A SAT Approach for Maximizing Satisfiability in Qualitative Spatial and Temporal Constraint Networks

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Abstract

In this paper, we focus on a recently introduced problem in the context of spatial and temporal qualitative reasoning, called the MAX-QCN problem. This problem involves obtaining a spatial or temporal configuration that maximizes the number of satisfied constraints in a qualitative constraint network (QCN). To efficiently solve the MAX-QCN problem, we introduce and study two families of encodings of the partial maximum satisfiability problem (PMAX-SAT). Each of these encodings is based on, what we call, a forbidden covering with regard to the composition table of the considered qualitative calculus. Intuitively, a forbidden covering allows us to express, in a more or less compact manner, the non-feasible configurations for three spatial or temporal entities. The experimentation that we have conducted with qualitative constraint networks from the Interval Algebra shows the interest of our approach.

1 Introduction

Qualitative spatial and temporal reasoning (QSTR) is a major field of study in Knowledge Representation that abstracts from numerical quantities of space and time by using qualitative descriptions instead (e.g., precedes, contains, is left of). The representational languages used in the qualitative approach have increasingly gained a lot of attention during the last decades, as they have the advantage of being conceptually concise and usually sufficiently expressive for a variety of applications in many areas, such as dynamic GIS, cognitive robotics, and spatiotemporal design (Hazarika 2012; Bhatt et al. 2011; Renz and Nebel 2007).

The Interval Algebra (IA) (Allen 1983) and a subset of the Region Connection Calculus (RCC) (Randell, Cui, and Cohn 1992), namely RCC8, are the dominant calculi in QSTR for representing qualitative temporal and spatial information respectively. In particular, IA encodes knowledge about the temporal relations between intervals in the timeline (see Figure 1a), and RCC8 encodes knowledge about the spatial relations between regions in some topological space. In addition to IA and RCC8, numerous other qualitative calculi have been proposed in the literature for representing spatial and temporal information (Ligozat and Renz 2004).

The problem of reasoning about qualitative spatial or temporal information can be modelled as a qualitative constraint network (QCN), i.e., a network comprising constraints corresponding to qualitative spatial or temporal relations between spatial or temporal variables respectively. Given a QCN $\mathcal{N}$, we are particularly interested in its *consistency problem*, which is the problem of deciding if there exists a spatial or temporal interpretation of the variables of $\mathcal{N}$ that satisfies its constraints, such an interpretation being called a *solution* of $\mathcal{N}$. The consistency problem is NP-hard in general. Further, it is closely related to other fundamental reasoning problems in the context of QSTR, such as the *minimality problem* studied in (Amaneddine, Condotta, and Sioutis 2013; Liu and Li 2012) and the *redundancy problem* studied in (Li et al. 2015; Sioutis, Li, and Condotta 2015), in the sense that such related problems exhibit functions that build on the core algorithms used for checking the consistency of a QCN.

In this paper, we focus on a recently introduced problem in the context of QSTR, called the MAX-QCN problem (Condotta et al. 2015). Given a QCN $\mathcal{N}$, the MAX-QCN problem is the problem of obtaining a spatial or temporal configuration that maximizes the number of satisfied constraints in $\mathcal{N}$. The motivation behind studying the MAX-QCN problem lies in the fact that representing spatial or temporal information may inevitably lead to inconsistencies. As illustration, due to the ever-increasing enrichment of the Semantic Web with geospatial data (Egenhofer 2002; Koubarakis et al. 2012), it is often the case that the geometries of geographical objects are not captured correctly due to contradictory data of different sources. Thus, we can obtain inconsistent topological information when extracting topological relations from such geometries (e.g., two overlapping regions may be stated to be identical to a third region, which is impossible as they would also have to be identical to each other if that was the case). With respect to temporal information, timetabling is an example of a scheduling problem where inconsistencies can naturally arise due to the lack of resources for certain tasks (Petrovic and Burke 2004). In particular, timetabling deals with finding suitable temporal intervals for a number of tasks that require limited resources. In the context of a university, an inconsistency can appear when two professors choose to teach the same class of students at overlapping temporal intervals. The incosis-

tency must then be repaired by taking into account the available temporal intervals and the preferences of the professors, and minimizing changes in the timetable so as to distort its structure as little as possible. Solving the MAX-QCN problem is clearly at least as difficult as solving the consistency problem. To solve this optimization problem, the authors in (Condotta et al. 2015) propose a branch and bound algorithm based on state of the art techniques for checking the consistency of a QCN, viz., the use of a triangulation of the constraint graph of the considered QCN to reduce the number of constraints to be treated, the use of a tractable subclass of relations to reduce the width of the search tree, and the use of partial ⋄-consistency to efficiently propagate constraints and prune non-feasible base relations during search.

We follow another approach for solving the MAX-QCN problem, by viewing it as a partial maximum satisfiability problem (PMAX-SAT) and proposing two related families of encodings. As such, our approach can be seen as similar to that concerning the consistency problem for which SAT encodings have been proposed to solve it (Pham, Thornton, and Sattar 2008). Each PMAX-SAT encoding that we propose is based on, what we call, a forbidden covering with regard to the composition table of the considered qualitative calculus. Intuitively, a forbidden covering is a compact set of triples that express all the non-feasible configurations for three spatial or temporal entities. Interestingly, in a way, the support SAT encoding and the forbidden SAT encoding proposed in (Pham, Thornton, and Sattar 2008) correspond to two particular coverings of our proposed forbidden coverings respectively. Using compact sets of tuples is not new in fields other than QSTR. In particular, in (Katsirelos and Walsh 2007) the authors express constraints of finite-domain constraint satisfaction problems (CSPs) through compact sets of tuples and propose a generalized arc consistency method adapted to this compact representation. The two families of PMAX-SAT encodings that we propose differ from one another in the use of auxiliary propositional variables that allow factorizing the number of obtained clauses. It should be noted that all of our encodings also use a proven technique with respect to the consistency problem of a given QCN, that considers a triangulation of the constraint graph of the QCN to reduce the number of constraints to be translated (Huang, Li, and Renz 2013). The experimentation conducted with QCNs from the Interval Algebra shows that a forbidden covering of ours can lead to a very compact representation of a MAX-QCN instance by a PMAX-SAT instance, and that a MAX-QCN instance can be efficiently solved by solving the corresponding PMAX-SAT instance.

The paper is organized as follows. After some preliminaries on QSTR and the MAX-QCN problem, we introduce the notion of forbidden coverings in Section 3. In Section 4, we define and study the two proposed families of PMAX-SAT encodings for solving the MAX-QCN problem. In Section 5, we present an algorithm for computing the forbidden coverings with regard to the composition table of a qualitative calculus, and characterize some forbidden coverings for IA. In Section 6, we report some experimental results about our encodings. Finally, we conclude and give some perspectives for future work in Section 7.

2   Preliminaries

Spatial and temporal qualitative calculi

A (binary) spatial or temporal qualitative calculus considers a domain D of spatial or temporal entities respectively and a finite set B of *jointly exhaustive and pairwise disjoint* (JEPD) relations defined on that domain (Ladkin and Maddux 1994). The elements of B are called base relations and represent the set of possible configurations between two spatial or temporal entities. The set B contains the identity relation Id, and is closed under the converse operation $(^{-1})$. Indefinite knowledge between two spatial or temporal entities can be described by a relation that corresponds to a union of base relations and is represented by the set containing them. Hence, 2^B represents the total set of relations. Given $x, y \in$ D and $r \in 2^B$, $x \, r \, y$ will denote that x and y satisfy a base relation $b \in r$. The set 2^B is equipped with the usual set-theoretic operations (union and intersection), the converse operation, the complement operation and the weak composition operation. The converse of a relation is the union of the converses of its base relations. The complement of a relation r, denoted by $\overline{r}$, is the relation $\{b \in B : b \notin r\}$. The weak composition $\diamond$ of two base relations $b, b' \in B$ is the relation of 2^B defined by $b \diamond b' = \{b'' : \exists x, y, z \in D$ such that $x \, b \, y$, $y \, b' \, z$ and $x \, b'' \, z\}$. For two relations $r, r' \in 2^B$, $r \diamond r'$ is the relation of 2^B defined by $r \diamond r' = \bigcup_{b \in r, b' \in r'} b \diamond b'$.

As an illustration, consider the well known temporal qualitative calculus of the Interval Algebra (IA) introduced by Allen (Allen 1983). IA considers time intervals (as its temporal entities) and the set of base relations $B_{IA} = \{eq, p, pi, m, mi, o, oi, s, si, d, di, f, fi\}$. Each base relation of B_{IA} represents a particular ordering of the four endpoints of two intervals in the timeline (see Figure 1a). Examples of spatial qualitative calculi include two subsets of the Region Connection Calculus (Randell, Cui, and Cohn 1992), namely, the RCC8 and RCC5 calculi, which serve for reasoning about mereotopological information, and consider spatial regions that are regular closed subsets of some topological space. RCC8 has eight base relations that allow us to specify how regions and their boundaries are related to each other. The RCC5 calculus is a fragment of RCC8 where no significance is attached to boundaries of regions. RCC5 comprises the set of base relations $B_{RCC5} = \{EQ, DC, PO, PP, PPi\}$, as shown in Figure 1b. Given the small number of base relations of RCC5, in the sequel, for simplicity, we will use this calculus to illustrate the various introduced concepts.

Qualitative constraint networks (QCNs)

Spatial or temporal information about the relative positions of a set of entities can be represented by a qualitative constraint network (QCN), which is a pair of a set of variables and a set of constraints. Each constraint is defined by a relation of 2^B and specifies the set of acceptable qualitative configurations between two spatial or temporal variables.

