Ranking-based Semantics for Abstract Argumentation

THÈSE

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Introduction

Argumentation is an integral part of everyday life. Expressing our opinions is common in our social worlds: from politics (in political debates), law (defense attorney vs. prosecutor), academia (debates between authors and reviewers), business (convincing others to buy a product) to our personal lives as shown in the following example:

Steven (x_1) *"The last movie of Christopher Nolan has received an overall rating of one out of five stars by The New York Times, so I do not know whether I will watch it."*

Emma (x_2) *"My friend John, who works in film industry, watched the movie and really loved it."*

Thus, we use arguments to defend an opinion, counter-arguments to attack another argument, etc. The existence of arguments for and arguments against an idea confronts us with conflicting information, and we can be forced to deal with the resulting inconsistencies. Often, we choose subconsciously in weighting information and select some items of information in preference to others. But, in a more conscious way, like when the consequences of a choice are "more" important, dealing with conflicting information is not so easy. For example, in our previous exchange between Steven and Emma, Steven, who must take a decision between watching the film or not, receives two conflicting information from The New York Times (it is a bad movie) and from Emma (it is a good movie). Thus, he must decide between the two options, which are inconsistent together (*i.e.* the movie is either good or bad), or seek additional information intended to increase or decrease the acceptability of the affirmation "the film is good". For example, if the following argument from Mike, which attacks the argument from Emma, is added, then the possibility that Steven considers it is a good movie decreases.

Mike (x_3) *"But John is a big fan of Christopher Nolan so he is not objective."*

Thus, argumentation has the effect of increasing or decreasing the acceptance of an opinion, usually with the aim of convincing someone with a different opinion.

By being as much at the heart of human reasoning and interactions, it is not surprising to find many researches related to argumentation in disciplines such as psychology, linguistics or philosophy. But argumentation is also by now an acknowledged branch of one of the main subfields of artificial intelligence (AI), namely knowledge representation and reasoning (KR), that is particularly concerned with reasoning with incomplete, uncertain and conflicting information. It is widely used in areas such as multi-agent systems, common-sense reasoning, decision making. Argumentation, in the field of artificial intelligence, is a formalism allowing to reason with contradictory information as well as to model an exchange of arguments between one or several agents.

A first step is sometimes needed to generate a set of arguments from "unstructured" data and determine in which ways these arguments interact. According to the type of data, a large number of works was introduced. For instance, some works [BESNARD & HUNTER 2008] investigate the formalization of arguments in the setting of classical logic. The data are represented by a knowledge base which is a finite set of formulae, and an argument is a pair whose the first item is a minimal consistent set of formulae (premises) that entails the second item (claim). Then an attack between arguments is captured for example when the claim of an argument refutes the premises of another argument [ELVANG-GØRANSSON *et al.* 1993]. Another family of formalization concerns argument mining (see [LIPPI & TORRONI 2016] for a recent state of the art) which aims at automatically recognizing argumentation structures in unstructured textual documents (legal text, scientific articles, student essays, discourse, . . .). The goal is segmenting texts into argumentative units, classifying the types of units (premise, claim) and identifying relations between them before evaluating automatically the quality of arguments.

The arguments and the conflicts between arguments involved in the argumentation process (and potentially revealed with the previously stated methods) can be represented with an argumentation framework. In this thesis, we focus on the one proposed in [DUNG 1995] which is very abstract and allows to grasp the characteristics of many other argumentation frameworks. An abstract argumentation framework is a set of abstract entities, called arguments, representing any piece of information (*e.g.* beliefs, statements, actions to be performed), linked with some attacks, which indicate the existence of conflicts between the arguments. Thus, an argumentation framework is often represented by a directed graph, in which nodes represent arguments and arrows represent the attack relation between arguments. The debate discussed at the beginning of this introduction can be represented by the argumentation framework shown in Figure 1 where the argument x_3, given by Mike, attacks the argument x_2, given by Emma, which attacks the argument x_1, given by Steven.

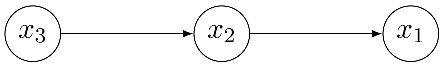


Figure 1 – Argumentation framework representing the movie example

The goal of the argumentation process is to evaluate the arguments, taking into account the existing conflicts between them, in order to determine their degree of acceptability. Given an argumentation framework, the first main reasoning task was to find sets of arguments which can be jointly considered as accepted [DUNG 1995]. These sets of arguments, called extensions, are computed using acceptability semantics defined by a set of conditions that a set of arguments must satisfy (*e.g.* the conflict-freeness) to be considered as acceptable with respect to a given semantics. Thus, if we reason with the argumentation framework depicted in Figure 1, we can first accept the argument x_3 because there is no argument against it (*i.e.* it is not attacked). This implies that x_2, which is directly attacked by x_3, must be rejected. Finally, the argument x_1 is attacked by x_2, but since x_2 is rejected, we can accept x_1 and deducing that the movie is bad.

Thus, we obtain one extension containing the arguments x_3 and x_1. From these extensions, several inference tasks were introduced to analyze if an argument is accepted or not. For example, it can be the case if this argument belongs to all extensions. Since Dung introduced his extension-based semantics [DUNG 1995], a lot of works have been made aiming to extend these semantics (see [BARONI *et al.* 2011] for an overview of the existing semantics), to define alternative ones (*e.g.* the labelling-based semantics [CAMINADA 2006a]), to study their behavior (*e.g.* [BARONI & GIACOMIN 2007]), to identify the complexity of computing the extensions [DUNNE & WOOLDRIDGE 2009], etc.

