, \mathcal{E}_{n+k}$ be* all *admissible extensions, i.e. $\mathcal{E}_1, \ldots, \mathcal{E}_n$ are complete and admissible and $\mathcal{E}_{n+1}, \ldots, \mathcal{E}_{n+k}$ are admissible but not complete. Here, we have $k \geq 0$.*

   We will show that $\mathcal{O}_r^{ad} \subseteq O_r^c$. Let $o \in \mathcal{O}$. Let us assume that $o \in \mathcal{O}_r^{ad}$ and $o \notin O_r^c$. Since $o \in \mathcal{O}_r^{ad}$, then $(\forall a \in \mathcal{H}(o))$ a is rejected under admissible semantics. According to Property 1, $(\forall a \in \mathcal{H}(o))$ $a \notin \bigcup_{i=1}^{n+k} \mathcal{E}_i$. Since $o \notin O_r^c$ then $\exists a' \in \mathcal{H}(o)$ such that a' is not rejected with respect to complete semantics. According to Property 1, $a' \in \bigcup_{i=1}^{n} \mathcal{E}_i$. But, according to Property 5, $\bigcup_{i=1}^{n} \mathcal{E}_i \subseteq \bigcup_{i=1}^{n+k} \mathcal{E}_i$. So, if $\exists a' \in \mathcal{H}(o)$ such that $a' \in \bigcup_{i=1}^{n} \mathcal{E}_i$, then $\exists a' \in \mathcal{H}(o)$ such that $a' \in \bigcup_{i=1}^{n+k} \mathcal{E}_i$. Contradiction with the fact that $(\forall a \in \mathcal{H}(o))$ $a \notin \bigcup_{i=1}^{n+k} \mathcal{E}_i$.

We will now show that $\mathcal{O}_r^c \subseteq O_r^{ad}$. Let $o \in \mathcal{O}$. Let us assume that $o \in \mathcal{O}_r^c$ and $o \notin O_r^{ad}$. Since $o \in \mathcal{O}_r^c$, then $(\forall a \in \mathcal{H}(o))$ a is rejected under complete semantics. Thus, $(\forall a \in \mathcal{H}(o))$ $a \notin \bigcup_{i=1}^{n} \mathcal{E}_i$. Since $o \notin O_r^{ad}$ then $\exists a' \in \mathcal{H}(o)$ such that a' is not rejected with respect to admissible semantics. According to Property 1, $a' \in \bigcup_{i=1}^{n+k} \mathcal{E}_i$. So, if $a' \in \bigcup_{i=1}^{n+k} \mathcal{E}_i$ and $a' \notin \bigcup_{i=1}^{n} \mathcal{E}_i$ then $\bigcup_{i=n+1}^{n+k} \mathcal{E}_i$. Thus $\exists \mathcal{E}_j$ with $n+1 \leq j \leq n+k$ such that $a' \in \mathcal{E}_j$ and $\mathcal{E}_j$ is an admissible extension but not a complete one. In [5], it has been shown that every admissible extension is contained in some complete extension. Hence, there exists a complete extension $\mathcal{E}_k$, with $1 \leq k \leq n$ such that $\mathcal{E}_j \subseteq \mathcal{E}_k$. So, $a' \in \mathcal{E}_k$. Consequently, $a' \in \bigcup_{i=1}^{n} \mathcal{E}_i$. Contradiction with the fact that $(\forall a \in \mathcal{H}(o))$ $a \notin \bigcup_{i=1}^{n} \mathcal{E}_i$.

Since $\mathcal{O}_r^{ad} \subseteq O_r^c$ and $\mathcal{O}_r^c \subseteq O_r^{ad}$, we have $\mathcal{O}_r^{ad} = O_r^c$.

2. *$\mathcal{O}_r^c = \mathcal{O}_r^p$. Let $\mathcal{E}_1, \ldots, \mathcal{E}_n$ be the preferred extensions. One may easily see that every preferred extension is complete, so $\mathcal{E}_1, \ldots, \mathcal{E}_n$ are also complete extensions. However, there can exist one or more complete extensions which are not preferred. So, let $\mathcal{E}_1, \ldots, \mathcal{E}_n, \ldots, \mathcal{E}_{n+k}$ be* all *complete extensions, i.e. $\mathcal{E}_1, \ldots, \mathcal{E}_n$ are complete and preferred and $\mathcal{E}_{n+1}, \ldots, \mathcal{E}_{n+k}$ are complete but not preferred. Here, we have $k \geq 0$.*

   We will show that $\mathcal{O}_r^c \subseteq O_r^p$. Let $o \in \mathcal{O}$. Let us assume that $o \in \mathcal{O}_r^c$ and $o \notin O_r^p$. Since $o \in \mathcal{O}_r^c$, then $(\forall a \in \mathcal{H}(o))$ a is rejected under complete semantics. According to Property 1, $(\forall a \in \mathcal{H}(o))$ $a \notin \bigcup_{i=1}^{n+k} \mathcal{E}_i$. Since $o \notin O_r^p$ then $\exists a' \in \mathcal{H}(o)$ a' is not rejected with respect to preferred semantics. According to Property 1, $a' \in \bigcup_{i=1}^{n} \mathcal{E}_i$. But, according to Property 5, $\bigcup_{i=1}^{n} \mathcal{E}_i \subseteq \bigcup_{i=1}^{n+k} \mathcal{E}_i$. So, if $\exists a' \in \mathcal{H}(o)$ such that $a' \in \bigcup_{i=1}^{n} \mathcal{E}_i$, then $\exists a' \in \mathcal{H}(o)$ such that $a' \in \bigcup_{i=1}^{n+k} \mathcal{E}_i$. Contradiction with the fact that $(\forall a \in \mathcal{H}(o))$ $a \notin \bigcup_{i=1}^{n+k} \mathcal{E}_i$. We will now show that $\mathcal{O}_r^p \subseteq \mathcal{O}_r^c$. Let $o \in \mathcal{O}$. Let us assume that $o \in \mathcal{O}_r^p$ and $o \notin O_r^c$. Since $o \in \mathcal{O}_r^p$, then $(\forall a \in \mathcal{H}(o))$ a is rejected under preferred semantics. Thus, $(\forall a \in \mathcal{H}(o))$ $a \notin \bigcup_{i=1}^{n} \mathcal{E}_i$. Since $o \notin O_r^c$ then $\exists a' \in \mathcal{H}(o)$ such that a' is not rejected with respect to complete semantics. According to Property 1, $a' \in \bigcup_{i=1}^{n+k} \mathcal{E}_i$. So, if $a' \in \bigcup_{i=1}^{n+k} \mathcal{E}_i$ and $a' \notin \bigcup_{i=1}^{n} \mathcal{E}_i$ then $\bigcup_{i=n+1}^{n+k} \mathcal{E}_i$. Thus $\exists \mathcal{E}_j$ with $n+1 \leq j \leq n+k$ such that $a' \in \mathcal{E}_j$ and $\mathcal{E}_j$ is a complete extension but not a preferred one. In [5], it has been shown that for every complete extension A, there exists a preferred extension B such that $A \in B$. So, there exists a preferred extension $\mathcal{E}_k$, with $1 \leq k \leq n$ such that $\mathcal{E}_j \subseteq \mathcal{E}_k$. So, $a' \in \mathcal{E}_k$. Consequently, $a' \in \bigcup_{i=1}^{n} \mathcal{E}_i$. Contradiction with the fact that $(\forall a \in \mathcal{H}(o))$ $a \notin \bigcup_{i=1}^{n} \mathcal{E}_i$. Since $\mathcal{O}_r^c \subseteq \mathcal{O}_r^p$ and $\mathcal{O}_r^p \subseteq O_r^c$, we have $\mathcal{O}_r^c = O_r^p$.

3. *$\mathcal{O}_r^p \subseteq \mathcal{O}_r^g$. Let $\mathcal{E}_1, \ldots, \mathcal{E}_n$ be the preferred extensions, and* `GE` *the grounded extension.*

   Let $o \in \mathcal{O}$. Let us assume that $o \in \mathcal{O}_r^p$ and $o \notin O_r^g$. Since $o \in \mathcal{O}_r^p$, then $\forall a \in \mathcal{H}(o)$ a is rejected under preferred semantics. According to Property 1, $(\forall a \in \mathcal{H}(o))$ $a \notin \bigcup_{i=1}^{n} \mathcal{E}_i$. Since $o \notin O_r^g$ then $\exists a' \in \mathcal{H}(o)$ a' is not rejected with respect to grounded semantics. Since there is always exactly one *grounded extension, $a' \in \texttt{GE}$. In [5], it has been shown that grounded extension is a subset of the intersection of all preferred extensions. So, $a' \in \texttt{GE}$ implies $a' \in \bigcap_{i=1}^{n} \mathcal{E}_i$. There is always at least one preferred extension, i.e. $i \geq 1$. Consequently, $a' \in \mathcal{E}_1$. According to Property 5, $\mathcal{E}_1 \subseteq \bigcup_{i=1}^{n} \mathcal{E}_i$. So, if $a' \in \mathcal{E}_1$ then $a' \in \bigcup_{i=1}^{n} \mathcal{E}_i$. Thus, $a' \in \bigcup_{i=1}^{n} \mathcal{E}_i$. Contradiction with the fact $(\forall a \in \mathcal{H}(o))$ $a \notin \bigcup_{i=1}^{n} \mathcal{E}_i$.*

■

Property 11 Let $\mathcal{O}$ be the set of options, and let $\mathcal{AF}$ be the argumentation system.

1. If $\mathcal{AF}$ has no stable extensions, then $\mathcal{O}_r^s = \mathcal{O}$, i.e. all the options are rejected.
2. If $\mathcal{AF}$ has stable extensions, then $\mathcal{O}_r^p \subseteq \mathcal{O}_r^s \subseteq \mathcal{O}_r^g$.

Proof

1. *If $\mathcal{AF}$ has no stable extensions then there are no skeptically accepted arguments under stable semantics, thus there are no accepted options under this semantics.*
2. *We will show that if there is at least one stable extension, then $\mathcal{O}_r^p \subseteq \mathcal{O}_r^s$.*

   Let us assume that $\mathcal{E}_1, \ldots, \mathcal{E}_n$ are stable extensions of $\mathcal{AF}$ and $\mathcal{E}_{n+1}, \ldots, \mathcal{E}_{n+k}$ are preferred extensions that are not stable. One may easily see that every stable extension is also a preferred one. Since $\mathcal{E}_1, \ldots, \mathcal{E}_n$ are stable, these are also preferred. So, stable extensions are $\mathcal{E}_1, \ldots, \mathcal{E}_n$ and preferred extensions are $\mathcal{E}_1, \ldots, \mathcal{E}_{n+k}$.

   Let us now assume that $\exists o \in \mathcal{O}$ such that $o \in \mathcal{O}_r^p$ and $o \notin \mathcal{O}_r^s$. Since $o \in \mathcal{O}_r^p$, then $(\forall a \in \mathcal{H}(o))$ a is rejected under preferred semantics. According to Property 1 we have $(\forall a \in \mathcal{H}(o))$ $a \notin \bigcup_{i=1}^{n+k} A_i$. Since $o \notin \mathcal{O}_r^s$, then $(\exists a' \in \mathcal{H}(o))$ a' is not rejected under stable semantics. According to Property 1 we have $(\exists a' \in \mathcal{H}(o))$ such that $a' \in \bigcup_{i=1}^{n} A_i$. According to Property 5, $\bigcup_{i=1}^{n} A_i \subseteq \bigcup_{i=1}^{n+k} A_i$. So, with $a' \in \bigcup_{i=1}^{n} A_i$, we have $a' \in \bigcup_{i=1}^{n+k} A_i$. Contradiction with the fact that $(\forall a \in \mathcal{H}(o))$ $a \notin \bigcup_{i=1}^{n+k} A_i$. So, $\mathcal{O}_r^p \subseteq \mathcal{O}_r^s$.

   We will now show that if there is at least one stable extension, then $\mathcal{O}_r^s \subseteq \mathcal{O}_r^g$.

   Let us assume that $\mathcal{E}_1, \ldots, \mathcal{E}_n$ are stable extensions of $\mathcal{AF}$ and $\mathcal{E}_{n+1}, \ldots, \mathcal{E}_{n+k}$ are complete extensions that are not stable. It is easy to see that every stable extension is also a complete one. Since $\mathcal{E}_1, \ldots, \mathcal{E}_n$ are stable, these are also complete. So, stable extensions are $\mathcal{E}_1, \ldots, \mathcal{E}_n$ and complete extensions are $\mathcal{E}_1, \ldots, \mathcal{E}_{n+k}$.