Definition 1 *A* QCN *is a pair* $\mathcal{N} = (V, C)$ *where V is a non-empty finite set of variables, and C is a mapping that associates a relation $C(v, v') \in 2^B$ with each pair (v, v') of*
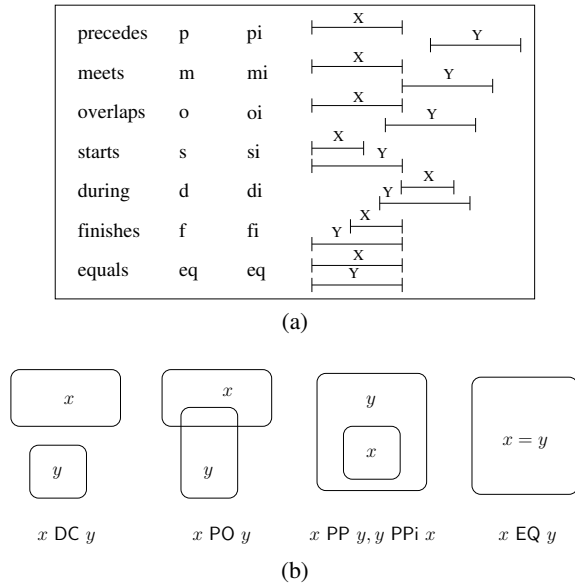
precedes	p	pi
meets	m	mi
overlaps	o	oi
starts	s	si
during	d	di
finishes	f	fi
equals	eq	eq

(a)

x DC y x PO y x PP y, y PPi x x EQ y

(b)

Figure 1: The base relations of IA (a) and RCC5 (b).

(a) $\mathcal{N}$ (b) G (c) $\mathcal{S}$

(d) $\mathcal{S}'$ (e) σ

Figure 2: An inconsistent QCN $\mathcal{N}$ of RCC5 (a), a triangulation G of $\mathsf{G}(\mathcal{N})$ (b), an optimal G-scenario $\mathcal{S}$ of $\mathcal{N}$ (c), an optimal scenario $\mathcal{S}'$ of $\mathcal{N}$ (d) and a solution σ of $\mathcal{N}$ (e).

$V \times V$. *Further, mapping C is such that $C(v, v) \subseteq \{\mathsf{Id}\}$ and $C(v, v') = (C(v', v))^{-1}$.*

Given a QCN $\mathcal{N} = (V, C)$ and $v, v' \in V$, the relation $C(v, v')$ will also be denoted by $\mathcal{N}[v, v']$. Concerning a QCN $\mathcal{N} = (V, C)$, we have the following definitions: An *instantiation* of V is a mapping σ defined from V to the domain D. A *solution* σ of $\mathcal{N}$ is an instantiation of V such that for every pair (v, v') of variables in V, $(\sigma(v), \sigma(v'))$ satisfies $C(v, v')$, *i.e.*, there exists a base relation $b \in C(v, v')$ such that $(\sigma(v), \sigma(v')) \in b$. $\mathcal{N}$ is *consistent* iff it admits a solution. A *sub*-QCN $\mathcal{N}'$ of $\mathcal{N}$, denoted by $\mathcal{N}' \subseteq \mathcal{N}$, is a QCN (V, C') such that $C'(v, v') \subseteq C(v, v') \ \forall v, v' \in V$. A scenario $\mathcal{S}$ is a QCN whose contraints are defined by a singleton relation, *i.e.*, $|C(v, v')| = 1$ for all pairs of variables of the QCN. A scenario $\mathcal{S}$ of $\mathcal{N}$ is a scenario which is a sub-QCN of $\mathcal{N}$. $\mathcal{N}$ is $\diamond$-*consistent* or *closed under weak composition* iff $\forall v, v', v'' \in V$, we have that $C(v, v') \subseteq C(v, v'') \diamond C(v'', v')$.

Given two (undirected) graphs $G = (V, E)$ and $G' = (V', E')$, G is a subgraph of G', denoted by $G \subseteq G'$, iff $V \subseteq V'$ and $E \subseteq E'$. A graph $G = (V, E)$ is a *chordal (or triangulated) graph* iff each of its cycles of length > 3 has a *chord*, *i.e.*, an edge joining two vertices that are not adjacent in the cycle. The constraint graph of a QCN $\mathcal{N} = (V, C)$ is the graph (V, E), denoted by $\mathsf{G}(\mathcal{N})$, for which we have that $(v, v') \in E$ iff $C(v, v') \neq \mathsf{B}$. Given a QCN $\mathcal{N} = (V, C)$ and a graph $G = (V, E)$, $\mathcal{N}$ is partially $\diamond$-consistent *w.r.t.* graph G or $\overset{\diamond}{G}$-consistent (Chmeiss and Condotta 2011) iff for $\forall (v, v'), (v, v''), (v'', v') \in E$, we have that $C(v, v') \subseteq C(v, v'') \diamond C(v'', v')$.

Example 1 *Consider the three QCNs $\mathcal{N}$, $\mathcal{S}$, and $\mathcal{S}'$ of RCC5 depicted in Figure 2. For these QCNs, a variable is represented by a node, and a constraint by an arc labeled with the associated relation;* B *universal relations, converse relations, and* Id *loops are omitted for simplicity. $\mathcal{N}$ is an inconsistent QCN, whereas $\mathcal{S}$ and $\mathcal{S}'$ are consistent QCNs. $\mathcal{S}'$*
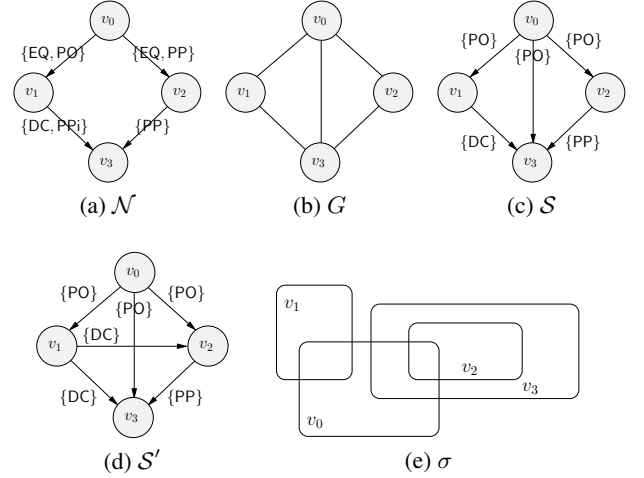
is a consistent scenario of $\mathcal{S}$. A solution σ of $\mathcal{S}$ and $\mathcal{S}'$ is represented in Figure 2e. Now, consider the graph G in Figure 2b. It can be verified that $\mathsf{G}(\mathcal{N}) \subseteq G$ and that G is a chordal graph. Hence, G is a triangulation of $\mathsf{G}(\mathcal{N})$.

The MAX-QCN problem

The MAX-QCN problem has been introduced in the context of QSTR in (Condotta et al. 2015). Given a QCN $\mathcal{N}$ over a set of variables V, the MAX-QCN problem is the problem of finding a consistent scenario over V that maximizes the number of satisfied constraints in $\mathcal{N}$, or, equivalently, the problem of finding a consistent scenario over V that minimizes the number of unsatisfied constraints in $\mathcal{N}$. Such consistent scenarios are called optimal scenarios of $\mathcal{N}$. In the sequel, we will not directly characterize scenarios that are solutions of the MAX-QCN problem of a given QCN, but particular QCNs that we call *partial* scenarios, formally defined in the following manner:

Definition 2 *Let $G = (V, E)$ be a graph. A* partial *scenario w.r.t. G is a QCN (V, C) such that $C(v, v) = \{\mathsf{Id}\}$ for all $v \in V$, $C(v, v') = \mathsf{B}$ for all $(v, v') \notin E$, and $|C(v, v')| = 1$ for all $(v, v') \in E$.*

In the sequel, a partial scenario *w.r.t.* a graph G will be denoted by G-scenario. Given a QCN $\mathcal{N} = (V, C)$ and a graph $G = (V, E)$, a G-scenario $\mathcal{S}$ will be said to be an optimal G-scenario of $\mathcal{N}$ iff $\mathcal{S}$ is consistent and there exists no other consistent G-scenario $\mathcal{S}'$ such that $|\{(v, v') \in E : \mathcal{S}[v, v'] \nsubseteq \mathcal{N}[v, v']\}| > |\{(v, v') \in E : \mathcal{S}'[v, v'] \nsubseteq \mathcal{N}[v, v']\}|$. Intuitively, an optimal G-scenario of $\mathcal{N}$ minimizes the number of unsatisfied constraints in $\mathcal{N}$ that correspond to the edges of G.

Theorem 1 *Let $\mathcal{N}$ be a QCN and G a graph such that $\mathsf{G}(\mathcal{N}) \subseteq G$. Any consistent scenario of any optimal G-scenario of $\mathcal{N}$ is a solution of the MAX-QCN problem of $\mathcal{N}$.*

For a qualitative calculus $\mathcal{Q}$ defined on the set of base relations B, we will say that partial $\diamond$-consistency is complete

for the consistency problem of a partial scenario iff for any $\overset{\diamond}{G}$-consistent G-scenario $\mathcal{S}$ of a QCN $\mathcal{N}$ where G is a chordal graph such that $\mathsf{G}(\mathcal{N}) \subseteq G$, we have that $\mathcal{S}$ is consistent. Note that for a qualitative calculus that has the aforementioned property for partial $\diamond$-consistency, a consistent scenario can be extracted from a consistent G-scenario in polynomial time. From (Huang 2012; Sioutis and Koubarakis 2012), we can assert that partial $\diamond$-consistency is complete for the consistency problem of a partial scenario for IA, PA, RCC5, and RCC8.

The PMAX-SAT problem

A literal is a propositional variable or its negation, and a clause is a disjunction of literals. The maximum satisfiability problem (MAX-SAT) is the problem of finding an assignment that satisfies as many clauses of a given set of clauses as possible (Johnson 1974). Hence, the MAX-QCN problem can be viewed as an exportation of the MAX-SAT problem in the context of QSTR. The partial maximum satisfiability problem (PMAX-SAT) is an extension of the MAX-SAT problem. An instance Ω of PMAX-SAT (Miyazaki, Iwama, and Kambayashi 1996; Cai et al. 2014) is a set of clauses composed of hard and soft clauses. A solution ω of Ω is an assignment that satisfies the hard clauses and maximizes the number of satisfied soft clauses.