The extension-based semantics can be used in applications like paraconsistent reasoning. However, there exists some other applications where they are not appropriate. Indeed, some aspects of these semantics, like the existence of multiple extensions, the non-existence of extensions or having only two levels of acceptability (accepted or not accepted), can be sometimes problematic. It is the case, for example, for decision-making problems (see the discussion in [AMGOUD & BEN-NAIM 2013]) or for online debate platforms, where additional information are available (see the discussion in [LEITE & MARTINS 2011]).
Thus, alternative semantics, with many levels of acceptability, have been introduced to evaluate arguments by directly reasoning on the arguments themselves by exploiting the topology of the argumentation framework: the scoring semantics, which assign a numerical acceptability degree to each argument, and the ranking-based semantics, which associate to any argumentation framework a ranking on the arguments. These two families of semantics are not independent because a scoring semantics can easily be transformed into a ranking one, since the scores assigned to each argument belong to an ordered scale, so it is possible to compare them in order to obtain a ranking between arguments. Contrary to the classical semantics, these two families of semantics are quite recent. Indeed, with few exceptions (*e.g.* [BESNARD & HUNTER 2001, CAYROL & LAGASQUIE-SCHIEX 2005b]), all the ranking-based semantics have been introduced from 2013 and have received more and more attention since then. However, these semantics have been proposed independently and are often compared using one or two nicely designed examples aiming to convince that, in some situations at least, they should be appropriate. But the behaviors of these semantics are not always clear, and thus make difficult the choice of a particular ranking-based semantics for a user. This thesis aims to study and compare the existing ranking-based semantics for abstract argumentation but also to propose new ones with alternative behaviors, more appropriate for some applications.

Thesis Overview

This thesis is articulated in two parts. We first give, in **Part I (State of the Art)**, the notions required for a good understanding of our work.
Chapter 1 (Abstract Argumentation) recalls the bases of the theory of abstract argumentation in focusing on Dung's framework [DUNG 1995]. We give a brief overview of the main acceptability semantics (extension-based semantics and labelling-based semantics) and a presentation of existing frameworks which extend or generalize Dung's framework.
Chapter 2 (Ranking-based Semantics) highlights first the limits of Dung's semantics for some applications (or problems) and explains why the ranking-based semantics are a better choice for those applications. Then, we provide a detailed presentation of the existing ranking-based se-

mantics and of existing properties for these semantics.

In **Part II (Contributions)**, we compare the ranking-based semantics studied in this thesis and propose new semantics with interesting behaviors.
In Dung's semantics, the non-attacked arguments have a great impact on the acceptability of the arguments they attack, while, in existing ranking-based semantics, they have no special impact. It is why we introduce, in **Chapter 3 (Ranking-based Semantics based on Propagation)**, a new family of ranking-based semantics, using the propagation principle, which allow us to control the influence of non-attacked arguments.
In **Chapter 4 (Comparative Study of Ranking-based Semantics)**, we apply two methods to compare the ranking-based semantics. An empirical comparison is first done to measure how similar (or different) are the ranking-based semantics on the basis of the rankings returned by these semantics from randomly generated argumentation frameworks. Then, we provide an axiomatic comparison of all these semantics with respect to the proposed properties.
Following the results obtained in the previous chapter, we questioned, in **Chapter 5 (Ranking-based Semantics for Persuasion)**, the ability of the existing ranking-based semantics to capture persuasion settings. This leads us to introduce a new parametrized ranking-based semantics which is more appropriate in this context.

Conclusion chapter summarizes the contributions of this thesis and points out some interesting future works which are related to the contributions described in this document.

Appendices A, B and **C** contain the proofs of the propositions in Chapter 3, 4 and 5 respectively. The proofs are separated from the contribution chapters for a matter of readability. **Appendix D** contains all the properties (basic idea and formal definition) for ranking-based semantics studied in this thesis. Finally, **Appendix E** contains a set of argumentation frameworks, more or less complex, with, for each of them, the rankings computed for the ranking-based semantics studied in this thesis.

Part I

State of the Art

Chapter 1

Abstract Argumentation

Over the last two decades, a lot of researches on the topic of argumentation are based on the abstract argumentation theory of Dung [DUNG 1995]. The central concept in this work concerns argumentation frameworks which formalize arguments together with a relation denoting conflicts between them. Such frameworks are represented by a directed graph in which the arguments are represented as nodes and the attack relation is represented by the arrows. Given an argumentation framework, one can then examine the question of which set(s) of arguments can be accepted together: answering this question corresponds to defining an argumentation semantics. Various proposals have been formulated in this respect. In this chapter, we introduce these notions of abstract argumentation and recall some approaches which extend Dung's framework with additional elements.

Contents

1.1 Dung's argumentation framework

Dung's argumentation framework [DUNG 1995] is an abstract framework, in which there is no assumption on the nature of the elements it contains. More precisely, neither the structure nor the origin of the arguments are required. Then, an argumentation framework is composed of a set of arguments and of a relation of conflict between them.

Definition 1.1.1 (Argumentation framework)**.**
An **(abstract) argumentation framework** (AF) is a pair $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ where $\mathcal{A}$ is a finite and non-empty set of (abstract) arguments and $\mathcal{R}$ is a binary relation on $\mathcal{A}$, i.e. $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$, called the attack relation. For two arguments $x, y \in \mathcal{A}$, the notation $(x, y) \in \mathcal{R}$ means that x attacks y.