   Let us now assume that $\exists o \in \mathcal{O}$ such that $o \in \mathcal{O}_r^s$ and $o \notin \mathcal{O}_r^g$. Since $o \in \mathcal{O}_r^p$, then $(\forall a \in \mathcal{H}(o))$ a is rejected. According to Property 1 we have $(\forall a \in \mathcal{H}(o))$ $a \notin \bigcup_{i=1}^{n} A_i$. Since $o \notin \mathcal{O}_r^s$, then $(\exists a' \in \mathcal{H}(o))$ such that a' is not rejected under grounded semantics. Thus, $(\exists a' \in \mathcal{H}(o))$ such that

$a' \in \texttt{GE}$. In [5], it has been shown that grounded extension is the intersection of all complete extensions. Hence, if $a' \in \texttt{GE}$ then $a' \in \bigcap_{i=1}^{n+k} A_i$. So, $a' \in \bigcap_{i=1}^{n+k} A_i$. According to Property 5, $\bigcap_{i=1}^{n+k} A_i \subseteq \bigcap_{i=1}^{n} A_i$. Consequently, $a' \in \bigcap_{i=1}^{n} A_i$. Recall that we supposed that $\exists \mathcal{E}_1$ such that $\mathcal{E}_1$ is a stable extension, i.e. $n \geq 1$. According to Property 5, $a' \in \bigcap_{i=1}^{n} A_i$ implies $a' \in \mathcal{E}_1$. using the same property one more time, $a' \in \mathcal{E}_1$ implies $a' \in \bigcup_{i=1}^{n} A_i$. Contradiction with the fact $(\forall a \in \mathcal{H}(o))\ a \notin \bigcup_{i=1}^{n} A_i$. So, $\mathcal{O}_r^s \subseteq \mathcal{O}_g^r$.

■

Property 13 $\mathcal{O}_n^g = \emptyset \subseteq \mathcal{O}_n^s \subseteq \mathcal{O}_n^p \subseteq \mathcal{O}_n^c \subseteq \mathcal{O}_n^{ad}$.

Proof

- *$\mathcal{O}_n^g = \emptyset$. Let us suppose that $\mathcal{O}_n^g \neq \emptyset$. So, $\exists o \in \mathcal{O}$ such that $\exists a \in \mathcal{H}(o)$ such that a is credulously accepted. According to Definition 4, $\exists \mathcal{E}_i$ such that $a \in \mathcal{E}_i$ and $\exists \mathcal{E}_j$ such that $a \notin \mathcal{E}_j$, where $\mathcal{E}_i$ and $\mathcal{E}_j$ are different grounded extensions. But, there is always exactly one grounded extension $\texttt{GE}$. So, $\mathcal{E}_i = \mathcal{E}_j = \texttt{GE}$. Thus, we have $a \in \texttt{GE}$ and $a \notin \texttt{GE}$. Contradiction.*
- *$\mathcal{O}_n^g \subseteq \mathcal{O}_n^s$. Since $\mathcal{O}_n^g = \emptyset$, then $\mathcal{O}_n^g \subseteq \mathcal{O}_n^s$.*
- *$\mathcal{O}_n^s \subseteq \mathcal{O}_n^p$. Let $o \in \mathcal{O}_n^s$ and assume that $o \notin \mathcal{O}_n^p$. Since $o \in \mathcal{O}_n^s$ then $\exists a \in \mathcal{H}(o)$ such that a is credulously accepted. So, there exist two stable extensions S_i, S_j such that $a \in S_i$ and $a \notin S_j$. One may easily see that every stable extension is a preferred one. Hence, S_i and S_j are also preferred extensions. Consequently, $a \in \mathcal{O}_n^p$.*
- *$\mathcal{O}_n^p \subseteq \mathcal{O}_n^c$. Let $o \in \mathcal{O}_n^p$. This means that $\exists a \in \mathcal{H}(o)$ such that there exist two preferred extensions S_i, S_j such that $a \in S_i$ and $a \notin S_j$. However, since every preferred extension is a complete one, S_i and S_j are also complete extensions. Thus, $o \in \mathcal{O}_n^c$.*
- *$\mathcal{O}_n^c \subseteq \mathcal{O}_n^{ad}$. Let $o \in \mathcal{O}_n^c$. So, $\exists a \in \mathcal{H}(o)$ such that there exist two complete extensions S_i, S_j such that $a \in S_i$ and $a \notin S_j$. On the other hand, it is easy to see that every complete extension is admissible. Since S_i and S_j are complete, they are also admissible extensions. Thus, $o \in \mathcal{O}_n^{ad}$.*

■

Property 14 Let e be a new practical argument. It holds that $\texttt{Sc}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b = \texttt{Sc}(\mathcal{AF}) \cap \mathcal{A}_b$.

Proof *Let $a \in \mathcal{A}_b$ such that $a \in \texttt{Sc}(\mathcal{AF})$ and $a \in \texttt{Rej}(\mathcal{AF} \oplus e)$. We will prove that:*

1. $(\exists i \in \{1, 2, 3, \ldots\})\ (\exists a_i \in \texttt{Sc}^i(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b)$
2. *if* $(\exists i \in \{2, 3, \ldots\})\ (\exists a_i \in \texttt{Sc}^i(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b)$ *then* $(\exists j \in \{1, 2, 3, \ldots\})\ (j < i) \wedge (\exists a_j \in \texttt{Sc}^j(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b)$.

Note that the 1 *is already proved. Let us now prove* 2. *Suppose that* $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b)$. *Since* $a_i \in \mathtt{Rej}(\mathcal{AF} \oplus e)$ *then* $(\exists x \in \mathcal{A} \cup \{e\})$ $(x, a_i) \in \mathtt{Def} \wedge (\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(b, x) \in \mathtt{Def}$. *Note that from* $a_i \in \mathcal{A}_b$ *and* $(x, a_i) \in \mathtt{Def}$ *we conclude that* $x \in \mathcal{A}_b$. *Since* e *is practical, then* $x \neq e$. *Thus,* x *has already existed before the agent has received the argument* e. *This implies* $(\exists x \in \mathcal{A}_b)$ $(x, a_i) \in \mathtt{Def}$. *From* $a_i \in \mathtt{Sc}^i(\mathcal{AF})$ *we conclude that some skeptically accepted argument defends argument* a_i*, i.e.,* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF}) \cap \mathcal{A}_b)$. *Since* $(\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(b, x) \in \mathtt{Def}$ *it must be that* $a_j \in \mathtt{Rej}(\mathcal{AF} \oplus e)$. *So, we proved* 2. *From* 1 *and* 2 *we obtain that:* $\exists a_1 \in \mathtt{Sc}^1(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b$. *Hence,* a_1 *is not defeated in* $\mathcal{AF}$ *and it is defeated in* $\mathcal{AF} \oplus e$. *So,* $(e, a_1) \in \mathtt{Def}$. *Contradiction, since* e *is practical and* a *is epistemic.*

Let $a \in \mathcal{A}_b$ *be an epistemic argument such that* $a \in \mathtt{Rej}(\mathcal{AF})$. *Let us suppose that* $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. *We will prove that:*

1. $(\exists i \in \{1,2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}) \cap \mathcal{A}_b)$
2. *if* $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}) \cap \mathcal{A}_b)$ *then* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}) \cap \mathcal{A}_b)$.

Note that the 1 *is already proved. Let us now prove* 2. *Suppose that* $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF}) \cap \mathcal{A}_b)$. *Since* $a_i \in \mathtt{Rej}(\mathcal{AF})$ *then* $(\exists x \in \mathcal{AF})$ $(x, a_i) \in \mathtt{Def} \wedge (\nexists b \in \mathtt{Sc}(\mathcal{AF})$ $(b, x) \in \mathtt{Def}$. *Since* $(x, a_i) \in \mathtt{Def}$ *and* $a_i \in \mathcal{A}_b$ *then* $x \in \mathcal{A}_b$. *But* $a_i \in \mathtt{Sc}^i(\mathcal{AF})$ *implies that* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i)$ $\wedge$ $(\exists a_j \in \mathtt{Sc}^j(\mathcal{AF} \oplus e) \cap \mathcal{A}_b) \wedge (a_j, x) \in \mathtt{Def}$. *From* $(a_j, x) \in \mathtt{Def}$ *and* $x \in \mathcal{A}_b$ *we have that* a_j *is also epistemic (since practical arguments cannot attack epistemic ones). The fact that* $a_j \in \mathcal{A}_b$ *and* e *is practical implies that* $a_j \neq e$. *Thus,* a_j *existed before agent has received the new argument* e. *Since* $(\nexists b \in \mathtt{Sc}(\mathcal{AF}))$ $(b, x) \in \mathtt{Def}$ *then* $a_j \in \mathtt{Rej}(\mathcal{AF})$. *Now we have proved* 1 *and* 2. *From* 1 *and* 2 *we have directly the following:* $(\exists a_1 \in \mathtt{Sc}^1(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}) \cap \mathcal{A}_b)$. *From* $a_1 \in \mathtt{Sc}^1(\mathcal{AF} \oplus e)$ *we have* $(\nexists y \in \mathcal{A} \cup \{e\})$ $(y, a_1) \in \mathtt{Def}$ *and from* $a_1 \in \mathtt{Rej}(\mathcal{AF})$ *we have* $(\exists y \in \mathcal{AF})$ $(y, a_1) \in \mathtt{Def}$. *Contradiction.* ■

Property 15 Let e be a new practical argument. If $(\exists a \in \mathcal{A}_b \cup \mathtt{Sc}(\mathcal{AF}))$ such that $(a, e) \in \mathtt{Def}$ then $\mathtt{Sc}(\mathcal{A} \oplus e) \cap \mathcal{A}_o = \mathtt{Sc}(\mathcal{AF}) \cap \mathcal{A}_o$.

Proof *According to Property 14, argument* $a \in \mathcal{A}_b$ *will remain skeptically accepted,* $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. *Since* e *is in conflict with* a *and* a *is in grounded extension,* e *cannot be in extension, since every extension is conflict-free. So,* e *is rejected,* $e \in \mathtt{Rej}(\mathcal{AF} \oplus e)$.

We will now prove that $\mathtt{Sc}(\mathcal{AF}) \subseteq \mathtt{Sc}(\mathcal{AF} \oplus e)$. *Suppose not. Then* $(\exists b \in \mathcal{A})$ $b \in \mathtt{Sc}(\mathcal{AF}) \wedge b \in \mathtt{Rej}(\mathcal{AF} \oplus e)$. *We will prove that:*

1. $(\exists i \in \{1,2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e))$
2. *if* $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e))$ *then* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e))$.

Note that the 1 is already proved. Let us now prove 2. Suppose that $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e))$. *Since* $a_i \in \mathtt{Rej}(\mathcal{AF} \oplus e)$ *then* $(\exists x \in \mathcal{A} \cup \{e\})$ $(x, a_i) \in \mathtt{Def} \wedge (\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(b, x) \in \mathtt{Def}$. *Suppose now that* $e = x$. *But* $(\exists a \in \mathcal{A}_b \cup \mathtt{Sc}(\mathcal{AF}))$ $(a, e) \in \mathtt{Def}$. *Contradiction with* $(\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(b, x) \in \mathtt{Def}$. *Thus,* $x \neq e$, *and* x *was present in the system* $\mathcal{AF}$. *Since* $x \in \mathcal{A}$ *and* $(x, a_i) \in \mathtt{Def}$, *from* $a_i \in \mathtt{Sc}^i(\mathcal{AF})$ *we conclude that some skeptically accepted argument defends argument* a_i *in* $\mathcal{AF}$, *i.e.,* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i)$ $\wedge$ $(\exists a_j \in \mathtt{Sc}^j(\mathcal{AF}) \cap \mathcal{A}_b)$ $\wedge$ $(a_j, x) \in \mathtt{Def}$. *Since* $(\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(b, x) \in \mathtt{Def}$ *it must be that* $a_j \in \mathtt{Rej}(\mathcal{AF} \oplus e)$. *So, we proved 2. As the consequence of 1 and 2 together, it holds that:* $\exists a_1 \in \mathtt{Sc}^1(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e)$. *This means that* $(\nexists b \in \mathcal{AF})$ $(b, a_1) \in \mathtt{Def}$ *and* $(\exists b \in \ \cup\{e\})$ $(b, x) \in \mathtt{Def}$. *So,* $b = e$, *i.e.,* $(e, a_1) \in \mathtt{Def}$. *Note that* e *is the* only *argument that defeats* a_1 *in* $\mathcal{AF} \oplus e$. *But* $(\exists a \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(a, e) \in \mathtt{Def}$. *Hence,* a_1 *is defended against all defeaters and, consequently,* $a_1 \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. *Contradiction.*

We will now prove that $\mathtt{Sc}(\mathcal{AF} \oplus e) \subseteq \mathtt{Sc}(\mathcal{AF})$. *Suppose not. Then* $(\exists a_i \in \mathcal{AF})$ $a_i \in \mathtt{Sc}(\mathcal{AF} \oplus e) \wedge a_i \in \mathtt{Rej}(\mathcal{AF})$. *We will prove that:*

1. $(\exists i \in \{1,2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$
2. *if* $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$ *then* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$.

Note that the 1 is already proved. Let us now prove 2. Suppose that $(\exists i \in \{2,3,\ldots\})$ $(\exists a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$. *Since* $a_i \in \mathtt{Rej}(\mathcal{AF})$ *then* $(\exists x \in \mathcal{AF})$ $(x, a_i) \in \mathtt{Def} \wedge (\nexists b \in \mathtt{Sc}(\mathcal{AF})$ $(b, x) \in \mathtt{Def}$. *Since* $(x, a_i) \in \mathtt{Def}$ *and* $a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e)$ *then* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$. *From* $(\nexists b \in \mathtt{Sc}(\mathcal{AF})$ $(b, x) \in \mathtt{Def}$ *we obtain that* $a_j \in \mathtt{Rej}(\mathcal{AF})$. *Now we have proved 1 and 2. From 1 and 2 we have directly the following:* $(\exists a_1 \in \mathtt{Sc}^1(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$. *From* $a_1 \in \mathtt{Sc}^1(\mathcal{AF} \oplus e)$ *we have* $(\nexists y \in \mathcal{A} \cup \{e\})$ $(y, a_1) \in \mathtt{Def}$ *and from* $a_1 \in \mathtt{Rej}(\mathcal{AF})$ *we have* $(\exists y \in \mathcal{A})$ $(y, a_1) \in \mathtt{Def}$. *Contradiction.* ■

Property 17 Let $o \in \mathcal{O}$, $a_i \in \mathcal{H}(o)$, $a_i \in \mathtt{Sc}^i(\mathcal{AF})$ and $x \in A$ such that $(x, a_i) \in \mathtt{Def}$.