3 Forbidden Coverings

In general, the operation of weak composition is defined by a dedicated table, called the *(weak) composition table*. This table has an entry for each pair of base relations $(b, b') \in \mathsf{B} \times \mathsf{B}$, totaling $|\mathsf{B}| \times |\mathsf{B}|$ entries. For an entry corresponding to a pair $(b, b') \in \mathsf{B} \times \mathsf{B}$, the composition table gives the relation of 2^{B} corresponding to $b \diamond b'$. In other words, given three entities $x, y, z \in \mathsf{D}$, the entry corresponding to (b, b') gives the set of possible base relations satisfied by x and z, when x and y satisfy b and y and z satisfy b'.

Example 2 *As an illustration, let us consider the composition table of* RCC5 *shown in Figure 3. By considering the entry corresponding to the pair of base relations* $(\mathsf{PO}, \mathsf{DC})$*, we can see that* $\mathsf{PO} \diamond \mathsf{DC} = \{\mathsf{DC}, \mathsf{PO}, \mathsf{PPi}\}$*. Hence, given three regions* x, y, z *such that* x *and* y *satisfy* PO *and* y *and* z *satisfy* DC*, there are three possible qualitative configurations concerning regions* x, y, z *that are derived by a base relation* $b'' \in \{\mathsf{DC}, \mathsf{PO}, \mathsf{PPi}\}$*, i.e.,* b'' *is such that it can be satisfied by* x *and* z*. Moreover, we can also deduce two non-possible qualitative configurations concerning* x, y, z *that are derived by a base relation* $b'' \in \overline{\{\mathsf{DC}, \mathsf{PO}, \mathsf{PPi}\}} = \{\mathsf{EQ}, \mathsf{PP}\}$*.*

In the sequel, a possible or a non-possible qualitative configuration between three entities will be represented by a triple of base relations (b, b', b''), with b, b', b'' corresponding to the base relation satisfied by x and y, the base relation satisfied by y and z, and the base relation satisfied by x and z respectively. Such a triple of base relations will be called a b-triple. An allowed (resp. forbidden) b-triple is a b-triple representing a possible (resp. a non-possible) configuration.

Example 3 *As an example, let us again consider the entry corresponding to the pair of base relations* $(\mathsf{PO}, \mathsf{DC})$*.*

	EQ	DC	PO	PP	PPi
EQ	{EQ}	{DC}	{PO}	{PP}	{PPi}
DC	{DC}	$\mathsf{B}_{\mathsf{RCC5}}$	{DC, PO, PP}	{DC, PO, PP}	{DC}
PO	{PO}	{DC, PO, PPi}	$\mathsf{B}_{\mathsf{RCC5}}$	{PO, PP}	{DC, PO, PPi}
PP	{PP}	{DC}	{DC, PO, PP}	{PP}	$\mathsf{B}_{\mathsf{RCC5}}$
PPi	{PPi}	{DC, PO, PPi}	{PO, PPi}	{EQ, PO, PP, PPi}	PPi

Figure 3: The composition table of RCC5.

From this entry, we can deduce the set of allowed b-triples $\{(\mathsf{PO}, \mathsf{DC}, \mathsf{DC}), (\mathsf{PO}, \mathsf{DC}, \mathsf{PO}), (\mathsf{PO}, \mathsf{DC}, \mathsf{PPi})\}$*, and the set of forbidden b-triples* $\{(\mathsf{PO}, \mathsf{DC}, \mathsf{EQ}), (\mathsf{PO}, \mathsf{DC}, \mathsf{PP})\}$*.*

In the sequel, $\mathcal{A}_{\mathsf{B}}$ and $\mathcal{F}_{\mathsf{B}}$ will denote the whole set of allowed b-triples and the whole set of forbidden b-triples defined on B respectively. Formally, $\mathcal{A}_{\mathsf{B}} = \{(b, b', b'') : b'' \in b \diamond b'\}$ and $\mathcal{F}_{\mathsf{B}} = \{(b, b', b'') : b'' \in \overline{b \diamond b'}\}$. It is clear that $\mathcal{A}_{\mathsf{B}} \cap \mathcal{F}_{\mathsf{B}} = \emptyset$ and $\mathcal{A}_{\mathsf{B}} \cup \mathcal{F}_{\mathsf{B}} = \mathsf{B} \times \mathsf{B} \times \mathsf{B}$. As an example, with regard to IA, $\mathcal{A}_{\mathsf{B}_{\mathsf{IA}}}$ contains 409 b-triples whereas $\mathcal{F}_{\mathsf{B}_{\mathsf{IA}}}$ contains 1788 b-triples of the 2197 possible b-triples.

Following the line of reasoning in (Katsirelos and Walsh 2007), we introduce a compact representation that allows representing a set of b-triples in a compact manner. A compact triple (c-triple) is a triple $(r, r', r'') \in 2^{\mathsf{B}} \times 2^{\mathsf{B}} \times 2^{\mathsf{B}}$. A c-triple $ct = (r, r', r'')$ represents the set of b-triples $r \times r' \times r''$, denoted by $\mathsf{btriples}(ct)$. Given a c-triple ct, ct will be said to be a forbidden c-triple iff $\mathsf{btriples}(ct) \subseteq \mathcal{F}_{\mathsf{B}}$.

Example 4 *As an illustration, let us consider the c-triples* $ct = (\{\mathsf{EQ}\}, \{\mathsf{PO}, \mathsf{PP}\}, \{\mathsf{EQ}, \mathsf{DC}, \mathsf{PPi}\})$ *and* $ct' = (\{\mathsf{PP}\}, \{\mathsf{DC}\}, \{\mathsf{DC}, \mathsf{PO}\})$*. We have that* $\mathsf{btriples}(ct) = \{$ $(\mathsf{EQ}, \mathsf{PO}, \mathsf{EQ}), (\mathsf{EQ}, \mathsf{PO}, \mathsf{DC}), (\mathsf{EQ}, \mathsf{PO}, \mathsf{PPi}), (\mathsf{EQ}, \mathsf{PP}, \mathsf{EQ}),$ $(\mathsf{EQ}, \mathsf{PP}, \mathsf{DC}), (\mathsf{EQ}, \mathsf{PP}, \mathsf{PPi})\}$*. As* $\mathsf{btriples}(ct)$ *contains only forbidden b-triples,* ct *is a forbidden c-triple. Further, we have that* $\mathsf{btriples}(ct') = \{(\mathsf{PP}, \mathsf{DC}, \mathsf{DC}), (\mathsf{PP}, \mathsf{DC}, \mathsf{PO})\}$*. As* $(\mathsf{PP}, \mathsf{DC}, \mathsf{DC})$ *is an allowed b-triple and* $(\mathsf{PP}, \mathsf{DC}, \mathsf{PO})$ *is a forbidden b-triple in* $\mathsf{btriples}(ct')$*, we have that* ct' *is neither an allowed nor a forbidden c-triple.*

In what follows, a set of c-triples representing all the forbidden b-triples, will be called a forbidden covering of the composition table on B, or, simply, a forbidden covering. Formally, forbidden coverings are defined as follows:

Definition 3 *A forbidden covering of the composition table on* B *is a set* $\mathcal{C}$ *of forbidden c-triples such that* $\mathcal{F}_{\mathsf{B}} = \bigcup_{ct \in \mathcal{C}} \mathsf{btriples}(ct)$*.*

Given a qualitative calculus $\mathcal{Q}$ based on a set of base relation B, the sets of c-triples $\mathcal{C}^{\alpha}_{\mathcal{Q}}$ and $\mathcal{C}^{\beta}_{\mathcal{Q}}$ are defined as follows:
- $\mathcal{C}^{\alpha}_{\mathcal{Q}} = \{(\{b\}, \{b'\}, \{b''\}) : (b, b', b'') \in \mathcal{F}_{\mathsf{B}}\}$;
- $\mathcal{C}^{\beta}_{\mathcal{Q}} = \{(\{b\}, \{b'\}, \overline{b \diamond b'}) : b, b' \in \mathsf{B} \text{ and } \overline{b \diamond b'} \neq \emptyset\}$.

Clearly, $\mathcal{C}^{\alpha}_{\mathcal{Q}}$ and $\mathcal{C}^{\beta}_{\mathcal{Q}}$ are examples of forbidden coverings.

Example 5 *With respect to* RCC5*, the forbidden coverings* $\mathcal{C}^{\alpha}_{\mathsf{RCC5}}$ *and* $\mathcal{C}^{\beta}_{\mathsf{RCC5}}$ *contain 71 and 22 c-triples respectively. However, there exist much smaller forbidden coverings, like the covering* $\mathcal{C}$ *containing the following 10 c-triples:*

1. ({EQ, PP}, {EQ, PP}, {DC, PO, PPi})   2. ({EQ, PPi}, {PO, PPi}, {EQ, DC, PP})
3. ({DC, PP}, {EQ, PO, PP}, {EQ, PPi})   4. ({PO, PPi}, {EQ, DC, PPi}, {EQ, PP})
5. ({EQ, PP}, {DC}, {EQ, PO, PP, PPi})   6. ({EQ, DC, PPi}, {EQ, PPi}, {PO, PP})
7. ({PO}, {EQ, PP}, {EQ, DC, PPi})   8. ({DC}, {PPi}, {EQ, PO, PP, PPi})
9. ({PPi}, {EQ, PO, PP, PPi}, {DC})   10. ({EQ}, {PO, PP}, {EQ, DC, PPi})

4 PMAX-SAT encodings for MAX-QCN

In this section we propose two families of encodings of the PMAX-SAT problem for solving the MAX-QCN problem. These encodings are parameterized by a forbidden covering and a triangulation of the constraint graph of the considered QCN. The proposed encodings are generic, like the ones given in (Pham, Thornton, and Sattar 2006), in the sense that they can be used in the context of any binary qualitative calculus. Nevertheless, as we will see in the sequel, particular properties of the considered qualitative calculus are necessary for ensuring the completeness of the approach. In what follows, for the proposed encodings, we consider a QCN $\mathcal{N}$ defined on a set of n variables $V = \{v_0, \ldots, v_{n-1}\}$, a chordal graph $G = (V, E)$, and a forbidden covering $\mathcal{C}$.

