On Revising Argumentation-Based Decision Systems

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Abstract. Decision making amounts to define a preorder (usually a complete one) on a set of options. Argumentation has been introduced in decision making analysis. In particular, an argument-based decision system has been proposed recently by Amgoud et al. The system is a variant of Dung's abstract framework. It takes as input a set of options, different arguments and a defeat relation among them, and returns as outputs a status for each option, and a total preorder on the set of options. The status is defined on the basis of the acceptability of their supporting arguments.
The aim of this paper is to study the revision of this decision system in light of a new argument. We will study under which conditions an option may change its status when a new argument is received and under which conditions this new argument is useless. This amounts to study how the acceptability of arguments evolves when the decision system is extended by new arguments.

1 Introduction

Decision making, often viewed as a form of reasoning toward action, has raised the interest of many scholars including economists, psychologists, and computer scientists for a long time. A decision problem amounts to selecting the "best" or sufficiently "good" action(s) that are feasible among different options, given some available information about the current state of the world and the consequences of potential actions. Available information may be incomplete or pervaded with uncertainty. Besides, the goodness of an action is judged by estimating how much its possible consequences fit the preferences of the decision maker.

Argumentation has been introduced in decision making analysis by several researchers only in the last few years (e.g. [2, 4, 7]). Indeed, in everyday life, decision is often based on arguments and counter-arguments. Argumentation can also be useful for explaining a choice already made. Recently, in [1], a decision model in which the pessimistic decision criterion was articulated in terms of an argumentation process has been proposed. The model is an instantiation of Dung's abstract framework ([6]). It takes as input a set of options, a set of arguments and a defeat relation among arguments. It assigns a status for each option on the basis of the acceptability of its supporting arguments. This paper studies deeply the revision of option status in light of a new argument. This amounts to study how the acceptability of arguments evolves when the decision system is extended by new arguments without computing the whole extensions. All the proofs are in [3].

This paper is organized as follows: Section 2 recalls briefly the decision model proposed in [1]. Section 3 studies the revision of option status when a new argument is received. In section 4 we study the revision of option status under some assumptions on the decision model. The last section concludes.

2 An Argumentation Framework for Decision Making

This section recalls briefly the argument-based framework for decision making that has been proposed in [1].

Let $\mathcal{L}$ denote a logical language. From $\mathcal{L}$, a finite set $\mathcal{O}$ of n distinct *options* is identified. Two kinds of arguments are distinguished: arguments supporting options, called *practical arguments* and arguments supporting beliefs, called *epistemic arguments*. Arguments supporting options are collected in a set $\mathcal{A}_o$ and arguments supporting beliefs are collected in a set $\mathcal{A}_b$ such that $\mathcal{A}_o \cap A_b = \emptyset$ and $\mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_o$. Note that the structure of arguments is assumed not known. Moreover, arguments in $\mathcal{A}_o$ highlight positive features of their conclusions, i.e., they are *in favor* of their conclusions. Practical arguments are linked to the options they support by a function $\mathcal{H}$ defined as follows:

$$\mathcal{H}\colon \mathcal{O} \rightarrow 2^{\mathcal{A}_o} \text{ s.t. } \forall i, j \text{ if } i \neq j \text{ then } \mathcal{H}(o_i) \cap \mathcal{H}(o_j) = \emptyset \text{ and } \mathcal{A}_o = \bigcup_{i=1}^{n} \mathcal{H}(o_i)$$

Each practical argument a supports only one option o. We say also that o is the conclusion of the practical argument a, and we write $\texttt{Conc}(a) = o$. Note that there may exist options that are not supported by arguments (i.e., $\mathcal{H}(o) = \emptyset$).

Example 1. Let us assume a set $\mathcal{O} = \{o_1, o_2, o_3\}$ of three options, a set $\mathcal{A}_b = \{b_1, b_2, b_3\}$ of three epistemic arguments, and finally a set $\mathcal{A}_o = \{a_1, a_2, a_3\}$ of three practical arguments. The arguments supporting the different options are summarized in table below.

$\mathcal{H}(o_1) = \{a_1\}$
$\mathcal{H}(o_2) = \{a_2, a_3\}$
$\mathcal{H}(o_3) = \emptyset$

Three binary relations between arguments have been defined. They express the fact that arguments may not have the same strength. The first preference relation, denoted by $\geq_b$, is a partial preorder[1] on the set $\mathcal{A}_b$. The second relation, denoted by $\geq_o$, is a partial preorder on the set $\mathcal{A}_o$. Finally, a third preorder, denoted by $\geq_m$ (m for *mixed* relation), captures the idea that any epistemic

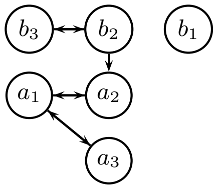
[1] Recall that a relation is a preorder iff it is *reflexive* and *transitive*.