1. If $x \in \mathcal{A}_b$ then $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathcal{A}_b \cap \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j, x) \in \mathtt{Def}$,
2. If $x \in \mathcal{A}_o$ then $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in (\mathcal{A}_b \cup \mathcal{H}(o)) \cap \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j, x) \in \mathtt{Def}$.

Proof *We will first prove that if* $a_i \in \mathcal{H}(o)$, $a_i \in \mathtt{Sc}^i(\mathcal{AF})$, $x \in A$ *and* $(x, a_i) \in \mathtt{Def}$, *then* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in (\mathcal{A}_b \cup \mathcal{H}(o)) \cap \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j, x) \in \mathtt{Def}$.

Suppose the contrary, $(\nexists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathcal{H}(o) \cup \mathcal{A}_b)$ $a_j \in \mathtt{Sc}^j(\mathcal{AF}) \wedge (a_j, x) \in \mathtt{Def}$. *Since* a_i *is skeptically accepted and defeated, then it is defended, so* $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathcal{A}_o \setminus \mathcal{H}(o))$ $a_j \in \mathtt{Sc}^j(\mathcal{AF}) \wedge (a_j, x) \in \mathtt{Def}$. *Hence,* $(\exists o' \in \mathcal{O})$ $(o' \neq o)$ *and* $a_j \in \mathcal{H}(o')$. *Since both* a_i *and* a_j *are in the grounded extension, there is not attack between them. Since*

$a_i \in \mathcal{H}(o)$ and $a_j \in \mathcal{H}(o')$, with $o' \neq o$, then $(a_i, a_j) \in \mathcal{R}$ and $(a_j, a_i) \in \mathcal{R}$. So, $(a_i, a_j) \in \geq$ and $(a_j, a_i) \in \geq$. Suppose that $(a_j, x) \in >$. Then, using the transitivity of preference relation, one can easily prove that $(a_i, x) \in >$. Contradiction, since $(x, a_i) \in \mathtt{Def}$. Hence, $(x, a_j) \in \mathtt{Def}$. We will prove that:

1. $(\exists j \in \{1, 2, 3, \ldots\})$ $(\exists a_j \in (\mathtt{Sc}^j(\mathcal{AF}) \cap \mathcal{A}_o) \setminus \mathcal{H}(o))$ $\wedge$ $(a_j, x) \in \mathtt{Def}$ $\wedge$ $(x, a_j) \in \mathtt{Def}$

2. *if* $(\exists j \in \{2, 3, \ldots\})$ $(\exists a_j \in (\mathtt{Sc}^j(\mathcal{AF}) \cap \mathcal{A}_o) \setminus \mathcal{H}(o))$ $\wedge$ $(a_j, x) \in \mathtt{Def}$ $\wedge$ $(x, a_j) \in \mathtt{Def}$ *then* $(\exists k \in \{1, 2, 3, \ldots\})$ $(k < j)$ $\wedge$ $(\exists a_k \in (\mathtt{Sc}^k(\mathcal{AF}) \cap \mathcal{A}_o) \setminus \mathcal{H}(o))$ $\wedge$ $(a_k, x) \in \mathtt{Def}$ $\wedge$ $(x, a_k) \in \mathtt{Def}$

Note that we have already proved 1. Let us prove 2. Suppose that $(\exists j \in \{2, 3, \ldots\})$ $(\exists a_j \in (\mathtt{Sc}^j(\mathcal{AF}) \cap \mathcal{A}_o) \setminus \mathcal{H}(o))$. Since a_j is skeptically accepted and defeated, then it is defended, so $(\exists k \in \{1, 2, 3, \ldots\})$ $(k < j)$ $\wedge$ $(\exists a_k \in \mathcal{A}_o \setminus \mathcal{H}(o))$ $a_k \in \mathtt{Sc}^k(\mathcal{AF}) \wedge (a_k, x) \in \mathtt{Def}$. Hence, $(\exists o'' \in \mathcal{O})$ $(o'' \neq o)$ and $a_k \in \mathcal{H}(o'')$. Recall that $o'' \neq o$ since we have supposed that $(\nexists m \in \{1, 2, 3, \ldots\})$ $(m < i)$ $\wedge$ $(\exists a_m \in \mathcal{H}(o) \cup \mathcal{A}_b)$ $a_m \in \mathtt{Sc}^m(\mathcal{AF}) \wedge (a_m, x) \in \mathtt{Def}$. Since both a_j and a_k are in the grounded extension, there is no defeat between them. Since $a_j \in \mathcal{H}(o)$ and $a_k \in \mathcal{H}(o'')$, with $o'' \neq o$, then $(a_j, a_k) \in \mathcal{R}$ and $(a_k, a_j) \in \mathcal{R}$. So, $(a_j, a_k) \in \geq$ and $(a_k, a_j) \in \geq$. Suppose that $(a_k, x) \in >$. Then, using the transitivity of preference relation, one can easily prove that $(a_j, x) \in >$. Contradiction, since $(x, a_j) \in \mathtt{Def}$. Hence, $(x, a_k) \in \mathtt{Def}$. Since we have proved 1 and 2, we conclude that $(\exists a_1 \in (\mathtt{Sc}^1(\mathcal{AF}) \cap \mathcal{A}_o) \setminus \mathcal{H}(o))$ $\wedge$ $(a_1, x) \in \mathtt{Def}$ $\wedge$ $(x, a_1) \in \mathtt{Def}$. Contradiction since a_1 is defeated by x and at the same time $a_1 \in \mathtt{Sc}^1(\mathcal{AF})$. So, we have proved that if $a_i \in \mathcal{H}(o)$, $a_i \in \mathtt{Sc}^i(\mathcal{AF})$, $x \in A$ and $(x, a_i) \in \mathtt{Def}$, then $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i)$ $\wedge$ $(\exists a_j \in (\mathcal{A}_b \cup \mathcal{H}(o)) \cap \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j, x) \in \mathtt{Def}$. Note that this ends the proof for the case 2, i.e. for the case when $x \in \mathcal{A}_o$. Suppose now that $x \in \mathcal{A}_b$. We have proved that $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i)$ $\wedge$ $(\exists a_j \in (\mathcal{A}_b \cup \mathcal{H}(o)) \cap \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j, x) \in \mathtt{Def}$. Suppose that $a_j \in \mathcal{H}(o)$. This means that a practical argument attacks an epistemic one. Contradiction. So, $a_j \in \mathcal{A}_b$. ■

Theorem 1 Let e be a new argument such that $\mathtt{Conc}(e) = o$. Then, $(\forall a \in \mathcal{H}(o))$ $a \in \mathtt{Sc}(\mathcal{AF}) \Rightarrow a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$.

Proof *Suppose the contrary, i.e., suppose that $\exists a \in \mathtt{Sc}(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e)$. We will prove that:*

1. $(\exists i \in \{1, 2, 3, \ldots\})$ $(\exists a_i \in (\mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$

2. *if* $(\exists i \in \{2, 3, \ldots\})$ $(\exists a_i \in (\mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$ *then* $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i)$ $\wedge$
   $(\exists a_j \in (\mathtt{Sc}^j(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$

Note that we have already proved 1. Let us now prove 2. Suppose that $(\exists i \in \{2, 3, \ldots\})$ $(\exists a_i \in (\mathtt{Sc}^i(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$. Since argument a_i is rejected in the new system, then $(\exists x \in \mathcal{A} \cup \{e\})$ $(x, a_i) \in \mathtt{Def}$ $\wedge$ $(\nexists y \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(y, x) \in \mathtt{Def}$. Note that $x \neq e$, because $e \in \mathcal{H}(o)$ and arguments in favor of same *offer do not attack each other. Since* $(a_i \in \mathtt{Sc}(\mathcal{AF}))$ *and* $(x, a_i) \in \mathtt{Def}$*, then according to the Property 17,* $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i) \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j \in \mathcal{H}(o) \cup \mathcal{A}_b) \wedge (a_j, x) \in \mathtt{Def}$*. Note that* $a_j \neq e$*, because* $a_j \in \mathtt{Sc}^j(\mathcal{AF})$ *and* $e \notin \mathtt{Sc}(\mathcal{AF})$*. Since* $(\nexists y \in \mathtt{Sc}(\mathcal{AF} \oplus e))$ $(y, x) \in \mathtt{Def}$*, then* $a_j \in \mathtt{Rej}(\mathcal{AF} \oplus e)$*. Argument* a_j *is practical, since* $a_j \in \mathcal{A}_b$*, according to Property 14, implies* $a_j \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ *which is in contradiction with the fact that* $a_j \in \mathtt{Rej}(\mathcal{AF} \oplus e)$*. So,* $a_j \in \mathcal{H}(o)$*. Now that we see that 1 and 2 are true, we may conclude that* $(\exists a_1 \in (\mathtt{Sc}^1(\mathcal{AF}) \cap \mathtt{Rej}(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$*. Since* a_1 *was not defeated in* $\mathcal{AF}$ *and it is defeated in* $\mathcal{AF} \oplus e$*, it holds that* $(e, a_1) \in \mathtt{Def}$*. Contradiction, since* $a_1 \in \mathcal{H}(o)$ *and* $e \in \mathcal{H}(o)$*, and arguments in favor of same offer do not defeat each other.* ■

Theorem 2 Let $o \in \mathcal{O}$, and $a \in \mathcal{H}(o)$. If $a \in \mathtt{Rej}(\mathcal{AF})$ and $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$, then $e \in \mathcal{H}(o)$.

Proof *Suppose that the theorem is not true. Then,* $(\exists a \in \mathtt{Rej}(\mathcal{AF}) \cap \mathtt{Sc}(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$ *and* $e \notin \mathcal{H}(o)$*. Since e is practical, it holds that* $(\exists o' \in \mathcal{O})$ $o' \neq o \wedge e \in \mathcal{H}(o')$*. We will prove that:*

1. $(\exists i \in \{1, 2, 3, \ldots\})$ $(\exists a_i \in (\mathtt{Rej}(\mathcal{AF}) \cap \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$
2. *if* $(\exists i \in \{2, 3, \ldots\})$ $(\exists a_i \in (\mathtt{Rej}(\mathcal{AF}) \cap \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$ *then* $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i) \wedge (\exists a_j \in \mathcal{H}(o) \cap \mathtt{Sc}^j(\mathcal{AF} \oplus e) \cap (\mathtt{Rej}(\mathcal{AF}))$

Since $a \in \mathcal{H}(o)$*,* $a \in \mathtt{Rej}(\mathcal{AF})$ *and* $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$*, we see that 1 is true. So, let us prove the 2. Suppose* $(\exists i \in \{2, 3, \ldots\})$ $(\exists a_i \in (\mathtt{Rej}(\mathcal{AF}) \cap \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$*. Since* a_i *was rejected,* $a_i \in \mathtt{Rej}(\mathcal{AF})$*, then* $(\exists x \in \mathcal{A})$ $(x, a_i) \in \mathtt{Def}$ $\wedge$ $(\nexists y \in \mathtt{Sc}(\mathcal{AF}))$ $(y, x) \in \mathtt{Def}$*. Since* $a_i \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ *then, according to Property 17,* $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i) \wedge (\exists a_j \in (\mathtt{Sc}^j(\mathcal{AF} \oplus e) \cap (\mathcal{H}(o) \cup \mathcal{A}_b)) \wedge$ $(a_j, x) \in \mathtt{Def}$*. We have* $a_j \neq e$ *because* $a_j \in \mathcal{H}(o)$ *and* $e \notin \mathcal{H}(o)$*. So,* $a_j \in \mathcal{A}$*. If* $a_j \in \mathcal{A}_b$*, then, according to the Property 14,* $a_j \in \mathtt{Sc}(\mathcal{AF})$*. Contradiction with the fact* $(\nexists y \in \mathtt{Sc}(\mathcal{AF}))$ $(y, x) \in \mathtt{Def}$*. So,* $a_j \in \mathcal{H}(o)$*. On the other hand, since* $a_i \in \mathtt{Rej}(\mathcal{AF})$ *then* $(\nexists y \in \mathtt{Sc}(\mathcal{AF}))$ $(y, x) \in \mathtt{Def}$*. Hence, since* $a_j \in \mathcal{A}$*, then, it must be the case that* $a_j \in \mathtt{Rej}(\mathcal{AF})$*. With 1 and 2 we have the following:* $(\exists a_1 \in (\mathtt{Rej}(\mathcal{AF}) \cap \mathtt{Sc}^1(\mathcal{AF} \oplus e) \cap \mathcal{H}(o))$*. So,* a_1 *is not defeated in* $\mathcal{AF} \oplus e$ *and* a_1 *is defeated in* $\mathcal{AF}$*. Contradiction.* ■

Property 18 It holds that $\mathtt{Dbe}(\mathcal{AF}) \subseteq \mathtt{Sc}(\mathcal{AF})$.