The FCTE encoding

We present the first encoding, denoted by FCTE (Forbidden Covering and Triangulation based Encoding). FCTE($\mathcal{N}$) is defined in the following way:

• For all $v_i, v_j \in V$ such that $i < j$ and $(v_i, v_j) \in E$, a propositional variable x_{ij}^b is introduced $\forall b \in$ B (*i.e.*, a propositional variable is indexed by i, j and the relation b).

• For all $v_i, v_j \in V$ such that $i < j$ and $(v_i, v_j) \in E$, to ensure that exactly one base relation of B holds between v_i and v_j, the at-least-one ALO-hard and at-most-one AMO-hard hard clauses are defined:

$$\bigvee_{b \in \mathsf{B}} x_{ij}^b \quad (\text{ALO-hard})$$

$$\forall \{b, b'\} \subseteq \mathsf{B} \text{ with } b \neq b', \neg x_{ij}^b \vee \neg x_{ij}^{b'} \quad (\text{AMO-hard})$$

• To exclude forbidden b-triples for each triangle of G, the FORB-hard hard clauses are introduced from the forbidden covering $\mathcal{C}$. To this end, for all $v_i, v_j, v_k \in V$ such that $\{(v_i, v_j), (v_j, v_k), (v_i, v_k)\} \subseteq E$ and $i < j < k$, one hard clause is introduced for each c-triple $(r, r', r') \in \mathcal{C}$:

$$\mathsf{comp}(r, i, j) \vee \mathsf{comp}(r', j, k) \vee \mathsf{comp}(r'', i, k) \quad (\text{FORB-hard})$$

where $\mathsf{comp}(s, l, m)$, with $s \in 2^\mathsf{B}$ and $l, m \in \{0, \ldots, n-1\}$, corresponds to a subclause encoding the complement of the relation s that holds between variables v_l and v_m. Formally, $\mathsf{comp}(s, l, m)$ is $\perp$ if $s = \mathsf{B}$, $\neg x_{lm}^b$ if $s = \{b\}$ with $b \in \mathsf{B}$, and $\bigvee_{b \in \bar{s}} x_{lm}^b$ in any other case. Note that when an obtained clause contains the propositional constant $\perp$, the clause is simplified by removing $\perp$.

• Next, we introduce the soft clauses representing the possible base relation instantiations of the constraints of the given QCN. For all $v_i, v_j \in V$ such that $i < j$, $(v_i, v_j) \in E$ and $C_{ij} \neq \mathsf{B}$, the following clause is added:

$$\bigvee_{b \in C_{ij}} x_{ij}^b \quad (\text{ALO-soft}).$$

Example 6 *Let us consider the QCN $\mathcal{N} = (V, C)$ of RCC5 and the triangulation $G = (V, E)$ of $\mathsf{G}(\mathcal{N})$ given in Figure 2, and the forbidden covering $\mathcal{C}$ of the composition table of RCC5 given in Example 5. We examine some clauses composing $\Omega = \mathsf{FCTE}(\mathcal{N}, G, \mathcal{C})$. First, we can note that the set of clauses Ω is defined on $|\mathsf{B}_{\mathsf{RCC5}}| \times |E| = 25$ propositional*

variables. Regarding the pair of variables $(v_0, v_1) \in E$, we have the following ALO-hard clause:

$$x_{01}^{\mathsf{EQ}} \vee x_{01}^{\mathsf{DC}} \vee x_{01}^{\mathsf{PO}} \vee x_{01}^{\mathsf{PP}} \vee x_{01}^{\mathsf{PPi}}.$$

For the same pair of variables $(v_0, v_1) \in E$, Ω contains the following AMO-hard clauses:

$$\neg x_{01}^{\mathsf{EQ}} \vee \neg x_{01}^{\mathsf{DC}}, \quad \neg x_{01}^{\mathsf{EQ}} \vee \neg x_{01}^{\mathsf{PO}}, \quad \neg x_{01}^{\mathsf{EQ}} \vee \neg x_{01}^{\mathsf{PP}},$$
$$\neg x_{01}^{\mathsf{EQ}} \vee \neg x_{01}^{\mathsf{PPi}}, \quad \neg x_{01}^{\mathsf{DC}} \vee \neg x_{01}^{\mathsf{PO}}, \quad \neg x_{01}^{\mathsf{DC}} \vee \neg x_{01}^{\mathsf{PP}},$$
$$\neg x_{01}^{\mathsf{DC}} \vee \neg x_{01}^{\mathsf{PPi}}, \quad \neg x_{01}^{\mathsf{PO}} \vee \neg x_{01}^{\mathsf{PP}}, \quad \neg x_{01}^{\mathsf{PO}} \vee \neg x_{01}^{\mathsf{PPi}},$$
$$\neg x_{01}^{\mathsf{PP}} \vee \neg x_{01}^{\mathsf{PPi}}.$$

By examining graph G, it becomes apparent that the FORB-hard clauses of Ω stem from the two triples of variables (v_0, v_1, v_3) and (v_0, v_2, v_3). Each of these triples yields $|\mathcal{C}| = 10$ FORB-hard clauses. With regard to (v_0, v_1, v_3) and the following particular c-triples of $\mathcal{C}$:

1. $(\{\mathsf{EQ}, \mathsf{PP}\}, \{\mathsf{EQ}, \mathsf{PP}\}, \{\mathsf{DC}, \mathsf{PO}, \mathsf{PPi}\})$,
5. $(\{\mathsf{EQ}, \mathsf{PP}\}, \{\mathsf{DC}\}, \{\mathsf{EQ}, \mathsf{PO}, \mathsf{PP}, \mathsf{PPi}\})$,
8. $(\{\mathsf{DC}\}, \{\mathsf{PPi}\}, \{\mathsf{EQ}, \mathsf{PO}, \mathsf{PP}, \mathsf{PPi}\})$,
10. $(\{\mathsf{EQ}\}, \{\mathsf{PO}, \mathsf{PP}\}, \{\mathsf{EQ}, \mathsf{DC}, \mathsf{PPi}\})$,

we obtain the following four FORB-hard clauses of Ω:

1. $x_{01}^{\mathsf{DC}} \vee x_{01}^{\mathsf{PO}} \vee x_{01}^{\mathsf{PPi}} \vee x_{13}^{\mathsf{DC}} \vee x_{13}^{\mathsf{PO}} \vee x_{13}^{\mathsf{PPi}} \vee x_{03}^{\mathsf{EQ}} \vee x_{03}^{\mathsf{PP}}$,
5. $x_{01}^{\mathsf{DC}} \vee x_{01}^{\mathsf{PO}} \vee x_{01}^{\mathsf{PPi}} \vee \neg x_{13}^{\mathsf{DC}} \vee x_{03}^{\mathsf{DC}}$,
8. $\neg x_{01}^{\mathsf{DC}} \vee \neg x_{13}^{\mathsf{PPi}} \vee x_{03}^{\mathsf{DC}}$,
10. $\neg x_{01}^{\mathsf{EQ}} \vee x_{13}^{\mathsf{EQ}} \vee x_{13}^{\mathsf{DC}} \vee x_{13}^{\mathsf{PPi}} \vee x_{03}^{\mathsf{PO}} \vee x_{03}^{\mathsf{PP}}$.

Further, Ω contains the following four ALO-soft clauses:

$$x_{01}^{\mathsf{EQ}} \vee x_{01}^{\mathsf{PO}}, \quad x_{02}^{\mathsf{EQ}} \vee x_{02}^{\mathsf{PP}}, \quad x_{13}^{\mathsf{DC}} \vee x_{13}^{\mathsf{PPi}}, \quad x_{23}^{\mathsf{PP}}.$$

For each assignment that satisfies the hard clauses of a set of clauses produced by the FCTE encoding, a partially $\diamond$-consistent partial scenario can be obtained, and vice versa, as described in the following definition:

Definition 4 *Let $\mathcal{N} = (V, C)$ be a QCN, $G = (V, E)$ a graph, and $\mathcal{C}$ a forbidden covering. Let Ω be the set of clauses corresponding to $\mathsf{FCTE}(\mathcal{N}, G, \mathcal{C})$.*
• For each assignment ω satisfying the hard clauses of Ω, the $\overset{\diamond}{G}$-consistent G-scenario $\mathsf{QCN}(\omega) = (V, C')$ is defined as follows: for all $(v_i, v_j) \in E$ such that $i < j$, $C'(v_i, v_j) = \{b\}$ and $C'(v_j, v_i) = \{b^{-1}\}$, where b is the unique base relation b of B such that $\omega(x_{ij}^b) = \mathsf{true}$. Moreover, $C(v_i, v_i) = \{\mathsf{Id}\}$ for all $v_i \in V$, and $C'(v_i, v_j) = C'(v_j, v_i) = \mathsf{B}$ for all $(v_i, v_j) \in (V \times V) \setminus E$ such that $v_i \neq v_j$.
• For each $\overset{\diamond}{G}$-consistent G-scenario $\mathcal{S} = (V, C')$, the assignment $\mathsf{sat}(\mathcal{S})$ that satisfies the hard clauses Ω is defined as follows: for all $(v_i, v_j) \in E$ such that $i < j$ and $b \in \mathsf{B}$, $\mathsf{sat}(\mathcal{S})(x_{ij}^b) = \mathsf{true}$ iff $C'(v_i, v_j) = \{b\}$.