Let us introduce some notation related to the argumentation frameworks.

Notation 1.1.1. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework.

- $Arg(AF) = \mathcal{A}$
- $Att(AF) = \mathcal{R}$
- For two argumentation frameworks $F = \langle \mathcal{A}, \mathcal{R} \rangle$ and $G = \langle \mathcal{A}', \mathcal{R}' \rangle$, we define the union $F \cup G = \langle \mathcal{A} \cup \mathcal{A}', \mathcal{R} \cup \mathcal{R}' \rangle$
- $\mathbb{AF}$ represents the set of all argumentation frameworks

Example 1.1.1 (Example from [CAYROL *et al.* 2006])**.**
Consider the arguments exchanged during a meeting of the editorial board of a newspaper:

(a) *Newspapers have no right to publish a private information about the person X because everybody has a right to privacy.*

(b) *This information is not private because X is a prime minister and all information concerning the prime minister is public.*

Clearly, a conflict occurs between a and b. Indeed, the argument b contradicts the private aspect of the information claimed by a in saying that the information is public because it concerns the prime minister. So b attacks a which means that newspapers are allowed to publish this information. Consider now an additional argument:

(c) *But he is no longer prime minister since he resigned yesterday.*

Here c attacks b because c contradicts the fact that X is a prime minister as suggested by b. Then, the information is really private and cannot be published. But, during the meeting, one last argument has been proposed which attacks c on the day for resignation:

(d) *The resignation will be announced officially this evening on TV, so he is still the prime minister.*

So, at the end of the exchange, we conclude that the person X is still the prime minister which is a public personality so newspapers are allowed to publish the information.

An argumentation framework can be represented as a directed graph whose nodes are arguments of the framework and the arcs represent the attacks between them.

Example 1.1.1 (cont.)**.** This dialogue can be formalized by an argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ with four arguments $\mathcal{A} = \{a, b, c, d\}$ and $\mathcal{R} = \{(b, a), (c, b), (d, c)\}$ the interactions between them. Its graphical representation is shown in Figure 1.1.

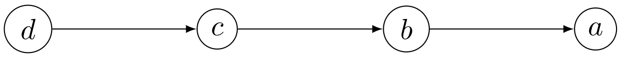


Figure 1.1 – Example of argumentation framework

If an attack between two arguments can be directly observed on an argumentation framework, another implicit relation, called defense, exists and corresponds to two consecutive attacks. The idea of a defense can be represented with the same principle than the famous expression "my enemy's enemy is my friend".

Definition 1.1.2 (Defense)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. Let $x, y \in \mathcal{A}$ be two arguments such that y attacks x. An argument $z \in \mathcal{A}$ defends x against y if z attacks y.

Example 1.1.1 (cont.)**.** Concretely, during the exchange, argument c has been proposed to defend a against b. This allowed to justify the publication ban of this information. But, d has been then introduced to defend b against c in order to finally conclude that the newspapers are allowed to publish this information.

The previous notions of the attack and of the defense only concern two arguments (an argument and its target). But these notions can be extended to a set of arguments which attacks or defends a single argument.

Definition 1.1.3 (Set attacks/defenses an argument)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $S \subseteq \mathcal{A}$.

- Let $x \in \mathcal{A}$ be an argument. S attacks x if $\exists y \in S$ such that $(y, x) \in \mathcal{R}$.
- Let $x, y \in \mathcal{A}$ be two arguments such that $(y, x) \in \mathcal{R}$. S defends x against y if S attacks y.

From a mathematical point of view, Dung's argumentation framework is a directed graph. Let us introduce different notions related to the graphs that we will use in this thesis. All these notions are illustrated in Example 1.1.1 (page 8).

Definition 1.1.4 (Path)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x, y \in \mathcal{A}$ be two arguments. A **path** P from y to x, noted $P(y, x)$, is a sequence $\langle x_0, \dots, x_n \rangle$ of arguments in $\mathcal{A}$ such that $x_0 = x$, $x_n = y$ and $\forall i < n, (x_{i+1}, x_i) \in \mathcal{R}$. The length of the path P is n (the number of attacks it is composed of) and is denoted by $l_P = n$.

According to the length of a path between two arguments, the argument at the beginning of this path can be an attacker or a defender of the argument at the end of the path.

Definition 1.1.5 (Attacker, Defender)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x, y \in \mathcal{A}$ be two arguments. Let $\mathcal{R}_n(x) = \{y \mid \exists P(y,x) \text{ with } l_P = n\}$ be the multiset of arguments that are bound by a path of length n to the argument x. Thus, an argument $y \in \mathcal{R}_n(x)$ is:

- a direct attacker of x if $n = 1$
- a direct defender of x if $n = 2$
- an **attacker** of x if n is odd
- a **defender** of x if n is even

Let us note $\mathcal{R}_+(x) = \bigcup_{n \in 2\mathbb{N}} \mathcal{R}_n(x)$ and $\mathcal{R}_-(x) = \bigcup_{n \in 2\mathbb{N}+1} \mathcal{R}_n(x)$ the multisets of all the defenders and all the attackers of x respectively.

Let us define two particular kinds of paths: branches and cycles.