Proof *Let* $\mathcal{AF} = \langle \mathcal{A}, \mathtt{Def} \rangle$ *and* $a \in \mathtt{Dbe}(\mathcal{AF})$*. Let* $Att(a) = \{x_i \in \mathcal{A} \mid (x_i, a) \in \mathtt{Def}\}$*. Since the set of arguments* $\mathcal{A}$ *is finite,* $Att(a) = \{x_1, \ldots, x_n\}$*. From* $a \in \mathtt{Dbe}(\mathcal{AF})$*, we obtain* $(\forall x_i \in \mathcal{A})$ *if* $(x_i, a) \in \mathtt{Def}$ *then* $(\exists \alpha \in \mathtt{Sc}(\mathcal{AF}) \cap \mathcal{A}_b)$ *such that* $(\alpha, x_i) \in \mathtt{Def}$*. Let* $Defends(a) = \{\alpha_1, \ldots, \alpha_k\}$ *be a set such that* $Defends(a) \subseteq \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF})$ *and* $(\forall x_i \in Att(a))$ $(\exists \alpha_j \in Defends(a))$ $(\alpha_j, x_j) \in \mathtt{Def}$*. Since* $Defends(a) \subseteq \mathtt{Sc}(\mathcal{AF})$ *then* $(\forall \alpha_i \in Defends(a))$ $(\exists m_i \in \{1, 2, 3, \ldots\})$ *s.t.* $\alpha_i \in \mathtt{Sc}^{m_i}(\mathcal{AF})$*. Let* $m = max\{m_1, \ldots, m_k\}$*. It holds that* $Defends(a) \subseteq \mathcal{F}^m(\emptyset)$*. Then, according to the definition of grounded semantics, it holds that* $a \in \mathcal{F}^{m+1}(\emptyset)$*, since argument a is defended by arguments of* $\mathcal{F}^m(\emptyset)$*against all attacks. From* $a \in \mathcal{F}^m(\emptyset)$*, we have* $a \in \mathtt{Sc}(\mathcal{AF})$*.*

■

Theorem 3 Let $o \in \mathcal{O}_a(\mathcal{AF})$ and let e be a new practical argument. $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$ iff $e \notin \mathcal{H}(o) \wedge (\nexists x \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF}))\ (x,e) \in \mathtt{Def} \wedge (\forall a \in \mathtt{Dbe}(\mathcal{AF}) \cap \mathcal{H}(o))\ (e,a) \in \mathtt{Def}$.

Proof $\Rightarrow$ *Since $o \in \mathcal{O}_a(\mathcal{AF})$, then $(\exists a \in \mathcal{H}(o))\ a \in \mathtt{Sc}(\mathcal{AF})$. Let $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$.*

1. *Suppose $e \in \mathcal{H}(o)$. Then, according to the Theorem 1, $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. Consequently, $o \in \mathcal{O}_a(\mathcal{AF} \oplus e)$, contradiction.*

2. *Suppose that $(\exists x \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF}))\ (x,e) \in \mathtt{Def}$. According to Property 15, $\mathtt{Sc}(\mathcal{AF} \oplus e) = \mathtt{Sc}(\mathcal{AF})$ and $\mathtt{Rej}(\mathcal{AF} \oplus e) = \mathtt{Rej}(\mathcal{AF}) \cup \{e\}$. So, $a \in \mathtt{Sc}(\mathcal{AF})$ implies $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. Contradiction with the fact that $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$.*

3. *Suppose that $(\exists a \in \mathtt{Dbe}(\mathcal{AF}) \cap \mathcal{H}(o))\ (e,a) \notin \mathtt{Def}$. Since $a \in \mathtt{Dbe}(\mathcal{AF})$, Property 18 implies that $a \in \mathtt{Sc}(\mathcal{AF})$. From $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$ we obtain $a \in \mathtt{Rej}(\mathcal{AF} \oplus e)$. So, $(\exists x \in \mathcal{A})\ (x,a) \in \mathtt{Def}\ (\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))\ (b,x) \in \mathtt{Def}$. Note that $x \neq e$ because $(x,a) \in \mathtt{Def}$ and $(e,a) \notin \mathtt{Def}$. So, $x \in \mathcal{A}$. From $a \in \mathtt{Dbe}(\mathcal{AF})$ we have $(\exists \alpha \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF})\ (\alpha, x) \in \mathtt{Def}$. From Property 14, we have $\alpha \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. Contradiction with $(\nexists b \in \mathtt{Sc}(\mathcal{AF} \oplus e))\ (b,x) \in \mathtt{Def}$.*

$\Leftarrow$ *Let $e \notin \mathcal{H}(o) \wedge (\nexists x \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF}))\ (x,e) \in \mathtt{Def} \wedge (\forall a \in \mathtt{Dbe}(\mathcal{AF}) \cap \mathcal{H}(o))\ (e,a) \in \mathtt{Def}$. Suppose that $o \notin \mathcal{O}_r(\mathcal{AF} \oplus e)$. Thus, $o \in \mathcal{O}_a(\mathcal{AF} \oplus e)$. This means that $(\exists a \in \mathcal{H}(o))\ a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. We will prove the following:*

1. *$(\exists i \in \{1,2,3,\ldots\})\ (\exists a_i \in \mathcal{H}(o))\ (a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e))$.*

2. *if $(\exists i \in \{2,3,\ldots\})\ (\exists a_i \in \mathcal{H}(o))\ (a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e))$ then $(\exists j \in \{1,2,3,\ldots\})$ $(j < i) \wedge (\exists a_j \in \mathcal{H}(o))\ (a_j \in \mathtt{Sc}^i(\mathcal{AF} \oplus e))$.*

Note that we have already proved 1, since $(\exists a \in \mathcal{H}(o))\ a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. Let us prove 2. Suppose that $(\exists i \in \{2,3,\ldots\})\ (\exists a_i \in \mathcal{H}(o))\ (a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e))$. Let us explore two possibilities: $a \in \mathtt{Dbe}(\mathcal{AF})$ and $a \notin \mathtt{Dbe}(\mathcal{AF})$. Suppose that $a_i \in \mathtt{Dbe}(\mathcal{AF})$. Since $a_i \in \mathtt{Dbe}(\mathcal{AF}) \cap \mathcal{H}(o)$ then $(e,a_i) \in \mathtt{Def}$. Since $a_i \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ and $(e,a) \in \mathtt{Def}$ then, according to Property 17, $(\exists j \in \{1,2,3,\ldots\})$ $j < i \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF} \oplus e))\ (a_j \in \mathcal{A}_b \cup \mathcal{H}(o)) \wedge (a_j, e) \in \mathtt{Def}$. We will now show that $a_j \in \mathcal{H}(o)$. Suppose that $a_j \in \mathcal{A}_b$. According to Property 14, $a_j \in \mathtt{Sc}(\mathcal{AF})$. Contradiction with $(\nexists x \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF}))\ (x,e) \in \mathtt{Def}$. Let us now explore the case when $a_i \notin \mathtt{Dbe}(\mathcal{AF})$. From Definition 6, we have $(\exists x \in \mathcal{A})\ (x,a_i) \in \mathtt{Def} \wedge$ $(\nexists a_j \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF} \oplus e))\ (a_j, x) \in \mathtt{Def}$. Since $a_i \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ and $(x, a_i) \in \mathtt{Def}$, Property 17 implies that $(\exists j \in \{1,2,3,\ldots\})\ j < i \wedge (\exists a_j \in \mathtt{Sc}^j(\mathcal{AF} \oplus e))$ $(a_j \in \mathcal{A}_b \cup \mathcal{H}(o)) \wedge (a_j, e) \in \mathtt{Def}$. Since $(\nexists a_j \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF} \oplus e))\ (a_j, x) \in \mathtt{Def}$ then $a_j \in \mathcal{H}(o)$.

Now, we have proved that 1 and 2. As the consequence, we have that: $(\exists a_1 \in \mathcal{H}(o))\ (a_1 \in \mathtt{Sc}^1(\mathcal{AF} \oplus e))$. This means that a_1 is not defeated by any argument in $\mathcal{AF} \oplus e$. This implies that a_1 is not defeated by any argument in $\mathcal{AF}$, i.e., $a_1 \in \mathtt{Sc}^1(\mathcal{AF})$. Consequently, $a_1 \in \mathtt{Dbe}(\mathcal{AF})$. So, $(e, a_1) \in \mathtt{Def}$. Contradiction with the fact that a_1 is not defeated in $\mathcal{AF} \oplus e$. ■

Theorem 4 Let $o \in \mathcal{O}_r(\mathcal{AF})$ and let e be a new practical argument. $o \in \mathcal{O}_a(\mathcal{AF} \oplus e)$ iff $e \in \mathcal{H}(o) \wedge e \in \mathtt{Sc}(\mathcal{AF} \oplus e)$.

Proof *$\Rightarrow$ Let $o \in \mathcal{O}_a(\mathcal{AF} \oplus e)$.*

1. *Let us prove that $e \in \mathcal{H}(o)$. Suppose not. Then $(\exists o' \in \mathcal{O})$ $o \neq o' \wedge e \in \mathcal{H}(o')$. But, according to Theorem 2, all rejected arguments in favor of o will remain rejected, i.e. $\mathcal{H}(o) \subseteq \mathtt{Rej}(\mathcal{AF} \oplus e)$. This means that $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$. Contradiction.*

2. *Let us now prove that $e \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. Suppose not. So, $e \in \mathtt{Rej}(\mathcal{AF} \oplus e)$. Since $o \in \mathcal{O}_a(\mathcal{AF} \oplus e)$ then $(\exists a \in \mathcal{H}(o))$ $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$. Note that $a \neq e$ because $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ and $ein\mathtt{Rej}(\mathcal{AF} \oplus e)$. We will prove the following:*

   (a) $(\exists i \in \{1, 2, 3, \ldots\})$ $(\exists a_i \in \mathcal{H}(o))$ $(a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$.

   (b) if $(\exists i \in \{2, 3, \ldots\})$ $(\exists a_i \in \mathcal{H}(o))$ $(a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$ *then* $(\exists j \in \{1, 2, 3, \ldots\})$ $(j < i) \wedge (\exists a_j \in \mathcal{H}(o))$ $(a_j \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$.

   We have already proved (a). Let us prove the (b). Suppose $(\exists i \in \{2, 3, \ldots\})$ $(\exists a_i \in \mathcal{H}(o))$ $(a_i \in \mathtt{Sc}^i(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$. Since $a_i \in \mathtt{Rej}(\mathcal{AF})$ then $(\exists x \in \mathcal{A})$ $(x, a_i) \in \mathtt{Def} \wedge (\nexists y \in \mathtt{Sc}(\mathcal{AF}))$ $(y, x) \in \mathtt{Def}$. Since $a_i \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ then, according to Property 17, $(\exists a_j \in \mathtt{Sc}^j(\mathcal{AF} \oplus e))$ $(a_j \in \mathcal{H}(o) \cup \mathcal{A}_b)$ $\wedge$ $(a_j, x) \in \mathtt{Def}$. Here, we have $a_j \neq e$ because $a_j \in \mathtt{Sc}(\mathcal{AF} \oplus e)$ and $e \notin \mathtt{Sc}(\mathcal{AF} \oplus e)$. So, a_j was already present before the agent has received the new argument e. Since $(\nexists y \in \mathtt{Sc}(\mathcal{AF}))$ $(y, x) \in \mathtt{Def}$ then $a_j \in \mathtt{Rej}(\mathcal{AF})$. Suppose that $a_j \in \mathcal{A}_b$. Then, according to Property 14, $a_j \in \mathtt{Rej}(\mathcal{AF} \oplus e)$, contradiction. So, $a_j \in \mathcal{H}(o)$. Now, when we have proved both (a) and (b), we conclude that $(\exists a_1 \in \mathcal{H}(o))$ $(a_1 \in \mathtt{Sc}^1(\mathcal{AF} \oplus e) \cap \mathtt{Rej}(\mathcal{AF}))$. Since a_1 is not defeated in $\mathcal{AF} \oplus e$, than it is not defeated in $\mathcal{AF}$. Contradiction with $a_1 \in \mathtt{Rej}(\mathcal{AF})$.

$\Leftarrow$ Suppose that an agent receives a skeptically accepted argument $e \in \mathcal{H}(o)$. Then, the offer o is acceptable according to Definition 5. ■

Property 19 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be a complete argumentation framework for decision making, and a be an arbitrary argument. Then:

1. a is skeptically accepted iff $(\forall x \in \mathcal{A}_o)$ $(a, x) \in \geq_o$.

2. a is rejected iff $(\exists x \in \mathcal{A})$ $(x, a) \in >_o$.

3. a is credulously accepted iff $((\exists x' \in \mathcal{A})$ $(a, x') \notin \geq_o) \wedge ((\forall x \in \mathcal{A})$ $((a, x) \notin \geq_o) \Rightarrow (x, a) \notin \geq_o))$.