Example 7 *As in Example 6, let us consider the QCN $\mathcal{N} = (V, C)$ of RCC5 and the triangulation $G = (V, E)$ of $\mathsf{G}(\mathcal{N})$ given in Figure 2, and the forbidden covering $\mathcal{C}$ of the composition table of RCC5 given in Example 5. Let $\Omega = \mathsf{FCTE}(\mathcal{N}, G, \mathcal{C})$. Now, let us consider the model ω of Ω defined by $\omega(x_{01}^{\mathsf{PO}}) = \omega(x_{02}^{\mathsf{PO}}) = \omega(x_{03}^{\mathsf{PO}}) = \omega(x_{13}^{\mathsf{PO}}) = \omega(x_{23}^{\mathsf{PP}}) = \mathsf{true}$, and $\omega(x) = \mathsf{false}$ for every other propositional variable x. We have that ω is an optimal model of Ω. Moreover, we have that $\mathsf{QCN}(w)$ corresponds to the optimal G-scenario $\mathcal{S}$ of $\mathcal{N}$ as shown in Figure 2.*

Theorem 2 *Let $\mathcal{Q}$ be a qualitative calculus for which partial $\diamond$-consistency is complete for the consistency problem of a partial scenario, $\mathcal{N}$ a QCN of $\mathcal{Q}$, $G = (V, E)$ a chordal graph such that $\mathsf{G}(\mathcal{N}) \subseteq G$, and $\mathcal{C}$ a forbidden covering. Further, let Ω be the set of clauses corresponding to $\mathsf{FCTE}(\mathcal{N}, G, \mathcal{C})$. We have that for all optimal G-scenarios $\mathcal{S}$ of $\mathcal{N}$, $\mathsf{sat}(\mathcal{S})$ is a solution of Ω, and for all solutions ω of Ω, $\mathsf{QCN}(\omega)$ is an optimal G-scenario of $\mathcal{N}$.*

Hence, given a qualitative calculus $\mathcal{Q}$ for which partial $\diamond$-consistency is complete for the consistency problem of a partial scenario and a QCN $\mathcal{N}$ of $\mathcal{Q}$, we showed that we can use the proposed FCTE encoding to characterize a solution of the MAX-QCN problem of $\mathcal{N}$. In particular, we first compute a triangulation G of $\mathsf{G}(\mathcal{N})$, then select a forbidden covering $\mathcal{C}$ of the composition table on B, and finally obtain a model w of the PMAX-SAT problem defined with respect to $\mathsf{FCTE}(\mathcal{N}, G, \mathcal{C})$; by Theorem 2, the G-scenario $\mathsf{QCN}(w)$ will be an optimal G-scenario of $\mathcal{N}$ and, consequently, a solution of the MAX-QCN problem of $\mathcal{N}$.

The FCTEX encoding

We present the second encoding, denoted by FCTEX (FCTE with Auxiliary Variables). The FCTEX encoding differs from the FCTE encoding in the use of FORB-hard clauses. In particular, for the new encoding we introduce auxiliary variables that represent the relations of 2^{B} implied by the set of c-triples of the forbidden covering at hand. These auxiliary variables allow for factorizing subclauses implied by forbidden clauses.

$\mathsf{FCTEX}(\mathcal{N})$ is composed of the variables and the sets of clauses ALO-hard, AMO-hard, and ALO-soft of $\mathsf{FCTE}(\mathcal{N})$, and the new sets of hard clauses AUX-hard and FORBX-hard that are defined in the following manner:

• To exclude forbidden b-triples for each triangle of G, the FORBX-hard hard clauses are introduced from the forbidden covering $\mathcal{C}$. To this end, for all $v_i, v_j, v_k \in V$ such that $\{(v_i, v_j), (v_j, v_k), (v_i, v_k)\} \subseteq E$ and $i < j < k$, one hard clause is introduced for each c-triple $(r, r', r') \in \mathcal{C}$:

$$\mathsf{c}'(r, i, j) \vee \mathsf{c}'(r', j, k) \vee \mathsf{c}'(r'', i, k) \quad \text{(FORBX-hard)}$$

where $\mathsf{c}'(s, l, m)$, with $s \in 2^{\mathsf{B}}$ and $l, m \in \{0, \dots, n-1\}$, corresponds to a subclause encoding the complement of the relation s that holds between variables v_l and v_m. Formally, $\mathsf{c}'(s, l, m)$ is $\perp$ if $s = \mathsf{B}$, $\neg x_{lm}^b$ if $s = \{b\}$ with $b \in \mathsf{B}$, and $x_{lm}^{\bar{s}}$ in any other case; in this last case, if x_{lm}^s is introduced for the first time, the hard clauses AUX-hard corresponding to $x_{lm}^{\bar{s}}$ are introduced. In particular:

• For each newly introduced propositional variable $x_{lm}^{\bar{s}}$ in the set of FORBX-hard clauses, with $(l, m) \in E$ and $s \in 2^{\mathsf{B}}$, the following hard clause is added:

$$\forall b \in s, \neg x_{lm}^b \vee \neg x_{lm}^{\bar{s}} \quad \text{(AUX-hard)}$$

Example 8 *Let us consider the QCN $\mathcal{N} = (V, C)$ of RCC5 and the triangulation $G = (V, E)$ of $\mathsf{G}(\mathcal{N})$ given in Figure 2, and the forbidden covering $\mathcal{C}$ of the composition table of RCC5 given in Example 5. Further, let $\Omega = \mathsf{FCTE}(\mathcal{N}, G, \mathcal{C})$, $\Omega' = \mathsf{FCTEX}(\mathcal{N}, G, \mathcal{C})$, and $ct =$*

$(\{\mathsf{EQ}, \mathsf{PP}\}, \{\mathsf{EQ}, \mathsf{PP}\}, \{\mathsf{DC}, \mathsf{PO}, \mathsf{PPi}\})$*, where ct is a particular c-triple of $\mathcal{C}$. With regard to the two triples of variables (v_0, v_1, v_3) and (v_0, v_2, v_3), ct leads to the addition of the following* FORB-hard *clauses in Ω:*

$$x_{01}^{\mathsf{DC}} \vee x_{01}^{\mathsf{PO}} \vee x_{01}^{\mathsf{PPi}} \vee x_{13}^{\mathsf{DC}} \vee x_{13}^{\mathsf{PO}} \vee x_{13}^{\mathsf{PPi}} \vee x_{03}^{\mathsf{EQ}} \vee x_{03}^{\mathsf{PP}},$$
$$x_{02}^{\mathsf{DC}} \vee x_{02}^{\mathsf{PO}} \vee x_{02}^{\mathsf{PPi}} \vee x_{23}^{\mathsf{DC}} \vee x_{23}^{\mathsf{PO}} \vee x_{23}^{\mathsf{PPi}} \vee x_{03}^{\mathsf{EQ}} \vee x_{03}^{\mathsf{PP}}.$$

Whereas, ct leads to the addition of the following FORBX-hard *and* AUX-hard *clauses in Ω':*

$$x_{01}^{\{\mathsf{DC},\mathsf{PO},\mathsf{PPi}\}} \vee x_{13}^{\{\mathsf{DC},\mathsf{PO},\mathsf{PPi}\}} \vee x_{03}^{\{\mathsf{EQ},\mathsf{PP}\}},$$
$$x_{02}^{\{\mathsf{DC},\mathsf{PO},\mathsf{PPi}\}} \vee x_{23}^{\{\mathsf{DC},\mathsf{PO},\mathsf{PPi}\}} \vee x_{03}^{\{\mathsf{EQ},\mathsf{PP}\}},$$
$$\neg x_{01}^{\mathsf{EQ}} \vee \neg x_{01}^{\{\mathsf{DC},\mathsf{PO},\mathsf{PPi}\}}, \quad \neg x_{01}^{\mathsf{PP}} \vee \neg x_{01}^{\{\mathsf{DC},\mathsf{PO},\mathsf{PPi}\}},$$
$$\neg x_{02}^{\mathsf{EQ}} \vee \neg x_{02}^{\{\mathsf{DC},\mathsf{PO},\mathsf{PPi}\}}, \quad \neg x_{02}^{\mathsf{PP}} \vee \neg x_{02}^{\{\mathsf{DC},\mathsf{PO},\mathsf{PPi}\}},$$
$$\neg x_{13}^{\mathsf{EQ}} \vee \neg x_{13}^{\{\mathsf{DC},\mathsf{PO},\mathsf{PPi}\}}, \quad \neg x_{13}^{\mathsf{PP}} \vee \neg x_{13}^{\{\mathsf{DC},\mathsf{PO},\mathsf{PPi}\}},$$
$$\neg x_{23}^{\mathsf{EQ}} \vee \neg x_{23}^{\{\mathsf{DC},\mathsf{PO},\mathsf{PPi}\}}, \quad \neg x_{23}^{\mathsf{PP}} \vee \neg x_{23}^{\{\mathsf{DC},\mathsf{PO},\mathsf{PPi}\}},$$
$$\neg x_{03}^{\mathsf{DC}} \vee \neg x_{03}^{\{\mathsf{EQ},\mathsf{PP}\}}, \neg x_{03}^{\mathsf{PO}} \vee \neg x_{03}^{\{\mathsf{EQ},\mathsf{PP}\}}, \neg x_{03}^{\mathsf{PPi}} \vee \neg x_{03}^{\{\mathsf{EQ},\mathsf{PP}\}}.$$

We can show that each solution of $\mathsf{FCTE}(\mathcal{N}, G, \mathcal{C})$ can be extended to a solution of $\mathsf{FCTEX}(\mathcal{N}, G, \mathcal{C})$, and that each solution of $\mathsf{FCTEX}(\mathcal{N}, G, \mathcal{C})$ can be restricted to a solution of $\mathsf{FCTE}(\mathcal{N}, G, \mathcal{C})$. Hence, by Theorem 2 we have the following result:

Theorem 3 *Let $\mathcal{Q}$ be a qualitative calculus for which partial $\diamond$-consistency is complete for the consistency problem of a partial scenario, $\mathcal{N}$ a QCN of $\mathcal{Q}$, $G = (V, E)$ a chordal graph such that $\mathsf{G}(\mathcal{N}) \subseteq G$, and $\mathcal{C}$ a forbidden covering. Further, let Ω be the set of clauses corresponding to $\mathsf{FCTEX}(\mathcal{N}, G, \mathcal{C})$. We have that for all solutions ω of Ω, $\mathsf{QCN}(\omega)$ is an optimal G-scenario of $\mathcal{N}$.*