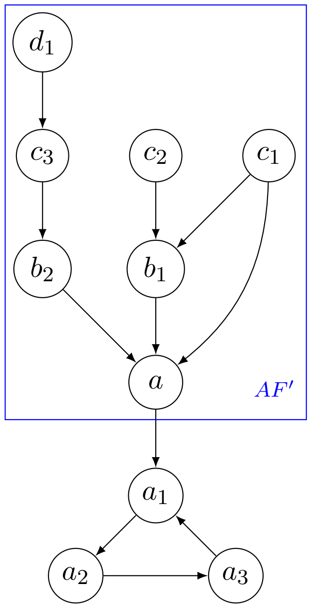
A branch is a path such that the argument at the beginning of the path is not attacked.

Definition 1.1.6 (Branch, Root)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x, y \in \mathcal{A}$ be two arguments.
A **defense root** (respectively **attack root**) of x is a defender (respectively attacker) of x which is not attacked. Let $\mathcal{B}_n(x) = \{y \in \mathcal{R}_n(x) \mid \mathcal{R}_1(y) = \emptyset\}$ be the multiset of roots that are bound by a path of length n to the argument x.
A path from y to x is a **defense branch** (respectively **attack branch**) for x if y is a defense root (respectively attack root) of x.
Let us note $\mathcal{B}_+(x) = \bigcup_{n \in 2\mathbb{N}} \mathcal{B}_n(x)$ and $\mathcal{B}_-(x) = \bigcup_{n \in 2\mathbb{N}+1} \mathcal{R}_n(x)$ the multiset of all defense roots and all the attack roots of x respectively.

A cycle is a path such that the first node is the same as the last one.

Definition 1.1.7 (Cycle, Loop)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x \in \mathcal{A}$. A **cycle** is a path from x to x and a **loop** is a cycle of length 1.

Example 1.1.2.



On this graph, one can find:

- a path of length 2 $\langle c_2, b_1, a\rangle$ from c_2 to a whereas $\langle b_1, a, b_2\rangle$ is not a path,
- a cycle $\langle a_1, a_2, a_3, a_1\rangle$ of length 3,
- c_1 and d_1 are the attack roots of a,
- c_1 and c_2 are the defense roots of a,
- $\langle c_1, a\rangle$ is an attack branch for a of length 1, $\langle d_1, c_3, b_2, a\rangle$ is an attack branch for a of length 3,
- $\langle d_1, c_3, b_2\rangle$, $\langle c_1, a, a_2\rangle$ and $\langle c_2, b_1, a\rangle$ are three possible defense branches of length 2,
- b_1, b_2 and c_1 are the direct attackers of a,
- c_1, c_2 and c_3 are the direct defenders of a.

We also need to introduce the notion of ancestors' graph of an argument x in an argumentation framework AF. It is a subgraph of AF, that contains x and all the attackers and defenders of x, as well as all the attack relations between these arguments.

Definition 1.1.8 (Ancestors' graph)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R}\rangle$ be an argumentation framework and $x \in \mathcal{A}$. The **ancestors' graph** of x is denoted by $Anc_{\text{AF}}(x) = \langle \mathcal{A}', \mathcal{R}'\rangle$ with $\mathcal{A}' = \{x\} \cup \mathcal{R}_+(x) \cup \mathcal{R}_-(x)$ and $\mathcal{R}' = \{(x_1, x_2) \in \mathcal{R} \mid x_1 \in \mathcal{A}' \textit{ and } x_2 \in \mathcal{A}'\}$.

Example 1.1.2 (cont.)**.** The argumentation framework AF', represented in the blue rectangle, is the ancestors' graph of a.

1.2 Acceptability semantics

Given an argumentation framework where conflicts are represented by the attack relation, the main reasoning problem is to determine the positions that a rational agent [1] should accept. Solutions to this problem can be represented by extensions, corresponding to sets of acceptable arguments that are coherent together. Such extensions are computed by using acceptability semantics that can be defined as a set of criteria that should be satisfied by a set of arguments in order to be acceptable. Examples of such semantics are the admissible, complete, preferred, stable, grounded semantics introduced by [DUNG 1995] as well as their refinements: semi-stable [CAMINADA 2006b], ideal [DUNG *et al.* 2007], prudent semantics [COSTE-MARQUIS *et al.* 2005], etc (see [BARONI *et al.* 2011] for an overview). We will concentrate on Dung's semantics in this section.

Among the set of criteria, the property of conflict-freeness seems necessary to obtain a coherent set of arguments such that no argument attacks another argument in the set.

1. In our case, a rational agent is an agent that always chooses to perform an action with the optimal expected outcome for itself from among all feasible actions.

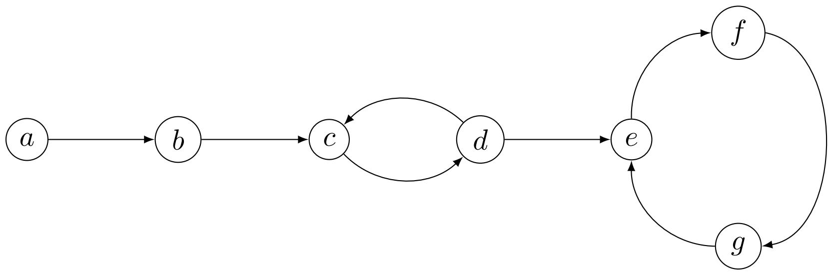


Figure 1.2 – An argumentation framework AF_a

Definition 1.2.1 (Conflict-free set)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. $S \subseteq \mathcal{A}$ is a **conflict-free** set in AF if and only if there exists no $x, y \in S$ such that $(x, y) \in \mathcal{R}$.