Proof

1. *$\Rightarrow$ Suppose that a is skeptically accepted. It can be shown that in this system an argument a is skeptically accepted iff $(\nexists x \in \mathcal{A}_o)$ $(x, a) \in \mathtt{Def}_o$. Suppose that there is an argument x' such that $(a, x') \notin \geq_o$. Since $\mathcal{R}_o$ is*

complete, then $(x', a) \in \mathcal{R}_o$*. Thus, according to definition of* $\texttt{Def}_o$*, we have* $(x', a) \in \texttt{Def}_o$*. Contradiction with the fact* $(\nexists x \in \mathcal{A}_o)$ $(x, a) \in \texttt{Def}_o$*.*

$\Leftarrow$ *Let us now suppose that* $(\forall x \in \mathcal{A}_o)$ $(a, x) \in \geq_o$ *and that a is not skeptically accepted. It can be shown that in this system an argument a is skeptically accepted iff* $(\nexists x \in \mathcal{A}_o)$ $(x, a) \in \texttt{Def}_o$*. Since a is not skeptically accepted,* $(\exists x')$ $(x, a) \in \texttt{Def}_o$*. Since* $(x', a) \in \texttt{Def}_o$ *then, according to definition of* $\texttt{Def}_o$*,* $(a, x') \notin \geq_o$*. Contradiction with the fact* $(\forall x \in \mathcal{A}_o)$ $(a, x) \in \geq_o$*.*

2. $\Rightarrow$ *Suppose that a is rejected. Then, there is no extension* $\mathcal{E}$ *such that* $a \in \mathcal{E}$*. It can be shown that for an arbitrary argument* $a \in \mathcal{A}$*, there exists an extension* $\mathcal{E}$ *such that* $a \in \mathcal{E}$ *iff a is self-defending. This means that a is not self-defending. So,* $(\exists x' \in \mathcal{A}_o)$ $((x', a) \in \texttt{Def}_o \wedge (a, x') \notin \texttt{Def}_o)$*. Since* $(x', a) \in \mathcal{R}_o$ *and* $(x', a) \in \texttt{Def}_o$ *then, according to definition of* $\texttt{Def}_o$*, we have* $(a, x') \notin \geq_o$*. Since* $(a, x') \in \mathcal{R}_o$ *and* $(a, x') \notin \texttt{Def}_o$ *then, according to definition of* $\texttt{Def}_o$*, we have* $(x', a) \in \geq_o$*. According to the definition of* $>_o$*,* $(a, x') \notin \texttt{Def}_o$ *and* $(x', a) \in \geq_o$ *give* $(x', a) \in >_o$*.*

   $\Leftarrow$ *Suppose now that* $(\exists x' \in \mathcal{A}_o)$ $(x', a) \in >_o$*. Since the relation* $\mathcal{R}_o$ *is complete, we have* $(x', a) \in \mathcal{R}_o$*. According to definition of* $>_o$*, we have* $(a, x') \notin \geq_o$*. These two facts, together with the the definition of* $\texttt{Def}_o$ *imply* $(x', a) \in \texttt{Def}_o$*. The fact that* $(x', a) \in \geq_o$ *implies that, according to definition of* $\texttt{Def}_o$*,* $(a, x') \notin \texttt{Def}_o$*. So,* $(x', a) \in \texttt{Def}_o$ *and* $(a, x') \notin \texttt{Def}_o$ *which means that a is not self-defending. Since it can be shown that for an arbitrary argument* $a \in \mathcal{A}$*, there exists an extension* $\mathcal{E}$ *such that* $a \in \mathcal{E}$ *iff a is self-defending, then there is no extension* $\mathcal{E}$ *such that* $a \in \mathcal{E}$*. So, a is rejected.*

3. $\Rightarrow$ *Let us suppose that a is credulously accepted. According to Definition 4, there is at least one extension* $\mathcal{E}_i$ *such that* $a \in \mathcal{E}_i$*. Since it can be shown that for an arbitrary argument* $a \in \mathcal{A}$*, there exists an extension* $\mathcal{E}$ *such that* $a \in \mathcal{E}$ *iff a is self-defending and since a is in* $\mathcal{E}_i$ *then a is self-defending. Suppose now that* $(a, x') \notin \geq_o$*. So,* $(x', a) \in \texttt{Def}_o$*. Since a is self-defending, we have* $(a, x') \in \texttt{Def}_o$*. So,* $(x', a) \notin \geq_o$*. Hence,* $((\forall x \in \mathcal{A})$ $((a, x) \notin \geq_o) \Rightarrow (x, a) \notin \geq_o))$*. We will now prove that* $((\exists x' \in \mathcal{A})$ $(a, x') \notin \geq_o)$*. Since a is not skeptically accepted, and it can be shown that an argument* $a \in \mathcal{A}_\wr$ *is skeptically accepted iff it is not attacked, then* $(\exists y' \in \mathcal{A}_o)$ $(y', a) \in \texttt{Def}_o$*. This means that* $(a, y') \notin \geq_o$*. So, we proved that* $((\exists x' \in \mathcal{A})$ $(a, x') \notin \geq_o)$ $\wedge$ $((\forall x \in \mathcal{A})$ $((a, x) \notin \geq_o) \Rightarrow (x, a) \notin \geq_o))$*.*

   $\Leftarrow$ *Let us now suppose that* $((\exists x' \in \mathcal{A})$ $(a, x') \notin \geq_o) \wedge ((\forall x \in \mathcal{A})$ $((a, x) \notin \geq_o$ $) \Rightarrow (x, a) \notin \geq_o))$*. We have* $((\exists x' \in \mathcal{A})$ $(a, x') \notin \geq_o)$*, so* $(x', a) \in \texttt{Def}_o$*. IT can be shown that an argument* $a \in \mathcal{A}_\wr$ *is skeptically accepted iff it is not attacked, so, a is not skeptically accepted. Suppose now that a is rejected. That means that* $((\exists x' \in \mathcal{A})$ $((x', a) \in \texttt{Def}_o)$ *and* $((a, x') \notin \texttt{Def}_o)$*. The fact* $((x', a) \in \texttt{Def}_o)$ *implies* $((a, x') \notin \geq_o)$*. According to the assumption* $((\forall x \in \mathcal{A})$ $((a, x) \notin \geq_o) \Rightarrow (x, a) \notin \geq_o))$*, we have* $((x', a) \notin \geq_o)$*. Thus,*

$((a, x') \in \texttt{Def}_o)$. Contradiction. Since a is neither skeptically accepted nor rejected, it is credulously accepted.

■

Property 20 The following equivalences hold.

1. There is at least one skeptically accepted argument iff there is at least one acceptable offer.
2. There is at least one credulously accepted argument iff there is at least one negotiable offer.

Proof

1. *$\Rightarrow$ Suppose that there is at least one skeptically accepted argument a. Since all the arguments are practical arguments, a is in favor of some offer o. Then, according to Definition 5, o is acceptable.*

   $\Leftarrow$ Let us now suppose that there is at least one acceptable offer o. Then, according to Definition 5, there is at least one skeptically accepted argument a such that $a \in \mathcal{H}(o)$.

2. *$\Rightarrow$ Suppose that there is at least one credulously accepted argument a. Then, according to Theorem 5, there are no skeptically accepted arguments. Since all the arguments are practical arguments, a is in favor of some offer o. Since there are no skeptically accepted arguments at all, there are no skeptically accepted arguments in favor of offer o. So, there is at least one credulously accepted argument a in favor of offer o and there are no skeptically accepted arguments in favor of offer o. According to Definition 5, o is negotiable.*

   $\Leftarrow$ Let us now prove the last part of the property. Suppose that there is at least one negotiable offer o. Then, according to Definition 5, there is at least one credulously accepted argument a in its favor.

■

Property 21 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \texttt{Def}_o \rangle$ be a complete argumentation framework for decision making and $a \in \mathcal{A}_o$. If $a \in \texttt{Rej}(\mathcal{AF}_o)$ then $(\exists x' \in \mathcal{A}_o)$ such that $x' \notin \texttt{Rej}(\mathcal{AF}_o) \wedge (x', a) \in >_o$.

Proof *Let $a \in \mathcal{A}_o$. Assume that a is rejected. Thus, from Property 19, there is at least one argument z_0 such that $(z_0, a) \in >_o$. Since z_0 is rejected, there exists at least one argument z_1 such that $(z_1, z_0) \in >_o$. Now, we can construct the sequence of arguments $z_0, \ldots, z_k$ such that $(\forall i \in \{1, \ldots, k\})$ $(z_i, z_{i-1}) \in >_o$. Let $z_0, \ldots, z_n$ be a maximal such a sequence. We will now prove that that all the arguments in this sequence are different. Suppose that $(\exists i, j \in \{0, \ldots, n\})$ $z_i = z_j$. Without loss of generality, suppose that $i > j$. Then, because of transitivity of the relation $>_o$, we have $(z_i, z_j) \in >_o$. On the other hand, $z_i = z_j$, so $(z_i, z_i) \in >_o$. This implies that $(z_i, z_i) \in \geq_o$ and $(z_i, z_i) \notin \geq_o$. Contradiction.*

Hence, all the arguments in this sequence are different. Since there is a finite number of arguments and all the arguments in the sequence are different, the sequence is finite. So, let z_n be the last argument in this sequence. Note that, because of the transitivity of relation $>_o$, it holds that $(z_n, x) \in >_o$. The argument z_n can be rejected or not. Suppose that it is rejected. Then, the fact that it is rejected implies that $(\exists z_{n+1})\ (z_{n+1}, z_n) \in >_o$. Contradiction with the fact that the sequence which ends with z_n is maximal. Suppose that z_n is not rejected. So, $(z_n, x) \in >_o$ and z_n is not rejected. Contradiction with the fact $(\forall x \in \mathcal{A}_o)$ $(x, a) \in >_o \Rightarrow x \in \mathtt{Rej}(\mathcal{AF}_o)$. In both cases we have a contradiction, so the assumption was false. Hence, $(\exists x' \in \mathcal{A}_o)\ (x', a) \in >_o \wedge x' \notin \mathtt{Rej}(\mathcal{AF}_o)$. ■

Property 22 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be a complete argumentation framework for decision making and $a \in \mathcal{A}_o$. If $\mathcal{A}_o \setminus \{a\} \subseteq \mathtt{Rej}(\mathcal{AF}_o)$ then $a \in \mathtt{Sc}(\mathcal{AF}_o)$.

Proof *Suppose that e is not skeptically accepted. Then, e is credulously accepted or rejected.*

1. *Suppose that e is rejected. According to Theorem 21, $(\exists x' \in \mathcal{A}_o)\ x' \notin \mathtt{Rej}(\mathcal{AF}_o)$ and $(x', e) \in >_o$. Contradiction with the fact that all the arguments are rejected.*

2. *Suppose that $e \in \mathtt{Cr}(\mathcal{AF}_o)$. According to Property 19, $((\exists x' \in \mathcal{A})\ (e, x') \notin \geq_o) \wedge ((\forall x \in \mathcal{A})\ ((e, x) \notin \geq_o \Rightarrow (x, e) \notin \geq_o))$. Since there are no self-attacking arguments, we have $x' \neq e$. Since $x' \neq e$ and all the arguments except e are rejected, then x' is rejected. According to Theorem 21, $(\exists y' \in \mathcal{A}_o)$ such that y' is not rejected and $(y', x') \in >_o$. Since y' is not rejected and all the arguments except e are rejected, then $y' = e$. Since $(y', x') \in >_o$ and $y' = e$, then $(e, x') \in >_o$. Since $(e, x') \in >_o$ then $(e, x') \in \geq_o$. Contradiction with the fact $(e, x') \notin \geq_o$.*

■

Property 23 Let $a, b \in \mathtt{Sc}(\mathcal{AF}_o)$. Then $(a, b) \in \geq_o$ and $(b, a) \in \geq_o$.

Proof *It can be shown that the framework has an accepted argument iff it has exactly one extension. Since the system has a skeptically accepted argument, there is exactly one extension $\mathcal{E}$. Since both a and b are accepted, then $a, b \in \mathcal{E}$. Since $\mathcal{E}$ is conflict-free, $(a, b) \notin \mathtt{Def}_o$ and $(b, a) \notin \mathtt{Def}_o$. The fact $(a, b) \notin \mathtt{Def}_o$ implies $(b, a) \in \geq_o$ and, similarly, $(b, a) \notin \mathtt{Def}_o$ implies $(a, b) \in \geq_o$.* ■

Theorem 5 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system. The following implications hold:

1. If $\mathtt{Sc}(\mathcal{AF}_o) \neq \emptyset$ then $\mathtt{Cr}(\mathcal{AF}_o) = \emptyset$.
2. If $\mathtt{Cr}(\mathcal{AF}_o) = \emptyset$ then $\mathtt{Sc}(\mathcal{AF}_o) \neq \emptyset$.

Proof

1. *It can be shown that the framework has an accepted argument iff it has exactly one extension. Since the framework has a skeptically accepted argument, then it has only one extension, say $\mathcal{E}$. Suppose that $(\exists a \in \mathcal{A}_o)$ a is credulously accepted. According to Definition 4, there are two different extensions $\mathcal{E}_1$ and $\mathcal{E}_2$ such that $a \in \mathcal{E}_1$ and $a \notin \mathcal{E}_2$. Contradiction with the fact that there is exactly one extension.*

2. *It can be shown that the argumentation framework $\mathcal{AF}_o$ always has at least one non-empty extension $\mathcal{E}_1$. Let $a \in \mathcal{E}_1$ be an arbitrary argument which belongs to this extension. Since $a \in \mathcal{E}_1$, according to Definition 4, a is skeptically accepted or credulously accepted. Since we have supposed that there are no credulously accepted arguments, then a is skeptically accepted.* ■

Property 24 Let x be an arbitrary argument.
If $(\exists a \in \mathtt{Sc}(\mathcal{AF}_o))$ such that $(a, x) \in \odot$ then $(\forall a' \in \mathtt{Sc}(\mathcal{AF}_o))$ $(a', x) \in \odot$.