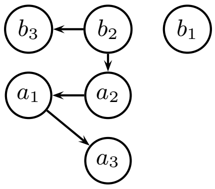
argument is stronger then any practical argument. The role of epistemic arguments in a decision problem is to validate or to undermine the beliefs on which practical arguments are built. Indeed, decisions should be made under certain information. Thus, $(\forall a \in \mathcal{A}_b)(\forall a' \in \mathcal{A}_o)\ (a, a') \in \geq_m \wedge\ (a', a) \notin \geq_m$. Note that $(a, a') \in \geq_x$ with $x \in \{b, o, m\}$ means that a is *at least as good as* a'. In what follows, $>_x$ denotes the strict relation associated with $\geq_x$. It is defined as follows: $(a, a') \in >_x$ iff $(a, a') \in \geq_x$ and $(a', a) \notin \geq_x$. We will sometimes write $(a, a') \in \odot$ to refer to one of the four possible situations: $(a, a') \in \geq_x \wedge (a', a) \in \geq_x$, meaning that the two arguments a and a' are *indifferent* for the decision maker, $(a, a') \in >_x$, meaning that a is *strictly preferred* to a', $(a', a) \in >_x$, meaning that a' is strictly preferred to a, $(a, a') \notin \geq_x \wedge (a', a) \notin \geq_x$, meaning that the two arguments are *incomparable*.
Generally arguments may be conflicting. These conflicts are captured by a binary relation on the set of arguments. Three such relations are distinguished. The first one, denoted by $\mathcal{R}_b$ captures the different conflicts between epistemic arguments. The second relation, denoted $\mathcal{R}_o$ captures the conflicts among practical arguments. Two practical arguments are conflicting if they support different options. Formally, $(\forall a, b \in \mathcal{A}_o)\ (a, b) \in \mathcal{R}_o$ iff $\texttt{Conc}(a) \neq \texttt{Conc}(b)$. Finally, practical arguments may be attacked by epistemic ones. The idea is that an epistemic argument may undermine the belief part of a practical argument. However, practical arguments are not allowed to attack epistemic ones. This avoids wishful thinking, i.e., avoids making decisions according to what might be pleasing to imagine instead of by appealing to evidence or rationality. This relation, denoted by $\mathcal{R}_m$, contains pairs (a, a') where $a \in \mathcal{A}_b$ and $a' \in \mathcal{A}_o$. Before introducing the framework, we need first to combine each preference relation $\geq_x$ (with $x \in \{b, o, m\}$) with the conflict relation $\mathcal{R}_x$ into a unique relation between arguments, denoted $\texttt{Def}_x$, and called *defeat* relation.

Definition 1. *(Defeat relation) Let* $a, b \in \mathcal{A}$. $(a, b) \in \texttt{Def}_x$ *iff* $(a, b) \in \mathcal{R}_x$ *and* $(b, a) \notin \geq_x$.

Let $\texttt{Def}_b$, $\texttt{Def}_o$ and $\texttt{Def}_m$ denote the three defeat relations corresponding to three attack relations. Since arguments in favor of beliefs are always preferred (in the sense of $\geq_m$) to arguments in favor of options, it holds that $\mathcal{R}_m = \texttt{Def}_m$.

Example 2. (Example 1 cont.) The graph on the left depicts different attacks among arguments. Let us assume the following preferences: $(b_2, b_3) \in \geq_b$, $(a_2, a_1) \in \geq_o$ and $(a_1, a_3) \in \geq_o$. The defeats are depicted on the right of figure below.





The different arguments of $\mathcal{A}_b \cup \mathcal{A}_o$ are evaluated within the system $\mathcal{AF} = \langle \mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_o, \texttt{Def} = \texttt{Def}_b \cup \texttt{Def}_o \cup \texttt{Def}_m \rangle$ using any Dung's acceptability semantics.

Definition 2. *(Conflict-free, Defense) Let* $\langle \mathcal{A}, \texttt{Def} \rangle$ *be an argumentation system*[2], $\mathcal{B} \subseteq \mathcal{A}$, *and* $a \in \mathcal{A}$.

– $\mathcal{B}$ *is* conflict-free *iff* $\nexists$ a, $b \in \mathcal{B}$ *s.t.* $(a, b) \in \texttt{Def}$.
– $\mathcal{B}$ defends a *iff* $\forall$ $b \in \mathcal{A}$, *if* $(b, a) \in \texttt{Def}$, *then* $\exists$ $c \in \mathcal{B}$ *s.t.* $(c, b) \in \texttt{Def}$.

The main semantics introduced by Dung are recalled in the following definition.

Definition 3. *(Acceptability semantics) Let* $\mathcal{AF} = \langle \mathcal{A}, \texttt{Def} \rangle$ *be an argumentation system, and* $\mathcal{E}$ *be a conflict-free set of arguments.*

– $\mathcal{E}$ *is a* preferred extension *iff* $\mathcal{E}$ *is a maximal (w.r.t set* $\subseteq$*) set that defends any element in* $\mathcal{E}$.
– $\mathcal{E}$ *is a* grounded extension, *denoted* GE, *iff* $\mathcal{E}$ *is the least fixpoint of function* $\mathcal{F}$ *where* $\mathcal{F}(S) = \{a \in \mathcal{A} \mid S \text{ defends } a\}$, *for* $S \subseteq \mathcal{A}$.

Using these acceptability semantics, the status of each argument can be defined.