Example 1.2.1. In the argumentation framework AF_a depicted in Figure 1.2, the sets of arguments $\{a\}$, $\{b, f\}$, $\{a, c, g\}$ are conflict-free, among others, but $\{a, d, e\}$ is not conflict-free because d attacks e.

A rational agent accepts an argument if it is not attacked, or if it can be defended against each attack targeting it. This idea of acceptability is transcribed in the following definition:

Definition 1.2.2 (Acceptable argument)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. An argument $x \in \mathcal{A}$ is **acceptable** with respect to $S \subseteq \mathcal{A}$ in AF if and only if for each $y \in \mathcal{A}$, if $(y, x) \in \mathcal{R}$ then S defends x against y.

Example 1.2.1 (cont.)**.** The argument f is acceptable with respect to $S = \{d\}$ because S defends f against its only attacker e.

In order to calculate the sets of acceptable arguments with respect to a given set of arguments, the following characteristic function has been introduced by Dung [DUNG 1995].

Definition 1.2.3 (Characteristic function)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. The **characteristic function** $\mathcal{F}_{\text{AF}} : 2^{\mathcal{A}} \to 2^{\mathcal{A}}$ of AF is defined such that for any $S \subseteq \mathcal{A}$:

$$\mathcal{F}_{\text{AF}}(S) = \{x \mid x \in \mathcal{A} \text{ is acceptable with respect to } S\}$$

In other words, $\mathcal{F}_{\text{AF}}(S)$ is the set containing all (and only) arguments in AF that S defends.

Therefore, the set of all arguments accepted by an agent is a conflict-free set of arguments that can defend itself against any attacks. This notion, called admissibility, combines the notions of conflict-freeness and of acceptability and constitutes a basic principle to build the Dung's semantics.

Definition 1.2.4 (Admissible set)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. $S \subseteq \mathcal{A}$ is an **admissible set** of AF if and only if S is conflict-free in AF and, for each $x \in S$, x is acceptable with respect to S in AF.

Example 1.2.1 (cont.)**.** The admissible sets of AF are $\emptyset$, $\{a\}$, $\{d\}$, $\{a, c\}$, $\{a, d\}$, $\{d, f\}$ and $\{a, d, f\}$.

Note that every argumentation framework has at least one admissible set because the empty set is admissible for every argumentation framework.

1.2.1 Extension-based semantics

Dung's semantics propose to refine the admissibility principle in order to select admissible sets satisfying some additional criteria.

Complete Semantics

Complete semantics can be regarded as a strengthening of the basic requirements enforced by the idea of admissibility. Intuitively, while admissibility requires one to be able to give reasons for accepted and rejected arguments but does not consider the eventuality to extend itself with potential acceptable arguments, complete semantics also requires one to include in the set, all the arguments which are acceptable with this set. For example, there is no particular reason to consider the empty set as an extension when it exists at least one non-attacked argument in the argumentation framework.

Definition 1.2.5 (Complete semantics)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. $S \subseteq \mathcal{A}$ is a **complete extension** of AF if and only if S is an admissible set of AF and each argument which is acceptable with respect to S belongs to S.

With the characteristic function, if S is conflict-free and $\mathcal{F}_{\text{AF}}(S) = S$ then S is a complete extension. Note that every argumentation framework has at least one complete extension.

Example 1.2.1 (cont.)**.** The argumentation framework AF_a, depicted in Figure 1.2 (page 12), contains exactly three complete extensions: $\{a\}$, $\{a, c\}$ and $\{a, d, f\}$.

Grounded Semantics

One can remark that the complete semantics may return more than one extension. But, if one regards each complete extension as a reasonable position one can take when there exists conflicting information in the argumentation framework, then a possible question is to examine what is the position which is least questionable.
The grounded semantics aims to return such unique extension which contains only the unquestionable arguments.

Definition 1.2.6 (Grounded Semantics)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. $S \subseteq \mathcal{A}$ is a **grounded extension** of AF if and only if S is the minimal (with respect to the set-theoretical inclusion $\subseteq$) complete extension.

The grounded extension of an argumentation framework AF can be calculated by iterative applications of characteristic function $\mathcal{F}_{\text{AF}}$ to the empty set, *i.e.* it is equal to $\bigcup_{i=0}^{\infty} \mathcal{F}^i_{\text{AF}}(\emptyset)$, where $\mathcal{F}^i_{\text{AF}}(S) = \underbrace{\mathcal{F}_{\text{AF}}(\mathcal{F}_{\text{AF}}(\dots \mathcal{F}_{\text{AF}}}_{i \text{ times}}(S)) \dots)$.

Example 1.2.1 (cont.)**.** The grounded extension of AF_a (see Figure 1.2 page 12) is calculated as follows: $\mathcal{F}_{\text{AF}_a}(\emptyset) = \{a\}$, $\mathcal{F}_{\text{AF}_a}(\{a\}) = \{a\}$. Thus, its grounded extension is the set $\{a\}$.

Preferred Semantics

While grounded semantics takes a skeptical standpoint, one can also consider the alternative view oriented at accepting as many arguments as reasonably possible. This may give rise to mutually exclusive alternatives for acceptance: for instance two arguments which attack each other can be reasonably resolved by accepting one of the conflicting arguments, but not both in order to keep conflict-freeness. The idea of maximizing accepted arguments is expressed by the preferred semantics.

Definition 1.2.7 (Preferred Semantics)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. $S \subseteq \mathcal{A}$ is a **preferred extension** of AF if and only if S is a maximal (with respect to the set-theoretical inclusion $\subseteq$) admissible set.