5 Computing Forbidden Coverings

With regard to the considered qualitative calculus, the cardinality of the whole set of c-triples can be very high. As such, whatever the criteria for the desired forbidden covering, it is not realistic to propose a generic algorithm that will review all of the forbidden coverings for the calculus at hand and select one of them. A more realistic approach is to generate a forbidden covering by choosing, in an iterative manner, some preferred c-triples among a restricted set of c-triples. In each iteration, the newly selected c-triple is chosen according to a heuristic that optimizes some general criteria associated with the desired forbidden covering. Following the aforementioned approach, we propose a generic algorithm, called computeFC, that intuitively allows for restricting the set of candidate c-triples by taking into account the size of the generated FORB-hard clauses through the use of the FCTE encoding. In particular, computeFC takes two integers as parameters, namely, maxRel and maxCl, such that maxRel ≥ 1 and maxCl ≥ 3. The integer maxRel defines the maximum allowed size for a subclause that corresponds to a relation belonging to a c-triple, whereas the integer maxCl defines the maximum allowed size for the whole clause that corresponds to all three relations belonging to the c-triple. Calculating the size of a subclause that corresponds to a non-empty relation $r \in 2^{\mathsf{B}}$ is made possible through the use of a mapping sizeCl that associates a size with each such subclause. Formally, sizeCl(r) is 0

$\mathcal{C}$	$\mathcal{C}_{IA}^{\alpha}$	$\mathcal{C}_{IA}^{\beta}$	$\mathcal{C}_{IA}^{1,\infty}$	$\mathcal{C}_{IA}^{3,\infty}$	$\mathcal{C}_{IA}^{5,\infty}$	$\mathcal{C}_{IA}^{6,\infty}$	$\mathcal{C}_{IA}^{7,\infty}$	$\mathcal{C}_{IA}^{8,\infty}$	$\mathcal{C}_{IA}^{9,\infty}$	$\mathcal{C}_{IA}^{10,\infty}$	$\mathcal{C}_{IA}^{11,\infty}$	$\mathcal{C}_{IA}^{5,7}$	$\mathcal{C}_{IA}^{5,8}$	$\mathcal{C}_{IA}^{5,9}$	$\mathcal{C}_{IA}^{5,10}$	$\mathcal{C}_{IA}^{6,10}$	$\mathcal{C}_{IA}^{6,11}$	$\mathcal{C}_{IA}^{6,12}$
$|\mathcal{C}|$	1788	166	279	207	73	63	51	49	34	34	31	133	107	103	97	79	73	69
$\sum_{ct \in \mathcal{C}} \mathsf{sizeCl}(ct)$	5364	702	837	721	585	589	523	530	695	623	666	567	519	525	507	443	583	568
$\mathsf{Min}_{ct \in \mathcal{C}} \mathsf{sizeCl}(ct)$	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
$\mathsf{Max}_{ct \in \mathcal{C}} \mathsf{sizeCl}(ct)$	3	11	3	5	11	13	19	22	27	27	27	7	8	9	10	10	11	12
$\mathsf{Avg}_{ct \in \mathcal{C}} \mathsf{sizeCl}(ct)$	3.0	4.22	3.0	3.48	8.01	9.34	10.25	10.81	20.44	18.35	21.48	4.26	4.85	5.09	5.22	5.60	7.98	8.23

Table 1: Some statistics about some forbidden coverings of IA.

Function computeFC(maxRel,maxCl)

    in : Two integers $maxRel \geq 1$ and $maxCl \geq 3$.
    output : A forbidden covering of the composition table on B.

1 **begin**
2    $\mathcal{S}_{rels} \leftarrow \{r \in 2^B : \mathsf{sizeCl}(r) \leq \mathsf{maxRel} \text{ and } r \neq \{\}\}$;
3    $\mathcal{C}_{forb} \leftarrow \emptyset; \mathcal{F} \leftarrow \mathcal{F}_B$;
4   **while** $\mathcal{F} \neq \emptyset$ **do**
5      $ct \leftarrow null; n \leftarrow 0$;
6   **foreach** $(r, r', r'') \in \mathcal{S}_{rels} \times \mathcal{S}_{rels} \times \mathcal{S}_{rels}$ *such that*
       $((r \diamond r') \cap r'') = \emptyset$ and $\mathsf{sizeCl}((r, r', r'')) \leq maxCl$ **do**
7        $ct' \leftarrow (r, r', r'')$;
8        $n' \leftarrow |\mathsf{btriples}(ct') \cap \mathcal{F}|$;
9   **if** $(n' > n)$ or $(n' = n$ and $\mathsf{sizeCl}(ct') < \mathsf{sizeCl}(ct))$
       then
10          $ct \leftarrow ct'; n \leftarrow n'$;
11
12      $\mathcal{C}_{forb} \leftarrow \mathcal{C}_{forb} \cup \{ct\}$;
13      $\mathcal{F} \leftarrow \mathcal{F} \setminus \mathsf{btriples}(ct)$;
14    $\mathcal{C}_{forb} \leftarrow \mathsf{minimize}(\mathcal{C}_{forb})$;
15   **return** $\mathcal{C}_{forb}$;

---

if $r = B$, 1 if $|r| = 1$, and $|\bar{r}|$ in any other case. Mapping sizeCl can be used to calculate the size of the whole clause that corresponds to all three relations belonging to a c-triple as well. In particular, given a c-triple $ct = (r, r', r'')$, $\mathsf{sizeCl}(ct) = \mathsf{sizeCl}(r) + \mathsf{sizeCl}(r') + \mathsf{sizeCl}(r'')$. Further, for a non-empty set of c-triples $\mathcal{C}$, we have that $\mathsf{sizeCl}(\mathcal{C}) = \sum_{ct \in \mathcal{C}} \mathsf{sizeCl}(ct)$.

Now we will describe function computeFC in detail. In line 2, the set of relations $\mathcal{S}_{rels}$ that will be used to define the candidate c-triples is computed. In line 3, the set of c-triples $\mathcal{C}_{forb}$ that will correspond to the resulting forbidden covering in the end of the treatment and the set of forbidden b-triples $\mathcal{F}$ are initialized to $\emptyset$ and $\mathcal{F}_B$ respectively. In lines 4–13, a forbidden covering of the composition table on B is computed in an iterative manner. In each iteration, a new c-triple with the maximum number of forbidden b-triples is selected, by considering the c-triples derived from the set of allowed relations $\mathcal{S}_{rels}$ and having a size smaller or equal to maxCl. Note that in each iteration there will always exist a c-triple that will engulf at least one not already considered forbidden b-triple, since the set of all c-triples derived from the set of allowed relations $\mathcal{S}_{rels}$ and having a size smaller or equal to maxCl contains the forbidden covering $\mathcal{C}_{\mathcal{Q}}^{\alpha}$ (with $\mathcal{Q}$ being the qualitative calculus corresponding to B). Hence, in each iteration, a new c-triple is added to $\mathcal{C}_{forb}$ (line 12) and at least one b-triple is removed from $\mathcal{F}$ (line 13). Upon completion of the iterative procedure, $\mathcal{C}_{forb}$ will be a forbidden covering of the composition table defined on B. In line 14, a final operation takes place to *minimize* the forbidden covering $\mathcal{C}_{forb}$ with the help of function minimize. This function, makes the follow-ing two treatments iteratively until a fixed-point is reached: first, it removes the redundant c-triples from $\mathcal{F}$; secondly, it checks if for every c-triple ct of $\mathcal{C}_{forb}$ there exists a c-triple ct' of $\mathcal{C}_{forb}$ with $ct' \neq ct$, $\mathsf{sizeCl}(ct') < \mathsf{sizeCl}(ct)$, and $\mathsf{btriples}((\mathcal{C}_{forb} \setminus \{\{ct\}\}) \cup \{ct'\}) = \mathsf{btriples}(\mathcal{C}_{forb})$ by considering all c-triples of $\mathcal{C}_{forb}$. If such a c-triple ct' exists, ct is replaced by ct' in $\mathcal{F}$. Finally, in line 15, the minimized forbidden covering is returned by function computeFC.

Table 1 reports some statistics about some forbidden coverings of IA computed with computeFC and about the forbidden coverings $\mathcal{C}_{IA}^{\alpha}$ and $\mathcal{C}_{IA}^{\beta}$. Each forbidden covering is denoted by $\mathcal{C}_{IA}^{\mathsf{maxRel},\mathsf{maxCl}}$, with maxRel and maxCl being the two integers used as parameters. Note that when maxCl is sufficiently large to allow c-triples of any size, the symbol ∞ is used instead of the integer maxCl. The number of c-triples of the coverings ranges from 31 to 279. The average size of the c-triples varies from 3 to 21.48.

6 Experiments

Regarding our experimentation, we considered QCNs from IA and the forbidden coverings computed as described in the previous section. We experimented with QCNs of IA because the MAX-QCN problem for IA seems to be in general more challenging than for other cited calculi (Condotta et al. 2015). The QCNs were randomly generated using the model A, which was first proposed by Nebel in (Nebel 1997) and has been widely used in the experimental studies in the context of QSTR ever since. This model generates QCNs according to the three following parameters: the number of variables n, the density of constraints defined by a non-trivial relation of IA (i.e., a relation other than B) d, and the average number of base relations in each constraint s. The reported experimental results concern QCNs of model $A(n, d, s)$, with $n = 20$, d varying from 8 to 14.5 with a step of 0.25, and $s = 6.5$. The relatively small number of variables was decided to be able to present results that are as complete as possible; this will become more clear in what follows. For each considered value of d, 10 instances were generated, giving a total of 280 randomly generated QCNs. Regarding triangulations of the constraint graphs of QCNs, a (linear) technique involves adding extra edges while eliminating vertices one by one in some order. Among the proposed heuristics to order the vertices, we used the Greedy-FillIn (GFI) heuristic (Bodlaender and Koster 2010).