Definition 4. *(Argument status) Let* $\mathcal{AF} = \langle \mathcal{A}, \texttt{Def} \rangle$ *be an argumentation system,* $\mathcal{E}_1, \ldots, \mathcal{E}_x$ *its extensions under a given semantics and let* $a \in \mathcal{A}$.

– a *is* skeptically accepted *iff exists at least one extension and* $(\forall \mathcal{E}_i)$ $a \in \mathcal{E}_i$.
– a *is* credulously accepted *iff* $(\exists \mathcal{E}_i)$ *s.t.* $a \in \mathcal{E}_i$ *and* $(\exists \mathcal{E}_j)$ *s.t.* $a \notin \mathcal{E}_j$.
– a *is* rejected *iff* $(\nexists \mathcal{E}_i)$ *s.t.* $a \in \mathcal{E}_i$.

Example 3. (Example 1 cont.) There is one preferred extension, which is also the grounded one, $\{a_1, b_1, b_2\}$. It is clear that a_1, b_1 and b_2 are skeptically accepted while other arguments are rejected.

Let $\mathcal{AF} = \langle \mathcal{A}, \texttt{Def} \rangle$ be an argumentation system. $\texttt{Sc}(\mathcal{AF})$, $\texttt{Cr}(\mathcal{AF})$ and $\texttt{Rej}(\mathcal{AF})$ denote respectively the sets of skeptically accepted arguments, credulously accepted arguments and rejected arguments of the system $\mathcal{AF}$. It can be shown that these three sets are disjoint. Moreover, their union is the set $\mathcal{A}$ of arguments.

Proposition 1. *Let* $\mathcal{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ *be an argumentation system and* $\texttt{Sc}(\mathcal{AF})$, $\texttt{Cr}(\mathcal{AF})$, $\texttt{Rej}(\mathcal{AF})$, *its sets of arguments.*

1. $\texttt{Sc}(\mathcal{AF}) \cap \texttt{Cr}(\mathcal{AF}) = \emptyset$, $\texttt{Sc}(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF}) = \emptyset$, $\texttt{Cr}(\mathcal{AF}) \cap \texttt{Rej}(\mathcal{AF}) = \emptyset$
2. $\texttt{Sc}(\mathcal{AF}) \cup \texttt{Cr}(\mathcal{AF}) \cup \texttt{Rej}(\mathcal{AF}) = \mathcal{A}$.

The status of an option is defined from the status of its arguments.

Definition 5. *(Option status) Let* $o \in \mathcal{O}$.

– o *is* acceptable *iff* $\exists a \in \mathcal{H}(o)$ *s.t.* $a \in \texttt{Sc}(\mathcal{AF})$.
– o *is* rejected *iff* $\mathcal{H}(o) \neq \emptyset$ *and* $\forall a \in \mathcal{H}(o)$, $a \in \texttt{Rej}(\mathcal{AF})$.
– o *is* negotiable *iff* $(\nexists a \in \mathcal{H}(o))$ $(a \in \texttt{Sc}(\mathcal{AF})) \wedge (\exists a' \in \mathcal{H}(o))$ $(a' \in \texttt{Cr}(\mathcal{AF}))$.
– o *is* non-supported *iff it is neither acceptable, nor rejected nor negotiable.*

Let $\mathcal{O}_a$ *(resp.* $\mathcal{O}_n$, $\mathcal{O}_{ns}$, $\mathcal{O}_r$*) be the set of acceptable (resp. negotiable, non-supported, rejected) options.*

[2] At some places, it will be referred to as a decision system.

Example 4. (Example 1 cont.) Option o_1 is acceptable, o_2 is rejected and o_3 is non-supported.

It can be checked that an option has only one status. This status may change in light of new arguments as we will show in next sections.

Proposition 2. *Let $o \in \mathcal{O}$. o has exactly one status.*

The choice of a semantics has an impact on the acceptability of arguments and, consequently, on the status of options. We have studied the impact of several semantics on the status of options. However, due to lack of space, we present only the results related to preferred and grounded semantics. Let $\mathcal{O}^x_y$ denote the set of options having status y under semantics x.

Proposition 3. *It holds that: $O^g_a \subseteq O^p_a$, $\mathcal{O}^p_r \subseteq \mathcal{O}^g_r$, $\mathcal{O}^p_{ns} = \mathcal{O}^g_{ns}$ and $\mathcal{O}^g_n = \emptyset$.*

In [1], the status of options makes it possible to compare them, thus to define a preference relation $\succeq$ on O. The basic idea is the following: acceptable options are preferred to negotiable ones. Negotiable options are themselves preferred to non-supported options, which in turn are better than rejected options.