Note that every argumentation framework has at least one preferred extension [DUNG 1995].

Example 1.2.1 (cont.)**.** The argumentation framework AF_a depicted in Figure 1.2 (page 12) contains exactly two preferred extensions: $\{a, c\}$ and $\{a, d, f\}$.

Stable Semantics

A stable extension is a conflict-free extension that makes a decision on all arguments. In other words, we want that every argument to be a member of the extension, or to be attacked by the extension.

Definition 1.2.8 (Stable Semantics)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. $S \subseteq \mathcal{A}$ is a **stable extension** of AF if and only if S is conflict-free and attacks each argument $x \in \mathcal{A} \backslash S$.

Note that some argumentation framework do not have any stable extension.

Example 1.2.1 (cont.)**.** The argumentation framework AF_a in Figure 1.2 contains exactly one stable extension: $\{a, d, f\}$. One can see that all the other arguments are attacked by this set: b is attacked by a, c and e are attacked by d and finally g is attacked by f.

We denote by $\mathcal{E}_\sigma(AF)$ the set of extensions of AF for the semantics $\sigma \in \{$**co**(mplete), **gr**(ounded), **pr**(eferred), **st**(able)$\}$.

1.2.2 Labelling-based semantics

An alternative way to represent the concepts of admissibility, as well as Dung's semantics, is by using a labelling-based approach [CAMINADA 2006a]. Indeed, rather than stating in terms of sets of arguments, a labelling function can be used to assign a label to each argument. The idea of a labelling is to associate exactly one label to each argument, which can either be in, out or $undec$. The label in indicates that the argument is explicitly accepted, the label out indicates that the argument is explicitly rejected, and the label $undec$ indicates that the status of the argument is undecided, meaning that one abstains from a judgment whether the argument is accepted or rejected.

Definition 1.2.9 (Labelling)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. $\mathcal{L}$ is a **labelling** of AF if and only if $\mathcal{L}$ is a mapping from $\mathcal{A}$ to $\{in, out, undec\}$. We define $in(\mathcal{L})$ as $\{x \in \mathcal{A} \mid \mathcal{L}(x) = in\}$, $out(\mathcal{L})$ as $\{x \in \mathcal{A} \mid \mathcal{L}(x) = out\}$ and $undec(\mathcal{L})$ as $\{x \in \mathcal{A} \mid \mathcal{L}(x) = undec\}$.
$\mathcal{L}$ can also be represented by the set of pairs $\{(x, \mathcal{L}(x)) \mid x \in \mathcal{A}\}$.

The notion of reinstatement labelling allows to ensure that the mapping takes the attack relation into account: an argument is labelled in if it is unattacked or if all its direct attackers are labelled out and an argument is labelled out if at least one of its direct attackers is labelled in, as it is stated in the following definition.

Definition 1.2.10 (Reinstatement Labelling)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. A labelling $\mathcal{L}$ is a **reinstatement labelling** of AF if and only if

- $\forall x \in \mathcal{A}$, $\mathcal{L}(x) = in$ if and only if $\forall y \in \mathcal{R}_1(x)$, $\mathcal{L}(y) = out$;
- $\forall x \in \mathcal{A}$, $\mathcal{L}(x) = out$ if and only if $\exists y \in \mathcal{R}_1(x)$, $\mathcal{L}(y) = in$;
- $\forall x \in \mathcal{A}$, $\mathcal{L}(x) = undec$ if and only if $\nexists y \in \mathcal{R}_1(x)$, $\mathcal{L}(y) = in$ and $\exists z \in \mathcal{R}_1(x)$, $\mathcal{L}(z) = undec$.

Example 1.2.2.
Let us compute the reinstatement labelings on the two argumentation frameworks illustrated in Figure 1.3.

Figure 1.3 – Two argumentation frameworks AF_1 and AF_2

For AF_1, there exists only one reinstatement labelling $\mathcal{L}_1$ with

$$in(\mathcal{L}_1) = \{a, c\}, out(\mathcal{L}_1) = \{b\}, undec(\mathcal{L}_1) = \emptyset$$

For AF_2, there exists three reinstatement labellings $\mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4$ with

$$in(\mathcal{L}_2) = \{d\}, out(\mathcal{L}_2) = \{e\}, undec(\mathcal{L}_2) = \emptyset,$$
$$in(\mathcal{L}_3) = \{e\}, out(\mathcal{L}_3) = \{d\}, undec(\mathcal{L}_3) = \emptyset,$$
$$in(\mathcal{L}_4) = \emptyset \quad , out(\mathcal{L}_4) = \emptyset \quad , undec(\mathcal{L}_4) = \{d, e\}.$$

Caminada defined two functions that, given an argumentation framework, allow a set of arguments to be converted to a labelling and vice versa [Caminada 2006a]. The function $Ext2Lab$ takes a conflict-free set of arguments and converts it to a labelling. The function $Lab2Ext$ takes a labelling and converts it to a set of arguments.