The first part of our analysis concerns the size of the PMAX-SAT instances obtained, more particularly, the average number of propositional variables, the average number of clauses, the average size of the clauses, and the average number of literals in the PMAX-SAT instances (i.e., the sum of the literals of the clauses). Let us examine the table in Figure 4. It is clear that the use of the GFI heuristic to triangulate the constraint graph of a given QCN al-

$\mathcal{C}$	#Vars				#Clauses$(\times 10^{-3})$				#Literals$(\times 10^{-3})$				|Clauses|			
	NONE		GFI		NONE		GFI		NONE		GFI		NONE		GFI	
	FCTE	FCTEX	FCTE	FCTEX	FCTE	FCTEX	FCTE	FCTEX	FCTE	FCTEX	FCTE	FCTEX	FCTE	FCTEX	FCTE	FCTEX
$\mathcal{C}_{IA}^{\alpha}$	2471.0	2471.0	1931.0	1931.0	2053.4	2053.4	1146.3	1146.3	6147.8	6147.8	3429.2	3429.2	3.0	3.0	3.0	3.0
$\mathcal{C}_{IA}^{\beta}$	2471.0	4694.0	1931.0	3581.5	204.4	224.2	117.2	131.9	833.1	640.2	471.2	371.2	4.1	2.9	4.0	2.8
$\mathcal{C}_{IA}^{1,\infty}$	2471.0	2471.0	1931.0	1931.0	333.2	333.2	188.9	188.9	987.0	987.0	556.9	556.9	3.0	3.0	2.9	2.9
$\mathcal{C}_{IA}^{3,\infty}$	2471.0	3911.0	1931.0	3024.0	251.1	265.5	143.2	154.1	854.8	769.6	483.3	441.7	3.4	2.9	3.4	2.9
$\mathcal{C}_{IA}^{5,\infty}$	2471.0	6907.0	1931.0	5343.7	98.3	137.2	58.2	88.1	699.7	360.2	397.0	224.6	7.1	2.6	6.8	2.6
$\mathcal{C}_{IA}^{6,\infty}$	2471.0	10397.0	1931.0	7951.7	86.9	151.8	51.8	101.1	704.3	378.1	399.5	244.3	8.1	2.5	7.7	2.4
$\mathcal{C}_{IA}^{7,\infty}$	2471.0	11918.0	1931.0	9065.7	73.3	143.0	44.2	97.0	629.1	346.8	357.7	228.5	8.6	2.4	8.1	2.4
$\mathcal{C}_{IA}^{8,\infty}$	2471.0	9740.0	1931.0	7409.4	71.0	121.9	42.9	81.3	637.0	302.3	362.1	195.9	9.0	2.5	8.5	2.4
$\mathcal{C}_{IA}^{9,\infty}$	2471.0	10249.0	1931.0	7851.1	53.9	99.0	33.4	67.7	825.1	239.3	466.8	159.0	15.3	2.4	14.0	2.4
$\mathcal{C}_{IA}^{10,\infty}$	2471.0	10191.0	1931.0	7783.9	53.9	96.2	33.4	65.5	744.2	233.7	421.7	154.7	13.8	2.4	12.7	2.4
$\mathcal{C}_{IA}^{11,\infty}$	2471.0	11256.0	1931.0	8589.3	50.5	97.1	31.5	66.9	792.1	232.1	448.4	155.7	15.7	2.4	14.3	2.3
$\mathcal{C}_{IA}^{5,7}$	2471.0	4707.0	1931.0	3657.5	166.7	187.6	96.2	112.3	679.2	529.4	385.6	311.1	4.1	2.8	4.0	2.8
$\mathcal{C}_{IA}^{5,8}$	2471.0	5017.0	1931.0	3880.4	137.1	161.4	79.7	98.3	624.5	447.4	355.1	266.7	4.6	2.8	4.5	2.7
$\mathcal{C}_{IA}^{5,9}$	2471.0	5849.0	1931.0	4552.2	132.5	164.4	77.2	101.9	631.3	448.9	358.9	271.3	4.8	2.7	4.7	2.7
$\mathcal{C}_{IA}^{5,10}$	2471.0	7289.0	1931.0	5645.6	125.7	169.8	73.4	107.4	610.8	452.8	347.5	278.5	4.9	2.7	4.7	2.6
$\mathcal{C}_{IA}^{6,10}$	2471.0	7245.0	1931.0	5591.0	105.2	147.4	62.0	94.3	537.9	387.4	306.9	240.8	5.1	2.6	5.0	2.6
$\mathcal{C}_{IA}^{6,11}$	2471.0	7933.0	1931.0	6102.8	98.3	145.4	58.2	94.1	697.5	376.6	395.7	236.7	7.1	2.6	6.8	2.5
$\mathcal{C}_{IA}^{6,12}$	2471.0	10378.0	1931.0	7927.1	93.8	160.8	55.6	106.5	680.4	402.9	386.2	258.9	7.3	2.5	7.0	2.4

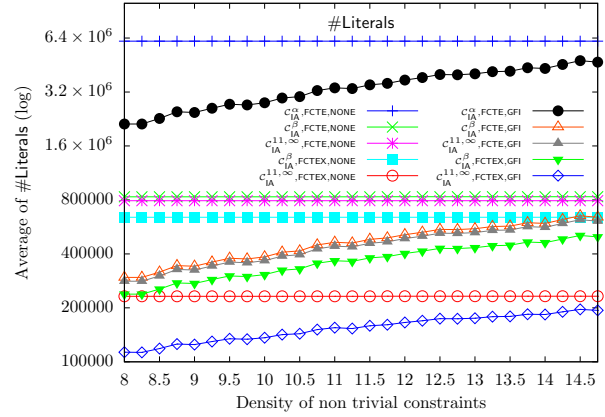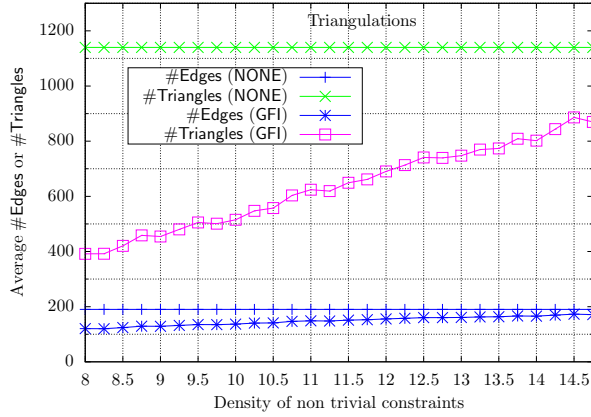Figure 4: Statistics about the PMAX-SAT instances of $A(20, d, 6.5)$ with $d \in \{8, \dots, 14.75\}$

Figure 5: Number of edges/triangles of the graphs obtained by the triangulation GFI and the number of literals for some encodings.

lows us to obtain PMAX-SAT instances of much smaller size compared with completing the constraint graph of the QCN (see Figure 5). Figure 4 shows that the denser a QCN with respect to the number of its non-trivial constraints is, the lesser the gain we have due to the use of the GFI heuristic, which is expected. Further, we note that forbidden coverings have a significant effect on the size of a PMAX-SAT instance. In particular, and as expected, the forbidden coverings $\mathcal{C}_{IA}^{9,\infty}$, $\mathcal{C}_{IA}^{10,\infty}$, $\mathcal{C}_{IA}^{11,\infty}$ lead to a much smaller number of clauses compared with $\mathcal{C}_{IA}^{\alpha}$ or $\mathcal{C}_{IA}^{\beta}$, which are the forbidden coverings proposed in (Pham, Thornton, and Sattar 2008). Concerning the FCTE encoding, the smaller numbers of literals are obtained through the use of forbidden coverings $\mathcal{C}_{IA}^{5,10}$ and $\mathcal{C}_{IA}^{6,10}$. However, the FCTEX encoding can lead to PMAX-SAT instances with a much smaller number of literals in many cases, as in the case of the forbidden coverings $\mathcal{C}_{IA}^{9,\infty}$, $\mathcal{C}_{IA}^{10,\infty}$, and $\mathcal{C}_{IA}^{11,\infty}$. The clause factorization used in FCTEX allows us to reduce the impact of oversized c-triples on the forbidden covering at hand. Concerning the

average size of the clauses, the FCTE encoding can lead to PMAX-SAT instances of a big average size of the clauses, e.g., 15.7 for $\mathcal{C}_{IA}^{11,\infty}$, whilst the FCTEX encoding leads to PMAX-SAT instances of an average size of the clauses that is always less than 3; this is compensated by a higher number of propositional variables used in the FCTEX encoding compared with the FCTE encoding.

The second part of our analysis concerns the time needed for solving the obtained PMAX-SAT instances. We used the QMaxSAT solver (Koshimura et al. 2012) in our experiments. We note that the obtained solving times do not include the time required for encoding a given QCN, as it was negligible *w.r.t.* the time required for solving that QCN with the PMAX-SAT solver. We only report results concerning GFI triangulation based encodings, as encodings not involving a triangulation of the constraint graph of the input QCN led to much worse results. For the first experiment, we considered a timeout of $1\,500$ seconds for solving the PMAX-SAT instances of model $A(20, d, 6.5)$ with $d \in \{8, \dots, 14.75\}$. We first consider the results uniquely

$\mathcal{C}$	#TO	#BT1	#BT2	#BT3
$\mathcal{C}_{IA}^{\alpha}$	30	3	7	11
$\mathcal{C}_{IA}^{\beta}$	**25**	**61**	**99**	**136**
$\mathcal{C}_{IA}^{1,\infty}$	23	65	124	152
$\mathcal{C}_{IA}^{3,\infty}$	23	55	102	137
$\mathcal{C}_{IA}^{5,\infty}$	27	0	2	4
$\mathcal{C}_{IA}^{6,\infty}$	36	0	1	3
$\mathcal{C}_{IA}^{7,\infty}$	32	1	1	3
$\mathcal{C}_{IA}^{8,\infty}$	35	0	0	0
$\mathcal{C}_{IA}^{9,\infty}$	67	0	0	0
$\mathcal{C}_{IA}^{10,\infty}$	65	0	0	0
$\mathcal{C}_{IA}^{11,\infty}$	68	0	0	0
$\mathcal{C}_{IA}^{max}$	62	0	0	0
$\mathcal{C}_{IA}^{5,7}$	23	36	75	120
$\mathcal{C}_{IA}^{5,8}$	24	26	54	106
$\mathcal{C}_{IA}^{5,9}$	26	8	24	44
$\mathcal{C}_{IA}^{5,10}$	31	5	17	31
$\mathcal{C}_{IA}^{6,10}$	**25**	7	19	33
$\mathcal{C}_{IA}^{6,11}$	31	0	4	7
$\mathcal{C}_{IA}^{6,12}$	33	0	3	6