3 Revising Option Status

Given a decision system $\mathcal{AF} = \langle \mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_o, \texttt{Def} = \texttt{Def}_b \cup \texttt{Def}_o \cup \texttt{Def}_m \rangle$ that defines a preorder on a set $\mathcal{O}$ of options, we study how the status of each option in $\mathcal{O}$ may change when a new argument is added to the set $\mathcal{A}$ of arguments. In this paper, we investigate the case where the new argument, say e, is practical. Let $\mathcal{AF} \oplus e = \langle \mathcal{A}', \texttt{Def}' \rangle$ denote the new decision system. When $e \in \mathcal{A}$, $\mathcal{A}' = \mathcal{A}$ and $\texttt{Def}' = \texttt{Def}$, all the arguments and all the options keep their original status (i.e., the one computed with $\mathcal{AF}$). Things are different when $e \notin \mathcal{A}$. In this case, $\mathcal{A}' = \mathcal{A} \cup \{e\}$ and $\texttt{Def}' = \texttt{Def} \cup \{(x, e) \mid x \in \mathcal{A}_b$ and $(x, e) \in \mathcal{R}^{\mathcal{L}}_m\}$[3] $\cup \{(e, y) \mid y \in \mathcal{A}_o$ and $\texttt{Conc}(y) \neq \texttt{Conc}(e)$ and $(y, e) \notin \geq_o\} \cup \{(y, e) \mid y \in \mathcal{A}_o$ and $\texttt{Conc}(y) \neq \texttt{Conc}(e)$ and $(e, y) \notin \geq_o\}$. Throughout the paper, we assume that $e \notin \mathcal{A}_o$. In this section we will use *grounded semantics* to compute acceptability of arguments. We will denote by $\mathcal{O}_x(\mathcal{AF})$, with $x \in \{a, r, ns\}$, the set of acceptable (resp. rejected and non-supported) options of the original system $\mathcal{AF}$ and $\mathcal{O}_x(\mathcal{AF} \oplus e)$ the corresponding sets of the new system. For example, $\mathcal{O}_r(\mathcal{AF} \oplus e)$ is the set of rejected options when argument e is added to the system $\mathcal{AF}$.
In this section, we will study the properties of an argument that can change the status of an option. For that purpose, we start by studying when an accepted argument in the system $\mathcal{AF}$ remains accepted (resp. becomes rejected) in $\mathcal{AF} \oplus e$. Then, we show under which conditions an option in $\mathcal{O}_x(\mathcal{AF})$ will move to $\mathcal{O}_y(\mathcal{AF} \oplus e)$ with $x \neq y$.

[3] $\mathcal{R}^{\mathcal{L}}_m$ contains all the attacks from epistemic arguments to practical arguments of a logical language $\mathcal{L}$.

The first results states that a new practical arguments e will have no impact on existing epistemic arguments. This is due to the fact that a practical argument is not allowed to attack an epistemic one. Formally:

Proposition 4. *Let e be a new practical argument. It holds that* $\mathtt{Sc}(\mathcal{AF} \oplus e) \cap \mathcal{A}_b = \mathtt{Sc}(\mathcal{AF}) \cap \mathcal{A}_b$.

This result in not necessarily true for the practical arguments of the set $\mathcal{A}_o$. However, this can be the case when the new argument is defeated by a skeptically accepted epistemic argument. In this case, the argument e is clearly useless.

Proposition 5. *Let e be a new practical argument. If* $(\exists a \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF}))$ *such that* $(a, e) \in \mathtt{Def}$ *then* $\mathtt{Sc}(\mathcal{A} \oplus e) \cap \mathcal{A}_o = \mathtt{Sc}(\mathcal{AF}) \cap \mathcal{A}_o$.

From the two above propositions, the following trivial result holds:

Proposition 6. *Let e be a new practical argument. If* $(\exists a \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF}))$ *such that* $(a, e) \in \mathtt{Def}$ *then* $\mathtt{Sc}(\mathcal{A} \oplus e) = \mathtt{Sc}(\mathcal{AF})$.

It can be shown that each skeptically accepted practical argument can be defended either by an epistemic argument or by another practical argument that supports the same option. Before presenting formally this result, let us first introduce a notation. Recall that $\mathtt{Sc}(\mathcal{AF}) = \bigcup_{i=1}^{\infty} \mathcal{F}^{(i)}(\emptyset)$. Let $\mathtt{Sc}^1(\mathcal{AF}) = \mathcal{F}(\emptyset)$ and let $(\forall i \in \{2, 3, \ldots\})$ $\mathtt{Sc}^i(\mathcal{AF})$ denote $\mathcal{F}^{(i)}(\emptyset) \setminus \mathcal{F}^{(i-1)}(\emptyset)$, i.e., the arguments reinstated at step i.

Proposition 7. *Let $o \in \mathcal{O}$, $a_i \in \mathcal{H}(o)$, $a_i \in \mathtt{Sc}^i(\mathcal{AF})$ and $x \in A$ such that* $(x, a_i) \in \mathtt{Def}$.

1. *If $x \in \mathcal{A}_b$ then* $(\exists j \geq 1)$ $(j < i) \wedge (\exists a_j \in \mathcal{A}_b \cap \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j, x) \in \mathtt{Def}$,
2. *If $x \in \mathcal{A}_o$ then* $(\exists j \geq 1)$ $(j < i) \wedge (\exists a_j \in (\mathcal{A}_b \cup \mathcal{H}(o)) \cap \mathtt{Sc}^j(\mathcal{AF}))$ $(a_j, x) \in \mathtt{Def}$.