Definition 1.2.11 ($Ext2Lab$, $Lab2Ext$)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $S \subseteq \mathcal{A}$, and $\mathcal{L}$ a labelling of AF. We denote:

- $Lab2Ext(\mathcal{L}) = in(\mathcal{L})$
- $Ext2Lab(S) = \text{IN} \cup \text{OUT} \cup \text{UNDEC}$ where:

$$\begin{aligned}
&\text{IN} = \{(x, in) \mid x \in S\}\\
&\text{OUT} = \{(x, out \mid S \text{ attacks } x\}\\
&\text{UNDEC} = \{(x, undec) \mid x \notin S \text{ and } S \text{ does not attack } x\}
\end{aligned}$$

Caminada showed that a link exists between extensions from Dung's semantics and some particular families of reinstatement labellings. Indeed, the complete extensions exactly match the reinstatement labellings. Then, the other extensions can also be caught with restricted reinstatement labellings (for example the stable semantics match the reinstatement labellings with no undecided argument).

Proposition 1. (Link between labellings and extensions [CAMINADA 2006a])
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. Given $\varepsilon \in \mathcal{E}_\sigma(AF)$ an extension of AF under the semantics σ and $\mathcal{L} = Ext2Lab(\varepsilon)$,

- if ε is a complete extension, then $\mathcal{L}$ is a reinstatement labelling of AF;
- if ε is a stable extension, then $\mathcal{L}$ is a reinstatement labelling of AF such that $undec(\mathcal{L}) = \emptyset$;
- if ε is a preferred extension, then $\mathcal{L}$ is a reinstatement labelling of AF such that $in(\mathcal{L})$ is maximal (with respect to the set-theoretical inclusion $\subseteq$);
- if ε is the grounded extension, then $\mathcal{L}$ is the reinstatement labelling of AF such that $in(\mathcal{L})$ is minimal (with respect to the set-theoretical inclusion $\subseteq$);

Given $\mathcal{L}$ a reinstatement labelling of AF and $\varepsilon = Lab2Ext(\mathcal{L})$,

- ε is a complete extension of AF;
- if $undec(\mathcal{L}) = \emptyset$, then ε is a stable extension of AF;
- if $in(\mathcal{L})$ is maximal (with respect to the set-theoretical inclusion $\subseteq$), then ε is a preferred extension of AF;
- if $in(\mathcal{L})$ is minimal (with respect to the set-theoretical inclusion $\subseteq$), then ε is the grounded extension of AF;

For each semantics σ, $Labs_\sigma(AF)$ denotes the set of labellings associated to the argumentation framework AF with respect to σ.

Example 1.2.1 (cont.).
Let us consider again the argumentation framework AF_a given at Figure 1.2 (page 12) and the three following reinstatement labellings $\mathcal{L}_1, \mathcal{L}_2$ and $\mathcal{L}_3$:

$$\begin{array}{lll} in(\mathcal{L}_1) = \{a\} & , out(\mathcal{L}_1) = \{b\} & , undec(\mathcal{L}_1) = \{c, d, e, f, g\} \\ in(\mathcal{L}_2) = \{a, c\} & , out(\mathcal{L}_2) = \{b, d\} & , undec(\mathcal{L}_2) = \{e, f, g\} \\ in(\mathcal{L}_3) = \{a, d, f\}, & out(\mathcal{L}_3) = \{b, c, e, g\}, & undec(\mathcal{L}_3) = \emptyset \end{array}$$

The labellings for the different semantics are:

- $Labs_{co}(AF) = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3\}$
- $Labs_{gr}(AF) = \{\mathcal{L}_1\}$
- $Labs_{pr}(AF) = \{\mathcal{L}_2, \mathcal{L}_3\}$
- $Labs_{st}(AF) = \{\mathcal{L}_3\}$

The benefit of using labellings over extensions is that arguments that are explicitly rejected (labelled out) and undecided (labelled $undec$) can be distinguished. By contrast, an extension only identifies the accepted arguments, and does not explicitly show a distinction between arguments that are rejected or undecided.

1.2.3 Gabbay's equational approach

The equational approach introduced by Gabbay [GABBAY 2012] is a numerical version of the Dung's approach. The goal is here to compute the extensions of an argumentation framework by assigning a numerical value to each argument instead of the classical qualitative labelling (in, out and $undec$ defined by Caminada [CAMINADA 2006a]). The graph of an abstract argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ is viewed as a system of equations Eq in which each argument $x \in \mathcal{A}$ is represented by a distinct variable $f(x)$ with a domain of real numbers in the interval $[0, 1]$. Solutions to these systems of equations assign to each variable a value from the domain. Thus, the evaluation function $f : \mathcal{A} \to [0, 1]$ is defined recursively because each variable associated to an argument depends of variables which represent the direct attackers of this argument. Some of the possible evaluation functions f as defined in the following definition.

Definition 1.2.12 (Possible equational systems [GABBAY 2012]).
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x \in \mathcal{A}$. Some equational systems Eq applied to f, denoted by $Eq(f)$ are defined as follows:

(1) $Eq_{inverse}(f)$

$$f(x) = \prod_{y \in \mathcal{R}_1(x)} (1 - f(y))$$

(2) $Eq_{geometrical}(f)$

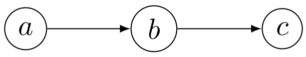
$$f(x) = \frac{\prod_{y \in \mathcal{R}_1(x)} (1 - f(y))}{\prod_{y \in \mathcal{R}_1(x)} (1 - f(y)) + \prod_{y \in \mathcal{R}_1(x)} (f(y))}$$

(3) $Eq_{max}(f)$

$$f(x) = 1 - \max_{y \in \mathcal{R}_1(x)} f(y)$$

Example 1.2.2. Let us define the system of equations of the two argumentation frameworks AF_1. The system of equations is the same in using $Eq_{inverse}$, $Eq_{geometrical}$ and Eq_{max}.