Proof *Let us suppose that $((\exists a \in \mathcal{A}_o)$ $a \in \mathtt{Sc}(\mathcal{AF}_o) \wedge (a, x) \in \odot)$. Let b be an arbitrary accepted argument. According to Property 23, $(a, b) \in \geq_o$ and $(b, a) \in \geq_o$. Now, using the transitivity of preference relation, it can easily be shown that $(a, x) \in \odot$ implies $(b, x) \in \odot$.* ■

Property 25 $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system and $\mathtt{Cr}(\mathcal{AF}_o)$ its set of credulously accepted arguments. Then $(\forall a, b \in \mathtt{Cr}(\mathcal{AF}_o)$ it holds that

$$((a, b) \in \geq_o \wedge (b, a) \in \geq_o) \vee ((a, b) \notin \geq_o \wedge (b, a) \notin \geq_o).$$

Proof *Suppose that the $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))(\exists b \in \mathtt{Cr}(\mathcal{AF}_o))$ $\neg((a, b) \in \geq_o \wedge (a, b) \in \geq_o$ $) \wedge \neg((a, b) \notin \geq_o \wedge (a, b) \notin \geq_o)$. Then, either $(a, b) \in >_o$ or $(b, a) \in >_o$ Without loss of generality, we can suppose that $(a, b) \in >_o$. Then, with $(a, b) \in \mathcal{R}_o$, we have $(a, b) \in \mathtt{Def}_o$ and $(b, a) \notin \mathtt{Def}_o$. So, the argument b is not self-defending. Since it can be shown that for an arbitrary argument $a \in \mathcal{A}$, there exists an extension $\mathcal{E}$ such that $a \in \mathcal{E}$ iff a is self-defending, then there is no extension $\mathcal{E}$ such that $b \in \mathcal{E}$. Consequently, b is not credulously accepted. Contradiction with the fact $b \in \mathtt{Cr}(\mathcal{AF}_o)$.* ■

Property 26 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system for decision making, and $\mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. Then it holds that: $(\forall a' \in \mathtt{Cr}(\mathcal{AF}_o))$ $(\exists a'' \in \mathtt{Cr}(\mathcal{AF}_o))$ $(a', a'') \notin \geq_o \wedge (a'', a') \notin \geq_o$.

Proof *Suppose the converse. Then $(\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))$ $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))$ $\neg$ $((a, a') \notin \geq_o \wedge (a', a) \notin \geq_o)$. Recall the result of the Property 25 which states that $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))(\forall b \in \mathtt{Cr}(\mathcal{AF}_o))$ $((a, b) \in \geq_o \wedge (b, a) \in \geq_o) \vee ((a, b) \notin \geq_o$ $\wedge (b, a) \notin \geq_o)$. So, if for two credulously accepted arguments a and a' it holds that $\neg$ $((a, a') \notin \geq_o \wedge (a', a) \notin \geq_o)$, then it must be the case that $((a, a') \in \geq_o \wedge$ $(a', a) \in \geq_o)$. So, $(\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))$ $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))$ $((a, a') \in \geq_o \wedge (a', a) \in \geq_o)$. Let $b, c \in \mathtt{Cr}(\mathcal{AF}_o)$. Since $(b, a') \in \geq_o$ and $(a', c) \in \geq_o$, then, because of the transitivity of the preference relation, $(b, c) \in \geq_o$. Similarly, since $(c, a') \in \geq_o$*

and $(a', b) \in \geq_o$, then $(c, b) \in \geq_o$. So, all the credulously accepted arguments are in the same class of equivalence with respect to $\geq_o$. This means that there is no attack in the sense of $\mathtt{Def}_o$ between arguments of $\mathtt{Cr}(\mathcal{AF}_o)$. So, $\mathtt{Cr}(\mathcal{AF}_o)$ is admissible. Since there are some credulously accepted arguments, according to Definition 4, there are at least two different non-empty preferred extensions $\mathcal{E}_1$ and $\mathcal{E}_2$. Since there are some credulously accepted arguments, then, according to Theorem 5, there are no skeptically accepted arguments. Since all the arguments in $\mathcal{E}_1$ and $\mathcal{E}_2$ are in some extension, they are not rejected. Since there are no skeptically accepted arguments, they are credulously accepted. Since it can be shown that all the extensions are pairwise disjoint, then $\mathcal{E}_1 \cap \mathcal{E}_2 = \emptyset$. All the arguments that are not in $\mathtt{Cr}(\mathcal{AF}_o)$ are not credulously accepted. Since there are no skeptically accepted arguments, they are rejected. Let us prove that $\mathcal{E}_1, \mathcal{E}_2 \subseteq \mathtt{Cr}(\mathcal{AF}_o)$. If $\neg(\mathcal{E}_1 \subseteq \mathtt{Cr}(\mathcal{AF}_o))$ then there is some argument which is credulously accepted (since it is in $\mathcal{E}_1$) and in the same time it is rejected (since it is not in $\mathtt{Cr}(\mathcal{AF}_o)$). Contradiciton. So, $\mathcal{E}_1 \subseteq \mathtt{Cr}(\mathcal{AF}_o)$. The same proof for $\mathcal{E}_2$. So, $\mathcal{E}_1$ and $\mathcal{E}_2$ are preferred extensions and $\mathcal{E}_1 \cap \mathcal{E}_2 = \emptyset$ and $\mathcal{E}_1 \neq \emptyset$ and $\mathcal{E}_2 \neq \emptyset$. Since $\mathcal{E}_2 \neq \emptyset$, then $\mathcal{E}_1 \neq \mathtt{Cr}(\mathcal{AF}_o)$. So, $\mathcal{E}_1$ is preferred and $\mathtt{Cr}(\mathcal{AF}_o)$ is admissible and $\mathcal{E}_1 \subseteq \mathtt{Cr}(\mathcal{AF}_o)$ and $\mathcal{E}_1 \neq \mathtt{Cr}(\mathcal{AF}_o)$. Contradiction, because, according to Definition 3, a preferred extension is a maximal admissible extension. ■

Property 27 Let $o \in \mathcal{O}$. The offer o is negotiable iff there is at least one credulously accepted argument in its favor.

Proof *⇒ Trivial, according to Definition 5.*

⇐ Let a be an credulously accepted argument in favor of o. Since there exists at least one credulously accepted argument, Theorem 5 implies that there are no skeptically accepted arguments. In particular, there are no skeptically accepted arguments in favor of o. According to Definition 5, o is negotiable. ■

Theorem 6 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be a complete argumentation framework for decision making. The following holds: $\mathcal{O}_a \neq \emptyset \Leftrightarrow \mathcal{O}_n = \emptyset$.

Proof *⇒ Let $\mathcal{O}_a \neq \emptyset$. According to Property 20, there is at least one skeptically accepted argument. Then, according to Theorem 5, there are no credulously accepted arguments. Using Property 20, we conclude that there are no negotiable offers.*

⇐ Let $\mathcal{O}_n = \emptyset$. According to Property 20, there are no credulously accepted arguments. Then, according to Theorem 5, there is at least one skeptically accepted argument. The Property 20 implies that there is at least one acceptable offer. ■

Theorem 28 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system. If $a \in \mathtt{Rej}(\mathcal{AF}_o)$, then $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$.

Proof *Let $a \in \mathcal{A}_o$. Assume that a is rejected in $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$. According to Property 19, $\exists x \in \mathcal{A}_o$ such that $(x, a) \in >_o$. Let $e \notin \mathcal{A}_o$. $\mathcal{AF}_o \oplus e$ is an argumentation system such that its set of arguments is $\mathcal{A}_o \cup \{e\}$. So, $a, x \in$*

$\mathcal{A}_o \cup \{e\}$, *which (according to Property 19) means that a is rejected in* $\mathcal{AF}_o \oplus e$. ■

Property 29 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system. If $a \in \mathtt{Cr}(\mathcal{AF}_o)$, then $a \notin \mathtt{Sc}(\mathcal{AF}_o \oplus e)$.

Proof *Assume that a is credulously accepted in* $\mathcal{AF}_o$. *Thus, according to Property 19,* $\exists x \in \mathcal{A}_o$ *such that* $(a, x) \notin \geq_o$. *It is clear that* $a, x \in \mathcal{A}_o \cup \{e\}$. *Assume that a is skeptically accepted in the system* $\mathcal{AF}_o \oplus e$. *According to Property 19,* $(\forall x \in \mathcal{A}_o \cup \{e\})$ $(a, x) \in \geq_o$. *Contradiction with the fact* $(a, x) \notin \geq_o$ ■

Property 30 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system for decision making.

1. If $a \in \mathtt{Sc}(\mathcal{AF}_o)$ then $a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ iff $(a, e) \in \geq_o$.

2. If $a \notin \mathtt{Rej}(\mathcal{AF}_o)$ then $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$ iff $(e, a) \in >_o$.

Proof

1. Let $a \in \mathtt{Sc}(\mathcal{AF}_o)$.
$\Rightarrow$ *Suppose that* $a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ *and* $(a, e) \notin \geq_o$. *Since the attack relation* $\mathcal{R}_o$ *is complete, then* $(a, e) \in \mathcal{R}_o$ *and* $(e, a) \in \mathcal{R}_o$. *With* $(a, e) \notin \geq_o$, *we have* $(e, a) \in \mathtt{Def}_o$. *Since* $(e, a) \in \mathtt{Def}_o$, *according to Property 19, we have that* $a \notin \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. *Contradiction.*

$\Leftarrow$ *Let* $(a, e) \in \geq_o$. *Since* $a \in \mathtt{Sc}(\mathcal{AF}_o)$, *according to Property 19,* $(\forall x \in \mathcal{A}_o)$ $(a, x) \in \geq_o$. *Suppose that* $a \notin \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. *Then, according to Property 19,* $(\exists x' \in \mathcal{A}_o \cup \{e\})$ $(a, x') \notin \geq_o$. *We will prove that* $x' \notin \mathcal{A}_o$. *Suppose the converse, i.e., suppose that* $x' \in \mathcal{A}_o$. *Since* $(\forall x \in \mathcal{A}_o)$ $(a, x) \in \geq_o$, *then* $(a, x') \in \geq_o$. *Contradiction, so it must be the case that* $x' \notin \mathcal{A}_o$. *With* $x' \in \mathcal{A}_o \cup \{e\}$ *and* $x' \notin \mathcal{A}_o$ *we have* $x' = e$, *and, consequently,* $(a, e) \notin \geq_o$. *Contradiction.*

2. Let $a \in A_o \setminus \mathtt{Rej}(\mathcal{AF}_o)$.
$\Rightarrow$ *Let a become rejected. Since* $a \notin \mathtt{Rej}(\mathcal{AF}_o)$, *then, according to Property 19,* $(\nexists x \in \mathcal{A}_o)$ $(x, a) \in >_o$. *Since* $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$, *then, according to Property 19,* $(\exists y \in \mathcal{A}_o \cup \{e\})$ $(y, a) \in >_o$. *We will prove that* $y = e$. *Suppose not. Then,* $y \in \mathcal{A}_o$ *and* $(y, a) \in >_o$. *Contradiction with the fact* $(\nexists x \in \mathcal{A}_o)$ $(x, a) \in >_o$. *So,* $y = e$ *and, consequently,* $(e, a) \in >_o$.

$\Leftarrow$ *Let* $(e, a) \in >_o$. *Since* $(e, a) \in >_o$, *then, according to Property 19, a is rejected.*

■

Property 31 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system and $a, b \in \mathtt{Sc}(\mathcal{AF}_o)$. Let $e \notin \mathcal{A}_o$.

1. If $a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ then $b \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$.

2. If $a \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$ then $b \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$.

3. If $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$ then $b \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$.