The following result states that a new practical argument will never influence the accepted arguments supporting the same option as the new argument e.

Theorem 1. *Let e be a new argument such that* $\mathtt{Conc}(e) = o$. *Then,* $(\forall a \in \mathcal{H}(o))$ $a \in \mathtt{Sc}(\mathcal{AF}) \Rightarrow a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$.

We can also show that if the new practical argument e induces a change in the status of a given practical argument from rejection to acceptance, then this argument supports the same option as e. This means that a new practical argument can improve the status of arguments supporting its own conclusion, thus it can improve the status of option it supports. However, it can never improve the status of other options.

Theorem 2. *Let $o \in \mathcal{O}$, and $a \in \mathcal{H}(o)$. If $a \in \mathtt{Rej}(\mathcal{AF})$ and $a \in \mathtt{Sc}(\mathcal{AF} \oplus e)$, then $e \in \mathcal{H}(o)$.*

Before continuing with the results on the revision of the status of options, let us define the set of arguments defended by epistemic arguments in $\mathcal{AF}$.

Definition 6. *(Defense by epistemic arguments) Let* $\mathcal{AF} = \langle \mathcal{A}, \mathtt{Def} \rangle$ *be an argumentation system and* $a \in \mathcal{A}$. *We say that* a *is defended by epistemic arguments in* $\mathcal{AF}$ *and we write* $a \in \mathtt{Dbe}(\mathcal{AF})$ *iff* $(\forall x \in \mathcal{AF})\ (x, a) \in \mathtt{Def} \Rightarrow (\exists \alpha \in \mathtt{Sc}(\mathcal{AF}) \cap \mathcal{A}_b)\ (\alpha, x) \in \mathtt{Def}$.

Note that, since elements of $\mathtt{Sc}^1(\mathcal{AF})$ are not attacked at all, they are also defended by epistemic arguments, i.e., $\mathtt{Sc}^1(\mathcal{AF}) \subseteq \mathtt{Dbe}(\mathcal{AF})$. We can prove that the set of arguments defended by epistemic arguments is skeptically accepted.

Proposition 8. *It holds that* $\mathtt{Dbe}(\mathcal{AF}) \subseteq \mathtt{Sc}(\mathcal{AF})$.

Given an option which is accepted in the system $\mathcal{AF}$, it becomes rejected in $\mathcal{AF} \oplus e$ if three conditions are satisfied: e is not in favor of the option o, there is no skeptically accepted epistemic argument that defeats e, and e defeats all the arguments in favor of option o that are defended by epistemic arguments.

Theorem 3. *Let* $o \in \mathcal{O}_a(\mathcal{AF})$ *and let agent receive new practical argument* e. *Then:* $o \in \mathcal{O}_r(\mathcal{AF} \oplus e)$ *iff* $e \notin \mathcal{H}(o) \wedge (\nexists x \in \mathcal{A}_b \cap \mathtt{Sc}(\mathcal{AF}))\ (x, e) \in \mathtt{Def} \wedge (\forall a \in \mathtt{Dbe}(\mathcal{AF}) \cap \mathcal{H}(o))\ (e, a) \in \mathtt{Def}$.

This result is important in a negotiation. It shows the properties of a good argument that may kill an option that is not desirable for an agent.

Similarly, we can show that it is possible for an option to move from a rejection to an acceptance. The idea is to send a practical argument that supports this option and that is accepted in the new system. Formally:

Theorem 4. *Let* $o \in \mathcal{O}_r(\mathcal{AF})$ *and let agent receive new practical argument* e. *Then:* $o \in \mathcal{O}_a(\mathcal{AF} \oplus e)$ *iff* $e \in \mathcal{H}(o) \wedge e \in \mathtt{Sc}(\mathcal{AF} \oplus e)$.

4 Revising Complete Decision Systems

So far, we have analyzed how an argument may change its status when a new practical argument is received, and similarly how an option may change its status without computing the new grounded extension. The decision system that is used assumes that an option may be supported by several arguments, each of them pointing out to a particular goal satisfied by the option. In some works on argument-based decision making, an argument in favor of an option refers to all the goals satisfied by that option. Thus, there is one argument per option. A consequence of having one argument per option is that all the practical arguments are conflicting. In this section, we will use this particular system, but we will allow multiple arguments in favor of an option under the condition that they all attack each other in the sense of $\mathcal{R}$. We assume also that the set of epistemic arguments is empty. The argumentation system that is used is then $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$, and we will use preferred semantics for computing the acceptability of arguments.

In this system, the status of each argument can be characterized as follows:

Proposition 9. *Let* $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ *be a complete argumentation framework for decision making, and* a *be an arbitrary argument. Then:*

1. *a is skeptically accepted iff* $(\forall x \in \mathcal{A}_o)\ (a, x) \in \geq_o$.
2. *a is rejected iff* $(\exists x \in \mathcal{A})\ (x, a) \in >_o$.
3. *a is credulously accepted iff* $((\exists x' \in \mathcal{A})\ (a, x') \notin \geq_o) \wedge ((\forall x \in \mathcal{A})\ ((a, x) \notin \geq_o) \Rightarrow (x, a) \notin \geq_o))$.