(a) FCTE, GFI

$\mathcal{C}$	#TO	#BT1	#BT2	#BT3
$\mathcal{C}_{IA}^{\alpha}$	30	0	1	1
$\mathcal{C}_{IA}^{\beta}$	21	12	33	50
$\mathcal{C}_{IA}^{1,\infty}$	23	25	42	64
$\mathcal{C}_{IA}^{3,\infty}$	27	16	31	50
$\mathcal{C}_{IA}^{5,\infty}$	**13**	**31**	**66**	**86**
$\mathcal{C}_{IA}^{6,\infty}$	21	5	10	14
$\mathcal{C}_{IA}^{7,\infty}$	21	9	15	20
$\mathcal{C}_{IA}^{8,\infty}$	**13**	**34**	**59**	**78**
$\mathcal{C}_{IA}^{9,\infty}$	18	3	4	8
$\mathcal{C}_{IA}^{10,\infty}$	18	4	4	8
$\mathcal{C}_{IA}^{11,\infty}$	22	1	2	7
$\mathcal{C}_{IA}^{max}$	19	2	5	7
$\mathcal{C}_{IA}^{5,7}$	20	23	55	**83**
$\mathcal{C}_{IA}^{5,8}$	**16**	**44**	**78**	**112**
$\mathcal{C}_{IA}^{5,9}$	18	15	38	58
$\mathcal{C}_{IA}^{5,10}$	19	9	24	39
$\mathcal{C}_{IA}^{6,10}$	22	15	28	46
$\mathcal{C}_{IA}^{6,11}$	**17**	18	32	55
$\mathcal{C}_{IA}^{6,12}$	22	3	11	19

(b) FCTEX, GFI

$\mathcal{C}$	Encoding	#BT1	#BT2	#BT3
$\mathcal{C}_{IA}^{\beta}$	FCTE	5	11	16
$\mathcal{C}_{IA}^{1,\infty}$	FCTE	7	14	19
$\mathcal{C}_{IA}^{3,\infty}$	FCTE	6	11	17
$\mathcal{C}_{IA}^{5,7}$	FCTE	**17**	**29**	**36**
$\mathcal{C}_{IA}^{5,8}$	FCTE	6	14	18
$\mathcal{C}_{IA}^{\beta}$	FCTEX	5	14	21
$\mathcal{C}_{IA}^{1,\infty}$	FCTEX	6	17	23
$\mathcal{C}_{IA}^{5,\infty}$	FCTEX	7	20	28
$\mathcal{C}_{IA}^{6,\infty}$	FCTEX	5	15	24
$\mathcal{C}_{IA}^{7,\infty}$	FCTEX	**16**	20	25
$\mathcal{C}_{IA}^{8,\infty}$	FCTEX	**21**	**22**	**31**
$\mathcal{C}_{IA}^{9,\infty}$	FCTEX	14	20	27
$\mathcal{C}_{IA}^{10,\infty}$	FCTEX	7	17	23
$\mathcal{C}_{IA}^{11,\infty}$	FCTEX	5	13	24
$\mathcal{C}_{IA}^{5,7}$	FCTEX	14	**25**	**37**
$\mathcal{C}_{IA}^{5,8}$	FCTEX	14	**22**	27
$\mathcal{C}_{IA}^{6,10}$	FCTEX	7	14	24
$\mathcal{C}_{IA}^{6,11}$	FCTEX	5	14	21

(c) FCTE + FCTEX, GFI

Figure 6: 280 instances of $A(20, d, 6.5)$ with $d \in \{8, \dots, 14.75\}$, timeout fixed to 1500 seconds, #TO corresponds to the number of timed-out instances, #BT$_i$ with $i \in \{1, 2, 3\}$ is the number of instances for which the encoding leads to a solving time belonging to the i first best times (considering all encodings). (a) concerns only FCTE encodings, (b) concerns only FCTEX encoding, and (c) concerns both FCTE and FCTEX encodings. For (c), only the results for the 18 best encodings are reported.
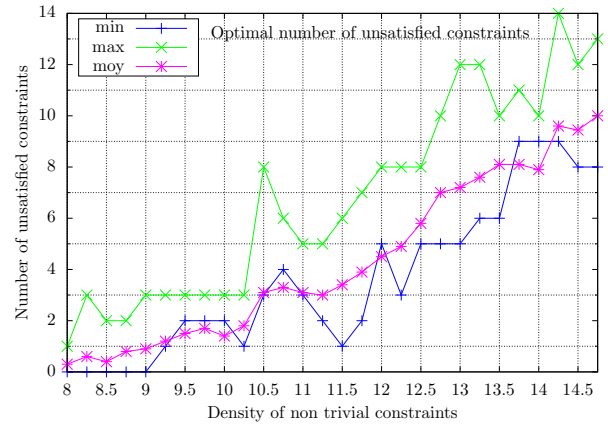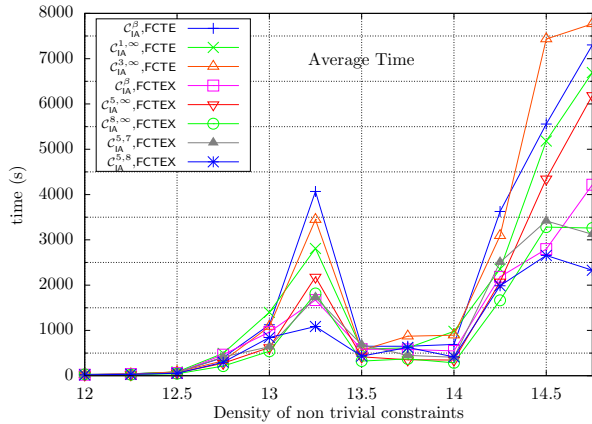

Figure 7: (left) Solving times for the PMAX-SAT instances of $A(20, d, 6.5)$ with $d \in \{8, \dots, 14.75\}$, timeout fixed to 24 hours; (right) Optimal number of unsatisfiable constraints.

concerning FCTE encodings, reported in Figure 6 (a). We note that the best-performing encodings correspond to encodings with clauses of a small size in average and with a reasonable number of clauses. Among these encodings, we have the ones corresponding to the forbidden coverings $\mathcal{C}_{IA}^{\beta}, \mathcal{C}_{IA}^{1,\infty}, \mathcal{C}_{IA}^{3,\infty}, \mathcal{C}_{IA}^{5,7}$, and $\mathcal{C}_{IA}^{5,8}$. By considering only the FCTEX encodings, we note that the ones with the lowest number of variables and a not too high number of literals are performing better than the others. In particular, among these FCTEX encodings, we have the encodings corresponding to the forbidden coverings $\mathcal{C}_{IA}^{5,\infty}, \mathcal{C}_{IA}^{8,\infty}, \mathcal{C}_{IA}^{5,7}$, and $\mathcal{C}_{IA}^{5,8}$. We note that in general the FCTEX encodings perform better than the FCTE encodings. Finally, in order to compare our approach with the one of (Condotta et al. 2015), we solved the QCN instances with the Medusa solver. For all PMAX-SAT encodings and for almost all QCN instances, the QMaxSAT

Density of non trivial constraints	#Solved for $\mathcal{C}_{IA}^{11,\infty}$,FCTE	#Solved for Medusa
8.00 / 8.25 / 8.50 / 8.75	10 / 10 / 10 / 10	8 / 6 / 7 / 7
9.00 / 9.25 / 9.50 / 9.75	10 / 10 / 10 / 10	6 / 6 / 3 / 3
10.00 / 10.25 / 10.50 / 10.75	10 / 10 / 10 / 10	4 / 2 / 2 / 1
11.00 / 11.25 / 11.50 / 11.75	10 / 10 / 10 / 10	0 / 0 / 0 / 0
12.00 / 12.25 / 12.50 / 12.75	10 / 10 / 8 / 4	0 / 0 / 0 / 0
13.00 / 13.25 / 13.50 / 13.75	5 / 5 / 4 / 1	0 / 0 / 0 / 0
14.00 / 14.25 / 14.50 / 14.75	3 / 2 / 0 / 0	0 / 0 / 0 / 0

Table 2: Number of solved instances for the PMAX-SAT instances of $A(20, d, 6.5)$ for Medusa and for the FCTE encoding $\mathcal{C}_{IA}^{11,\infty}$ (with the triangulation GFI) with $d \in \{8, \dots, 14.75\}$, timeout fixed to 1500 seconds.

solver outperforms Medusa. Moreover, Medusa solved only 55 out of the 280 instances (see Table 2).

For the last part of our analysis we selected some encodings that were previously found to be better performing and solved the corresponding PMAX-SAT instances with a timeout of 24h in order to obtain results as complete as possible

(see Figure 7). First, we note that even with this high time-out, some PMAX-SAT instances are not solved for some encodings. Among the 280 QCN instances, only two instances are not solved in the given timeout for some encodings; the solving times and the optimal number of unsatisfiable constraints for these two QCNs are not taken into account in Figure 7 for any of the encodings. Clearly, the more the density of non-trivial constraints increases, the more the optimal number of unsatisfiable constraints increases and, consequently, the more difficult the instances can be to solve. Further, we remind the reader that the denser a QCN with respect to the number of its non-trivial constraints is, the lesser the gain we have due to the use of the GFI heuristic. The better performing encodings are the FCTEX encodings. In particular, the FCTEX encodings corresponding to the forbidden coverings $\mathcal{C}_{\text{IA}}^{8,\infty}, \mathcal{C}_{\text{IA}}^{5,7}$ and $\mathcal{C}_{\text{IA}}^{5,8}$ have superior performance for hard instances.

7 Conclusions

In this work, we introduced two families of encodings of the partial maximum satisfiability problem (PMAX-SAT), called FCTE and FCTEX, for solving the MAX-QCN problem. These encodings are based on forbidden coverings of the composition table of the considered qualitative calculus and a triangulation of the constraint graph of the QCNs at hand. Depending on the chosen forbidden covering, we can obtain PMAX-SAT instances that are very compact. Moreover, some encodings can lead to efficient solving. We have experimentally shown that our approach outperforms the one presented in (Condotta et al. 2015). Future work consists of conducting experiments with several PMAX-SAT solvers to compare their behavior against the instances obtained through the FCTE and FCTEX encodings. Another perspective consists of using forbidden coverings in the context of SAT encodings for the consistency problem of QCNs.

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