Proof

1. *Since $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$, then, according to Property 30, $(a, e) \in \geq_o$. According to Property 24, $(b, e) \in \geq_o$. According to Property 30, $b \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$.*

2. *Since $a \notin \texttt{Sc}(\mathcal{AF}_o \oplus e)$, then, according to Property 30, $(a, e) \notin \geq_o$. Since $a \notin \texttt{Rej}(\mathcal{AF}_o \oplus e)$, then, according to Property 30, $(e, a) \notin >_o$. According to Property 24, $(b, e) \notin \geq_o$ and $(e, b) \notin >_o$. Since $(b, e) \notin \geq_o$, then, according to Property 30, $b \notin \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Since $(e, b) \notin >_o$, then, according to Property 30, we have $b \notin \texttt{Rej}(\mathcal{AF}_o \oplus e)$. Hence, according to Property 2, $b \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$.*

3. *Since $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$, then, according to Property 30, $(e, a) \in >_o$. According to Property 24, $(e, b) \in >_o$. According to Property 30, $b \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$.*

■

Theorem 7 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \texttt{Def}_o \rangle$ be a complete argumentation framework for decision making, $a \in \texttt{Sc}(\mathcal{AF}_o)$ and $e \notin \mathcal{A}_o$. The following holds:

1. $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e) \wedge e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$ iff $((a, e) \in \geq_o) \wedge ((e, a) \in \geq_o)$

2. $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e) \wedge e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$ iff $(e, a) \in >_o$

3. $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e) \wedge e \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$ iff $(a, e) \in >_o$

4. $a \in \texttt{Cr}(\mathcal{AF}_o \oplus e) \wedge e \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$ iff $((a, e) \notin \geq_o) \wedge ((a, e) \notin \geq_o)$

Proof

1. *Let $((a, e) \in \geq_o) \wedge ((e, a) \in \geq_o)$. Let us prove $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Suppose not. So, a changed its status. According to Property 30, $(a, e) \notin \geq_o$. Contradiction. Thus, $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$.*

   We will now prove that $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Suppose not. Then, according to Property 19, $(\exists x' \in \mathcal{A}_o \cup \{e\})$ $(e, x) \notin \geq_o$. Since we proved that $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$, then, according to Property 19, $(\forall x \in \mathcal{A}_o \cup \{e\})$ $(a, x) \in \geq_o$. In particular, $(a, x') \in \geq_o$. Since $(e, a) \in \geq_o$ and $(a, x') \in \geq_o$, the transitivity of the preference relation $\geq_o$ implies that $(e, x') \in \geq_o$. Contradiction. So, $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$.

2. *Let $(e, a) \in >_o$. According to Property 19, it holds that $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$, since there is now at least one argument which is strictly preferred to it.*

   Let us now prove that $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Suppose not. Then, according to Property 19, $(\exists x' \in \mathcal{A}_o \cup \{e\})$ $(e, x') \notin \geq_o$. Since there are no self-attacking arguments, we have $x' \neq e$. So, $x' \in \mathcal{A}_o$. Since $a \in \texttt{Sc}(\mathcal{AF}_o)$, it holds *that* $(\forall x \in \mathcal{A}_o)$ $(a,x) \in \geq_o$. *In particular,* $(a,x') \in \geq_o$. *So,* $(e,a) \in >_o$ *and* $(a,x') \in \geq_o$. *One can easily see that* $(e,x') \in >_o$. *Consequently, we have* $(e,x') \in \geq_o$. *Contradiction with the fact* $(e,x') \notin \geq_o$. *Hence,* $e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$.

3. $(a,e) \in >_o$. *We will prove that* $a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. *Suppose not. So, a changed its status. According to Property 30,* $(a,e) \notin \geq_o$. *Contradiction with the fact* $(a,e) \in >_o$. *So,* $a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$.

   We will now prove that $e \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. *Since* $(a,e) \in >_o$, *then, according to Property 19, it holds that* $e \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$.

4. *Let* $((a,e) \notin \geq_o) \wedge ((a,e) \notin \geq_o)$. *We will prove that* $a \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$. *Suppose that* $a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. *So, according to Property 19,* $(\forall x \in \mathcal{A}_o \cup \{e\})$ $(a,x) \in \geq_o$. *But,* $(a,e) \notin \geq_o$. *Contradiction. So,* $a \notin \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. *Suppose that* $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. *Then, according to Property 19,* $(\exists x' \in \mathcal{A}_o \cup \{e\})$ $(x',a) \in >_o$. $a \in \mathtt{Sc}(\mathcal{AF}_o)$. *So, according to Property 19,* $(\forall x \in \mathcal{A}_o)$ $(a,x) \in \geq_o$. *Suppose that* $x' \in \mathcal{A}_o$. *Then,* $(x',a) \in >_o$ *and* $(a,x') \in \geq_o$. *Contradiction, so* $x' \notin \mathcal{A}_o$. *The fact that* $x' \in \mathcal{A}_o \cup \{e\}$ *and* $x' \notin \mathcal{A}_o$ *implies that* $x' = e$. *So,* $(e,a) \in >_o$. *Contradiction. Hence,* $a \notin \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. *Since we proved that* $a \notin \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ *and* $a \notin \mathtt{Rej}(\mathcal{AF}_o \oplus e)$, *then, according to Property 2* $a \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$.

   Let us now prove that $e \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$. *Suppose that* $e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. *According to Property 19,* $(\forall x \in \mathcal{A}_o)$ $(e,x) \in \geq_o$. *But,* $(e,a) \notin \geq_o$. *Contradiction. So,* $e \notin \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. *Suppose now that* $e \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. *Then, according to Property 19,* $(\exists y' \in \mathcal{A})$ $(y',e) \in >_o$. *Since* $(y',e) \in >_o$ *then* $(e,y') \notin \geq_o$. *Since* $\geq_o$ *is reflexive, then* $y' \neq e$. *So,* $y' \in \mathcal{A}_o$. $a \in \mathtt{Sc}(\mathcal{AF}_o)$. *So, according to Property 19,* $(\forall x \in \mathcal{A}_o)$ $(a,x) \in \geq_o$. *Since* $y' \in \mathcal{A}_o$, *then* $(a,y') \in \geq_o$. *So, we have* $(a,y') \in \geq_o$ *and* $(y',e) \in >_o$. *Now, it is easy to see that* $(a,e) \in >_o$. *Contradiction. Since we proved that* $e \notin \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ *and* $e \notin \mathtt{Rej}(\mathcal{AF}_o \oplus e)$, *then, according to Property 2, it must be the case that e is credulously accepted.*

■

Property 32 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system such that $\mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. The following result holds: $((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \in >_o)$ iff $((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a) \in \geq_o)$.

Proof $\Rightarrow$ *Trivial, according to definition of* $>_o$.
$\Leftarrow$ *Let us suppose that* $(\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))$ $((e,a') \notin >_o \wedge (e,a') \in \geq_o)$. *So, according to definition of* $>_o$, $(a',e) \in \geq_o$. *According to Property 26,* $(\exists a'' \in \mathtt{Cr}(\mathcal{AF}_o))$ $((a',a'') \notin \geq_o \wedge (a'',a') \notin \geq_o)$. *Since* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))$ $(e,a) \in \geq_o$, *then, in particular,* $(e,a'') \in \geq_o$. *With* $(a',e) \in \geq_o$ *and* $(e,a'') \in \geq_o$ *we have* $(a',a'') \in \geq_o$. *Contradiction.* ■

Property 33 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be and argumentation system s.t. $\mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. The following holds: $((\forall a \in \mathtt{Cr}(A_o))\ a \in \mathtt{Rej}(A_o \oplus e))$ iff $((\forall a \in \mathtt{Cr}(A_o))\ (e,a) \in >_o)$.

Proof $\Rightarrow$ *Let all the credulously accepted arguments become rejected. Suppose that* $a' \in \mathtt{Cr}(\mathcal{AF}_o)$. *According to Property 30, since* $a' \in \mathtt{Cr}(\mathcal{AF}_o)$ *and* $a' \in \mathtt{Rej}(\mathcal{A}_o \oplus e)$, *it holds that* $(e, a') \in >_o$.

$\Leftarrow$ *Let* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in >_o$. *Suppose that* $a' \in \mathtt{Cr}(\mathcal{AF}_o)$. *According to Property 19, since* $(e, a') \in >_o$ *then* $a' \in \mathtt{Rej}(\mathcal{A}_o \oplus e)$. ■

Theorem 8 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation framework such that $\mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. Then, the following holds:

1. $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in >_o$ iff $e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e) \land \mathcal{A}_o = \mathtt{Rej}(\mathcal{AF}_o \oplus e)$.
2. $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \notin >_o \land (\nexists a' \in \mathtt{Cr}(\mathcal{AF}_o))$
   $(a', e) \in >_o$ iff $e \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$
3. $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (a, e) \in >_o$ iff $e \in \mathtt{Rej}(\mathcal{AF}_o \oplus e) \land \mathcal{A}_o = \mathtt{Cr}(\mathcal{AF}_o \oplus e)$.

Proof *During the proof, we will sometimes use the following fact. Since, according to Property 32,* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in >_o$ *is equivalent to* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in \geq_o$, *then the negation of* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in >_o$ *is equivalent to negation of* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in \geq_o$. *So,* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \notin >_o$ *is equivalent to* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \notin \geq_o$.

1. $\Rightarrow$ *Let* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in >_o$. *Let* $a \in \mathtt{Cr}(\mathcal{AF}_o)$. *Since* $(e, a) \in >_o$, *then, Property 19 implies that* $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. *So,* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. *Since, according to Property 28, rejected arguments cannot change their status, then* $\mathcal{A}_o \subseteq \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. *So, as the consequence of Property 22, we have that e is skeptically accepted.*

   $\Leftarrow$ *Let* $a \in \mathtt{Cr}(\mathcal{AF}_o)$. *Since* $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$, *then, according to Property 30, it holds that* $(e, a) \in >_o$. *Since* $a \in \mathtt{Cr}(\mathcal{AF}_o)$ *was arbitrary, we have* $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in >_o$.

2. $\Rightarrow$ *Since* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \notin >_o$ *then we have* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \notin \geq_o$. *Since it holds that* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \notin \geq_o$, *then, according to Property 19, e is not skeptically accepted. Since* $(\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'', e) \in >_o$, *then, according to the same property, e is not rejected. Since e is neither skeptically accepted nor rejected, according to Property 2, it is credulously accepted.*

   $\Leftarrow$ *Let e be credulously accepted. Since e is credulously accepted, according to Property 2, it is neither skeptically accepted, nor rejected. Since e is not rejected, then, according to Property 19, it holds that* $(\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'', e) \in >_o$. *Since e is not skeptically accepted, then, according to the same property,* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \notin \geq_o$. *Since* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \notin \geq_o$ *then* $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \notin >_o$.

3. $\Rightarrow$ *Let* $(\exists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'', e) \in >_o$. *According to Property 19, e is rejected. Let us prove that* $\mathtt{Cr}(\mathcal{AF}_o) \subseteq \mathtt{Cr}(\mathcal{AF}_o \oplus e)$. *Suppose not. So,* $(\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))$ *such that* a' *changes its status. Since, according to*

Property 29, no argument can become skeptically accepted, then a' becomes rejected. According to Property 30, it holds that $(e, a') \in >_o$. Since $(a'', e) \in >_o$ and $(e, a') \in >_o$ then $(a'', a') \in >_o$. Since the preference relation between the arguments does not change, this means that $(a'', a') \in >_o$ was true in the moment when a' and a'' were both credulously accepted. Contradiction with Property 25. So, we proved that e is rejected and that no other argument changes its status.

$\Leftarrow$ Let e be rejected. So, according to Theorem 21, $(\exists a' \in \mathcal{A}_o)$ such that $(a', e) \in >_o$ and $a' \notin \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. Since $a' \neq e$ then $a' \in \mathcal{A}_o$. So, $a \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$. Since $a \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$, then, according to Property 28, $a \notin \mathtt{Rej}(\mathcal{AF}_o)$. Since $\mathtt{Sc}(\mathcal{AF}_o) = \emptyset$, then $a \in \mathtt{Cr}(\mathcal{AF}_o)$. So, $(\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))$ $(a', e) \in >_o$.

■

Theorem 9 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system and $o \in \mathcal{O}_a(\mathcal{AF}_o)$. Suppose that $a \in \mathtt{Sc}(\mathcal{AF}_o)$ is an arbitrary skeptically accepted argument. Then:

1. $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ iff
   $((a, e) \in \geq_o) \vee (e \in \mathcal{H}(o)) \wedge ((e, a) \in >_o)$
2. $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ iff
   $((a, e) \notin \geq_o) \wedge ((e, a) \notin \geq_o))$
3. $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ iff
   $(e \notin \mathcal{H}(o)) \wedge (e, a) \in >_o)$

Proof

1. *$\Rightarrow$ According to Definition 5, offer o was acceptable, so there was already at least one skeptically accepted argument a' in its favor before that agent received the argument e. Suppose that the offer o remains acceptable. Since, according to Property 28 and Property 29, no argument can become skeptically accepted, then either some skeptically accepted argument in favor of o remained skeptically accepted or e is skeptically accepted and e is in favor of o. Let us explore the first possibility. So, $\exists a'' \in \mathcal{H}(o) \cap \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. The argument a'' remained skeptically accepted, so, according to Property 31, a will remain skeptically accepted as well. Since $(a'', e) \in \geq_o$ and, according to Property 24, all the skeptically accepted arguments are in the same relation with e, then $(a, e) \in \geq_o$. Suppose now that $e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e) \cap \mathcal{H}(o)$. Since e is skeptically accepted, according to Theorem 7, we have $(e, a) \in \geq_o$. If $(a, e) \in \geq_o$ then the first part of the disjunction is true, i.e., $(a, e) \in \geq_o$. If $(a, e) \notin \geq_o$ then $(e, a) \in >_o$. So, the second part of the disjunction is true, i.e., $(e, a) \in >_o$ $\wedge$ $e \in \mathcal{H}(o)$.*

$\Leftarrow$ *Suppose now that* $(a, e) \in \geq_o \vee ((e, a) \in >_o \wedge e \in \mathcal{H}(o))$. *Suppose that the first part of the disjunction is true, i.e.,* $(a, e) \in \geq_o$. *According to Theorem 7,* $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. *Consequently, o remains acceptable. Suppose now that the second part of the disjunction is true, i.e.,* $(e, a) \in >_o \wedge e \in \mathcal{H}(o)$. *Since* $(e, a) \in >_o$, *then, according to Theorem 7,* $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. *Since* $e \in \mathcal{H}(o)$ *then o is acceptable.*