It can be checked that all skeptically accepted arguments in this system are equally preferred.

Proposition 13. *Let* $a, b \in \texttt{Sc}(\mathcal{AF}_o)$. *Then* $(a, b) \in \geq_o$ *and* $(b, a) \in \geq_o$.

We will now prove that in this particular system, there are two possible cases: the case where there exists at least one skeptically accepted argument but there are no credulously accepted arguments, and the case where there are no skeptically accepted arguments but there is "at least" one credulously accepted argument. This means that one cannot have a state with both skeptically accepted and credulously accepted arguments. Moreover, it cannot be the case that all the arguments are rejected. Formally:

Theorem 5. *Let* $\mathcal{AF}_o = \langle \mathcal{A}_o, \texttt{Def}_o \rangle$ *be an argumentation system. The following implications hold:*

1. *If* $\texttt{Sc}(\mathcal{AF}_o) \neq \emptyset$ *then* $\texttt{Cr}(\mathcal{AF}_o) = \emptyset$.
2. *If* $\texttt{Cr}(\mathcal{AF}_o) = \emptyset$ *then* $\texttt{Sc}(\mathcal{AF}_o) \neq \emptyset$.

We will now show that an arbitrary argument e is in the same relation with all accepted arguments. Recall that we use the notation $(e, a) \in \odot$ to refer to one particular relation between the arguments e and a.

Proposition 14. *Let e be an arbitrary argument.*
If $(\exists a \in \texttt{Sc}(\mathcal{AF}_o))$ *such that* $(a, e) \in \odot$ *then* $(\forall a' \in \texttt{Sc}(\mathcal{AF}_o))\ (a', e) \in \odot$.

Let us now have a look at credulously accepted arguments. While all the skeptically accepted arguments are in the same class with respect to the preference relation $\geq_o$, this is not always the case with credulously accepted arguments. The next proposition shows that credulously accepted arguments are either incomparable or indifferent with respect to $\geq_o$.

Proposition 15. $\mathcal{AF}_o = \langle \mathcal{A}_o, \texttt{Def}_o \rangle$ *be an argumentation system and* $\texttt{Cr}(\mathcal{AF}_o)$ *its set of credulously accepted arguments. Then* $(\forall a, b \in \texttt{Cr}(\mathcal{AF}_o)$ *it holds that*

$$((a, b) \in \geq_o \wedge (b, a) \in \geq_o) \vee ((a, b) \notin \geq_o \wedge (b, a) \notin \geq_o).$$

The next proposition shows that if a' is credulously accepted then there exists another credulously accepted argument a'' such that they are incomparable in the sense of preference relation.

Proposition 16. *Let* $\mathcal{AF}_o = \langle \mathcal{A}_o, \texttt{Def}_o \rangle$ *be an argumentation system for decision making, and* $\texttt{Cr}(\mathcal{AF}_o) \neq \emptyset$. *Then it holds that:* $(\forall a' \in \texttt{Cr}(\mathcal{AF}_o))\ (\exists a'' \in \texttt{Cr}(\mathcal{AF}_o))\ (a', a'') \notin \geq_o \wedge (a'', a') \notin \geq_o$.

The next proposition will make some reasoning easier, because it shows that, in this particular framework, the definition of negotiable options can be simplified.

Proposition 17. *Let $o \in \mathcal{O}$. The option o is negotiable iff there is at least one credulously accepted argument in its favor.*

As a consequence of the above propositions, the following result shows that negotiable options and acceptable ones cannot exist at the same time.

Theorem 6. *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \texttt{Def}_o \rangle$ be a complete argumentation framework for decision making. The following holds: $\mathcal{O}_a \neq \emptyset \Leftrightarrow \mathcal{O}_n = \emptyset$.*

4.1 Revising the Status of an Argument

Like in the previous section, we assume that an agent receives a new practical argument e. The question is, how the status of an argument given by the system $\mathcal{AF}_o$ may change in the system $\mathcal{AF} \oplus e$ without having to compute the preferred extensions of $\mathcal{AF}_o \oplus e$.

The first result states that rejected arguments in $\mathcal{AF}_o$ remain rejected in the new system $\mathcal{AF}_o \oplus e$. This means that rejected arguments cannot be "saved".

Proposition 18. *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \texttt{Def}_o \rangle$ be an argumentation system. If $a \in \texttt{Rej}(\mathcal{AF}_o)$, then $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$.*

We can also show that an argument that was credulously accepted in $\mathcal{AF}_o$ can never become skeptically accepted in $\mathcal{AF}_o \oplus e$. It can either remain credulously accepted, either become rejected.

Proposition 19. *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \texttt{Def}_o \rangle$ be an argumentation system. If $a \in \texttt{Cr}(\mathcal{AF}_o)$, then $a \notin \texttt{Sc}(\mathcal{AF}_o \oplus e)$.*

The next proposition is simple but will be very useful later in this section.

Proposition 20. *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \texttt{Def}_o \rangle$ be a decision system.*

1. *If $a \in \texttt{Sc}(\mathcal{AF}_o)$ then $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$ iff $(a, e) \in \geq_o$.*
2. *If $a \notin \texttt{Rej}(\mathcal{AF}_o)$ then $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$ iff $(e, a) \in >_o$.*

The next proposition shows that all the skeptically accepted arguments will have the "same destiny" when a new argument is recieved.

Proposition 21. *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \texttt{Def}_o \rangle$ be an argumentation system and $a, b \in \texttt{Sc}(\mathcal{AF}_o)$. Let $e \notin \mathcal{A}_o$.*

1. *If $a \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$ then $b \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$.*
2. *If $a \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$ then $b \in \texttt{Cr}(\mathcal{AF}_o \oplus e)$.*
3. *If $a \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$ then $b \in \texttt{Rej}(\mathcal{AF}_o \oplus e)$.*

The next theorem analyzes the status of all skeptically accepted arguments after a new argument has arrived.

Theorem 7. *Let $\mathcal{AF}_o = \langle \mathcal{A}_o, \texttt{Def}_o \rangle$ be a complete argumentation framework for decision making, $a \in \texttt{Sc}(\mathcal{AF}_o)$ and $e \notin \mathcal{A}_o$. The following holds:*

1. *$a \in \texttt{Sc}(\mathcal{AF}_o \oplus e) \wedge e \in \texttt{Sc}(\mathcal{AF}_o \oplus e)$ iff $((a, e) \in \geq_o) \wedge ((e, a) \in \geq_o)$*

2. $a \in \mathtt{Rej}(\mathcal{AF}_o \oplus e) \wedge e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e)$ *iff* $(e, a) \in >_o$
3. $a \in \mathtt{Sc}(\mathcal{AF}_o \oplus e) \wedge e \in \mathtt{Rej}(\mathcal{AF}_o \oplus e)$ *iff* $(a, e) \in >_o$
4. $a \in \mathtt{Cr}(\mathcal{AF}_o \oplus e) \wedge e \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$ *iff* $((a, e) \notin \geq_o) \wedge ((a, e) \notin \geq_o)$

Note that, according to Proposition 14, all skeptically accepted arguments are in the same relation with e as a is. Formally, if a and e are in a particular relation i.e., $(a, e) \in \odot$, then $(\forall b \in \mathcal{A}_o)$ $((b \in \mathtt{Sc}(\mathcal{AF}_o)) \Rightarrow (b, e) \in \odot)$. Hence, the condition "let $a \in \mathtt{Sc}(\mathcal{AF}_o)$ and $(a, e) \in \odot$" in the previous theorem is equivalent to the condition $(\forall a \in \mathcal{A}_o)$ $((a \in \mathtt{Sc}(\mathcal{AF}_o)) \Rightarrow (a, e) \in \odot)$.

Theorem 7 stands as a basic tool for reasoning about the status of new arguments as well as about the changes in the status of other arguments. Once the argument status is known, it is much easier to determine the status of options.

We will now analyze the relation between credulously accepted arguments and new arguments. The next result shows that if there are credulously accepted arguments in $\mathcal{AF}_o$ and the new argument e is preferred to all of them, then it is strictly preferred to all of them.

Proposition 22. *Let* $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ *s.t.* $\mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. *The following result holds:* $((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))$ $(e, a) \in >_o)$ *iff* $((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))$ $(e, a) \in \geq_o)$.

Proposition 23. *Let* $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ *s.t.* $\mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. *The following holds:* $((\forall a \in \mathtt{Cr}(A_o))$ $a \in \mathtt{Rej}(A_o \oplus e))$ *iff* $((\forall a \in \mathtt{Cr}(A_o))$ $(e, a) \in >_o)$.

The next theorem analyzes the case when there are no skeptically accepted arguments in $\mathcal{AF}_o$.

Theorem 8. *Let* $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ *be an argumentation framework such that* $\mathtt{Cr}(\mathcal{AF}_o) \neq \emptyset$. *Then, the following holds:*

1. $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))$ $(e, a) \in >_o$ *iff* $e \in \mathtt{Sc}(\mathcal{AF}_o \oplus e) \wedge \mathcal{A}_o = \mathtt{Rej}(\mathcal{AF}_o \oplus e)$.
2. $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))$ $(e, a) \notin >_o \wedge (\nexists a' \in \mathtt{Cr}(\mathcal{AF}_o))$
$(a', e) \in >_o$ *iff* $e \in \mathtt{Cr}(\mathcal{AF}_o \oplus e)$
3. $(\exists a \in \mathtt{Cr}(\mathcal{AF}_o))$ $(a, e) \in >_o$ *iff* $e \in \mathtt{Rej}(\mathcal{AF}_o \oplus e) \wedge \mathcal{A}_o = \mathtt{Cr}(\mathcal{AF}_o \oplus e)$.

Recall that, according to Proposition 22, the condition $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))$ $(e, a) \in >_o$ in the previous theorem is equivalent to the condition $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))$ $(e, a) \in \geq_o$. While all the skeptically accepted arguments have the "same destiny" after a new argument arrives, this is not the case with credulously accepted arguments. Some of them may remain credulously accepted while the others may become rejected.