2. $\Rightarrow$ *Since the offer o becomes negotiable, according to the Definition 5, there is at least one credulously accepted argument in its favor. The Property 28 states that rejected arguments cannot become credulously accepted. So, either an skeptically accepted argument* a' *in favor of o has become credulously accepted or e is credulously accepted and e is in favor of o. The first possibility, with respect to Theorem 7, implies that* $(a, e) \notin \geq_o$ *and* $(e, a) \notin \geq_o$. *The second possibility, according to the same theorem, leads to the same conclusion:* $(a, e) \notin \geq_o$ *and* $(e, a) \notin \geq_o$, *which ends the proof.*

   $\Leftarrow$ *Let* $(a, e) \notin \geq_o \wedge (e, a) \notin \geq_o$. *Theorem 7 together with the fact that* $(a, e) \notin \geq_o \wedge (e, a) \notin \geq_o$ *lead to the conclusion that* $a, e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. *Since we have* $\texttt{Cr}(\mathcal{AF}_o \oplus e) \neq \emptyset$, *according to Property 20,* $\texttt{Sc}(\mathcal{AF}_o \oplus e) = \emptyset$. *So, there will be no skeptically accepted arguments in favor of o, and there will be at least one credulously accepted argument in its favor. According to Definition 5, o becomes negotiable.*

3. $\Rightarrow$ *Let o be an acceptable offer that becomes rejected. The offer o was acceptable, so, according to Definition 5, there were at least one skeptically accepted argument* a' *in its favor. Since o has become rejected, according to the same definition,* $\mathcal{H}(o) \subseteq \texttt{Rej}(\mathcal{AF}_o \oplus e)$, *so* a' *must have become rejected. So,* a' *was not rejected but it is rejected now. Let* a'' *be an arbitrary skeptically accepted argument.* $a' \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$, *so, according to Property 31,* $a'' \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$. *Since* a'' *has become rejected, the Property 30 implies that* $(e, a'') \in >_o$. *Let us now prove that* $e \notin \mathcal{H}(o)$. *Suppose that the converse,* $e \in \mathcal{H}(o)$, *is true. The fact* $(e, a) \in >_o$, *according to Theorem 7, implies that e is skeptically accepted. Since* $e \in \mathcal{H}(o)$, *then there is at least one skeptically accepted argument in favor of the offer o, which, according to Definition 5, contradicts the fact that o became rejected. So, the assumption* $e \in \mathcal{H}(o)$ *is false. Hence,* $e \notin \mathcal{H}(o)$.

   $\Leftarrow$ *Let* $(e, a) \in >_o \wedge e \notin \mathcal{H}(o)$. *The fact* $(e, a) \in >_o$, *according to Theorem 7, implies that* $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$ *and* $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$. *Let* a' *be the arbitrary skeptically accepted argument. According to Property 31,* a' *will become rejected, too. So, an arbitrary skeptically accepted argument becomes rejected. This means that all skeptically accepted arguments will become rejected,* $\texttt{Sc}(\mathcal{AF}_o) \subseteq \texttt{Rej}(\mathcal{AF}_o \oplus e)$. *Since* $\texttt{Sc}(\mathcal{AF}_o) \neq \emptyset$, *according to Theorem 5,* $\texttt{Cr}(\mathcal{AF}_o) = \emptyset$. *According to Property 28, rejected arguments cannot change their status. Since there were no credulously accepted arguments and all skeptically accepted arguments became rejected and all the rejected arguments remain rejected, we conclude that all the arguments except e are rejected,* $\mathcal{A}_o \subseteq \texttt{Rej}(\mathcal{AF}_o \oplus e)$. *Recall that* $e \notin \mathcal{H}(o)$. *All the* *arguments in favor of o are rejected. Since there is at least one argument in favor of o and all the arguments in its favor are rejected, according to Definition 5, o is rejected.*

■

Theorem 10 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be an argumentation system and $o \in \mathcal{O}_n\mathcal{AF}$. Then:

1. $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ iff
   $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathtt{Cr}(\mathcal{A}_o))\ (e, a) \in >)$

2. $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ iff
   $((e \in \mathcal{H}(o)) \wedge (\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a') \notin >_o \wedge$
   $(\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'', e) \in >_o)$
   $\vee$
   $((\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a' \in \mathcal{H}(o) \wedge (e, a') \notin >_o))$

3. $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ iff $((e \notin \mathcal{H}(o)) \wedge$
   $((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (a \in \mathcal{H}(o)) \Rightarrow (e, a) \in >_o))$.

Proof

1. *$\Rightarrow$ Let o become acceptable. According to Definition 5, this means that there will be at least one skeptically accepted argument in its favor. According to Property 28 and Property 29, no argument can become skeptically accepted. So, in order to make o become acceptable, agent must receive a new argument in favor of o. Hence, $e \in \mathcal{H}(o)$ and $e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. Since e is skeptically accepted, then, according to Theorem 5, $\mathtt{Cr}(\mathcal{AF}_o \oplus e) = \emptyset$. So, all the credulously accepted arguments have changed their status. With respect to Property 29, they are all rejected. So, all arguments in $\mathcal{A}_o \setminus \{e\}$ are rejected. Property 22 states that in this case, e must be skeptically accepted. Since $\mathtt{Cr}(\mathcal{AF}_o) \subseteq \mathtt{Rej}(\mathcal{AF}_o \oplus e)$, then, according to Property 33, $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in >_o$.*

   $\Leftarrow$ Let $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in >_o)$. The fact $((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in >_o)$ is, according to Property 33, equivalent to $\mathtt{Cr}(\mathcal{AF}_o) \subseteq \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. So, all the credulously accepted arguments have become rejected. There were no skeptically accepted arguments. According to the Property 28, all the rejected arguments remain rejected. So, all the arguments except e are rejected. According to the Property 22, $e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$. Since $(e \in \mathcal{H}(o))$, then there is exactly one accepted argument in favor of the offer o. According to Definition 5, o is acceptable.

2. *$\Rightarrow$ Let o stay negotiable. According to Property 27, this means that there is at least one credulously accepted argument in favor of o. If $((\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ a' \in \mathcal{H}(o) \wedge (e, a') \notin >_o)$ then that fact ends the proof. Suppose that $((\nexists a \in \mathtt{Cr}(\mathcal{AF}_o))\ a \in \mathcal{H}(o) \wedge (e, a) \notin >_o)$. According to Property 28, all the rejected arguments remain rejected. Since $((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ a \in$*

$\mathcal{H}(o) \Rightarrow (e,a) \in >_o$), this means that for all the credulously accepted arguments in favor of o, it holds that $(e,a) \in >_o$. According to Property 19, this means that all the credulously accepted arguments in favor of o will become rejected. Since o remains negotiable, according to Property 27, this means that there is at least one credulously accepted argument in its favor. So, it must be the case that $e \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$ and $e \in \mathcal{H}(o)$. According to Theorem 8, since e is credulously accepted then $(\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a') \notin >_o \wedge (\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'',e) \in >_o$.

$\Leftarrow$ Let $(e \in \mathcal{H}(o)) \wedge (\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a') \notin >_o \wedge (\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'',e) \in >_o)$ or $((\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ a' \in \mathcal{H}(o) \wedge (e,a') \notin >_o)$. Suppose that $(e \in \mathcal{H}(o)) \wedge (\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (e,a') \notin >_o \wedge (\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'',e) \in >_o)$. According to Theorem 8, $e \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$. Since $e \in \mathcal{H}(o)$, according to the Property 27, o is negotiable. Let us now suppose that $(\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ a' \in \mathcal{H}(o) \wedge (e,a') \notin >_o$ is true. The fact $(e,a') \notin >_o$ and Property 30 imply that $a' \notin \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. Since, according to Property 28 and Property 29, no argument cannot become skeptically accepted, a' is neither rejected nor skeptically accepted. According to Proposition 2, it is credulously accepted. Property 27 implies that o is negotiable.

3. *$\Rightarrow$ Since o becomes rejected, according to Definition 5, this means that $\mathcal{H}(o) \subseteq \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. Suppose that $(\exists a' \in \mathcal{H}(o) \cap \mathtt{Cr}(\mathcal{AF}_o))\ (e,a') \notin >_o$. According to Property 30, $a \notin \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. So, there is at least one argument in favor of o which is not rejected. According to Definition 5, o is not rejected. Contradiction. Suppose now that $e \in \mathcal{H}(o)$. Since o is rejected, then $e \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. Since e is rejected, according to Property 21, $(\exists x' \in \mathcal{A}_o)\ x' \notin \mathtt{Rej}(\mathcal{AF}_o \oplus e)$ and $(x',e) \in >_o$. Since o was negotiable, $\mathcal{H}(o) \cap \mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. Let $a'' \in \mathcal{H}(o) \cap \mathtt{Cr}(\mathcal{AF}_o)$. It holds that $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (a \in \mathcal{H}(o)) \Rightarrow ((e,a) \in >_o)$. In particular, $(e,a'') \in >_o$. It also holds that $(x',e) \in >_o$. From the transitivity of preference relation one can easily conclude that $(x',a'') \in >_o$. So, a'' was not self-defending in $\mathcal{AF}_o$ (before the agent has received the argument e), so $a'' \in \mathtt{Rej}(\mathcal{AF}_o)$. Contradiction. So, $e \notin \mathcal{H}(o)$.*

   $\Leftarrow$ Since $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (a \in \mathcal{H}(o) \Rightarrow (e,a) \in >_o)$, then, as a consequence of Property 30, $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (a \in \mathcal{H}(o)) \Rightarrow a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. So, $\mathtt{Cr}(\mathcal{AF}_o) \cap \mathcal{H}(o) \subseteq \mathtt{Rej}(\mathcal{AF}_o \oplus e)$ and, according to the Property 28, $\mathtt{Rej}(\mathcal{AF}_o) \subseteq \mathtt{Rej}(\mathcal{AF}_o \oplus e)$. So, since $e \notin \mathcal{H}(o)$, all the arguments in favor of o are rejected. Since o was negotiable, then $\mathcal{H}(o) \neq \emptyset$. So, according to Definition 5, o becomes rejected.

■

Theorem 11 Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ be a complete argumentation framework for decision making and $o \in \mathcal{O}$ an rejected offer. Suppose that $e \notin \mathcal{A}_o$. Then:

1. Option o will become acceptable iff
   $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)\ (e,a) \in \geq_o)$

2. Option o will become negotiable iff
   $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)\ (a, e) \notin >_o) \wedge ((\exists a \in \mathcal{A}_o)\ (e, a) \notin >_o)$

3. Option o will rest rejected iff
   $(e \notin \mathcal{H}(o)) \vee ((e \in \mathcal{H}(o)) \wedge (\exists a \in \mathcal{A}_o)(a, e) \in >)$

Proof

1. *$\Rightarrow$ Suppose that offer o becomes acceptable. This means that there is at least one skeptically accepted argument in its favor. Since it was rejected, and, according to Property 28, all rejected arguments remain rejected, it must be that $e \in \mathcal{H}(o)$ and $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Property 19 now implies that $(\forall a \in \mathcal{A}_o)\ (e, a) \in \geq_o$.*

   $\Leftarrow$ Suppose that $e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)\ (e, a) \in \geq_o$. According to Property 19, $e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$. Since $e \in \mathcal{H}(o)$, we have one skeptically accepted argument in favor of offer o, hence it is acceptable.

2. *$\Rightarrow$ Suppose that offer o becomes negotiable. According to Property 27, there is at least one credulously accepted argument in its favor. Since it was rejected, and, according to Property 28, all rejected arguments remain rejected, it must be that $e \in \mathcal{H}(o)$ and $e \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$. From Property 19, we have $((\forall a \in \mathcal{A}_o)\ (a, e) \notin >_o) \wedge ((\exists a \in \mathcal{A}_o)\ (e, a) \notin >_o)$.*

   $\Leftarrow$ Suppose that $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)\ (a, e) \notin >_o) \wedge ((\exists a \in \mathcal{A}_o)\ (e, a) \notin >_o)$. According to Property 19, $e \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$. Since $e \in \mathcal{H}(o)$, we have one credulously accepted argument in favor of offer o, which together with Property 27 means that o is negotiable.

3. *$\Rightarrow$ Suppose that offer o stays rejected. This means that all arguments in its favor are rejected. If $e \notin \mathcal{H}(o)$ the proof is over. Let us suppose that $e \in \mathcal{H}(o)$. Since $e \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$ then Property 19 implies that $(\exists a \in \mathcal{A}_o)(a, e) \in >$.*

   $\Leftarrow$ Let $(e \notin \mathcal{H}(o)) \vee ((e \in \mathcal{H}(o)) \wedge (\exists a \in \mathcal{A}_o)(a, e) \in >)$. If $e \notin \mathcal{H}(o)$, then, according to Property 28, all rejected arguments remain rejected, so the offer remains rejected. If $e \notin \mathcal{H}(o)$ then $(\exists a \in \mathcal{A}_o)(a, e) \in >$. Property 19 implies that e is rejected, so with $\mathcal{H}(o) \subseteq \texttt{Rej}(\mathcal{AF}_o)$ we have that o is rejected.

■