4.2 Revising the Status of an Option

We will now show under which conditions an option can change its status. We start by studying acceptable options.

Theorem 9. *Let* $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ *be an argumentation system and* $o \in \mathcal{O}_a(\mathcal{AF}_o)$. *Suppose that* $a \in \mathtt{Sc}(\mathcal{AF}_o)$ *is an arbitrary skeptically accepted argument. Then:*

1. $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ *iff* $((a, e) \in \geq_o) \vee (e \in \mathcal{H}(o)) \wedge ((e, a) \in >_o)$
2. $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ *iff* $((a, e) \notin \geq_o) \wedge ((e, a) \notin \geq_o))$
3. $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ *iff* $(e \notin \mathcal{H}(o)) \wedge (e, a) \in >_o)$

Recall that, according to Proposition 14, all skeptically accepted arguments are in the same relation with an arbitrary argument. Hence, the condition $(\exists a \in \mathtt{Sc}(\mathcal{AF}_o))\ (a, e) \in \odot)$ in the previous theorem is equivalent to the condition $(\forall a \in \mathtt{Sc}(\mathcal{AF}_o))\ (a, e) \in \odot)$.

A similar characterization is given bellow for negotiable options.

Theorem 10. *Let* $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ *be an argumentation system and* $o \in \mathcal{O}_n\mathcal{AF}$. *Then:*

1. $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ *iff* $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathtt{Cr}(\mathcal{A}_o))\ (e, a) \in >)$
2. $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ *iff* $((e \in \mathcal{H}(o)) \wedge (\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a') \notin >_o \wedge (\nexists a'' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a'', e) \in >_o) \vee ((\exists a' \in \mathtt{Cr}(\mathcal{AF}_o))\ (a' \in \mathcal{H}(o) \wedge (e, a') \notin >_o))$
3. $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ *iff* $((e \notin \mathcal{H}(o)) \wedge ((\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (a \in \mathcal{H}(o)) \Rightarrow (e, a) \in >_o))$.

Note that, according to Proposition 22, the condition $(\forall a \in \mathtt{Cr}(\mathcal{A}_o))\ (e, a) \in >$ in the previous theorem is equivalent to condition $(\forall a \in \mathtt{Cr}(\mathcal{AF}_o))\ (e, a) \in \geq_o$.

Let us now analyze when a rejected option in $\mathcal{AF}_o$ may change its status in $\mathcal{AF} \oplus e$.

Theorem 11. *Let* $\mathcal{AF}_o = \langle \mathcal{A}_o, \mathtt{Def}_o \rangle$ *be an argumentation system and* $o \in \mathcal{O}_r(\mathcal{AF})$. *Then:*

1. $o \in \mathcal{O}_a(\mathcal{AF}_o \oplus e)$ *iff* $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)\ (e, a) \in \geq_o)$
2. $o \in \mathcal{O}_n(\mathcal{AF}_o \oplus e)$ *iff* $(e \in \mathcal{H}(o)) \wedge ((\forall a \in \mathcal{A}_o)\ (a, e) \notin >_o) \wedge ((\exists a \in \mathcal{A}_o)\ (e, a) \notin >_o)$
3. $o \in \mathcal{O}_r(\mathcal{AF}_o \oplus e)$ *iff* $(e \notin \mathcal{H}(o)) \vee ((e \in \mathcal{H}(o)) \wedge (\exists a \in \mathcal{A}_o)(a, e) \in >)$

5 Conclusion

This paper has tackled the problem of revising argument-based decision models. To the best of our knowledge, in this paper we have proposed the first investigation on the impact of a new argument on the outcome of a decision system. The basic idea is to check when the status of an option may shift when a new argument is received without having to compute the whole new ordering on options. For that purpose, we have considered a decision model that has recently been proposed in the literature. This model computes a status for each option on the basis of the status of their supporting arguments. We have studied two cases: the case where an option may be supported by several arguments and the case where an option is supported by only one argument. In both cases, we assumed that the new argument is practical, i.e., it supports an option. We have provided a full characterization of acceptable options that become rejected, negotiable or remain accepted. Similarly, we have characterized any shift from one status to another. These results are based on a characterization of a shift of the status of arguments themselves.

These results may be used to determine strategies for negotiation, since at a given step of a dialog an agent has to choose an argument to send to another agent in order to change the status of an option. Moreover, they may help to understand which arguments are useful and which arguments are useless in a given situation, which allows us to understand the role of argumentation in a negotiation.

Note that a recent work has been done on revision in argumentation systems in [5]. That paper addresses the problem of revising the set of extensions of an abstract argumentation system. It studies how the extensions of an argumentation system may evolve when a new argument is received. Nothing is said on the revision of a particular argument. In our paper, we are more interested by the evolution of the status of a given argument without having to compute the extensions of the new argumentation system. We have also studied how the status of an option changes when a new argument is received. Another main difference with this work is that in [5] only the case of adding an argument having only one interaction with an argument of the initial argumentation system is studied. In our paper we have studied the more general case, i.e., the new argument may attack and be attacked by an arbitrary number of arguments of the initial argumentation system.

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