AF_1

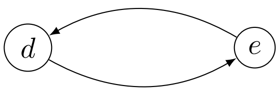


$$\begin{cases} f(a) = 1 \\ f(b) = 1 - f(a) \\ f(c) = 1 - f(b) \end{cases}$$

This system of equations admits one solution where $f(a) = 1$ which implies that $f(b) = 0$ and so $f(c) = 1$.

Example 1.2.3. Let us define the system of equations of the argumentation framework AF_2. The system of equations is the same in using $Eq_{inverse}$, $Eq_{geometrical}$ and Eq_{max}.

AF_2



$$\begin{cases} f(d) = 1 - f(e) \\ f(e) = 1 - f(d) \end{cases}$$

This system of equations admits an infinite number of solution. Among them, $f(d) = 1$ and $f(e) = 0$, $f(d) = 0$ and $f(e) = 1$ or $0 < f(d) < 1$ and $0 < f(e) < 1$ with $f(d) + f(e) = 1$.

Gabbay proved [GABBAY 2012, Theorem 2.7] that the variable assignment with Eq_{max} corresponds to a complete labelling. Indeed, for a given solution, if the variable of an argument is mapped to 1, then this argument is labelled in, if it is mapped to 0 then it is labelled out and undecided otherwise (between 0 and 1). For example, the solution of the system of equations associated to AF_1 allows to define a labelling which is exactly the only complete labelling where a and c are in (because $f(a) = f(c) = 1$), b is out (because $f(b) = 0$) and there is no argument labelled $undec$. The same reasoning holds for AF_2 because it is possible to define the three complete labellings from the solution of its system of equations. Indeed, the solution where $f(d) = 1$ and $f(e) = 0$ corresponds to the labelling where d is in and e is out, the solution where $f(d) = 0$ and $f(e) = 1$ corresponds to the labelling where d is out and e is in and all the other solutions where $f(d)$ and $f(e)$ are between 0 and 1 corresponds to the labelling where d and e are $undec$. This allows for easy identification of the labellings by inspecting the variable assignments.

1.2.4 Status of arguments

In argumentation frameworks, two problems are conventionally associated with semantics which returned several extensions (like the complete, preferred and stable semantics). The first one consists in determining whether a given argument belongs to at least one extension of an argumentation framework given a semantics. If it is the case, this argument is considered as credulously accepted by the semantics. Such argument can be accepted if the agent does not need an absolute certainty about its status.

The second problem allows to know whether a given argument belongs to each extension of an argumentation framework given a semantics. Such argument is considered as skeptically accepted by the semantics and will be selected by an agent who wants no doubt about its acceptability.

Definition 1.2.13 (Status of an argument)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x \in \mathcal{A}$ be an argument. Given a semantics σ and its set of extensions $\mathcal{E}_\sigma(AF)$:

- x is **skeptically accepted** if and only if it belongs to each extension of AF: $\forall \varepsilon \in \mathcal{E}_\sigma(AF), x \in \varepsilon$
- x is **credulously accepted** if and only of it belongs to at least one extension of AF: $\exists \varepsilon \in \mathcal{E}_\sigma(AF), x \in \varepsilon$
- x is **rejected** if and only if it does not belong to any extension of AF: $\forall \varepsilon \in \mathcal{E}_\sigma(AF), x \notin \varepsilon$

The set of credulously accepted arguments and the set of skeptically accepted arguments, for a semantics σ and an argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$, are respectively defined like this:

$$ca_\sigma(AF) = \bigcup_{\varepsilon \,\in\, \mathcal{E}_\sigma(AF)} \varepsilon$$

$$sa_\sigma(AF) = \bigcap_{\varepsilon \,\in\, \mathcal{E}_\sigma(AF)} \varepsilon$$

Note that when only one extension exists, any argument belonging to this extension will be both skeptically and credulously accepted. More generally, any argument skeptically accepted is also credulously accepted ($\forall AF \in \mathbb{AF}, sa_\sigma(AF) \subseteq ca_\sigma(AF)$), the converse is not true.

Example 1.2.3 (cont.)**.** Each set of credulously accepted arguments and skeptically accepted arguments for the complete, grounded, preferred and stable semantics computed from AF_a are shown in Table 1.1.

	Complete	Grounded	Preferred	Stable
$\mathcal{E}_\sigma(AF)$	$\{a\}, \{a,c\}, \{a,d,f\}$	$\{a\}$	$\{a,c\}, \{a,d,f\}$	$\{a,d,f\}$
$ca_\sigma(AF)$	$\{a,c,d,f\}$	$\{a\}$	$\{a,c,d,f\}$	$\{a,d,f\}$
$sa_\sigma(AF)$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a,d,f\}$

Table 1.1 – Set of credulously accepted arguments and skeptically accepted arguments from Dung's semantics on AF_a

1.2.5 Link between Dung's semantics

As can be observed in Table 1.2, the extensions returned by the different Dung's semantics are not always disconnected.

$\mathcal{E}_\sigma(AF)$	Complete	Grounded	Preferred	Stable
$\{a\}$	✓	✓		
$\{a, c\}$	✓		✓	
$\{a, d, f\}$	✓		✓	✓

Table 1.2 – List of extensions from Dung's semantics on AF_a

Indeed, in this example, each extension is shared by at least two semantics. In addition, one can see that the stable extension is included in the set of preferred extensions which is included in the set of complete extensions. However, the grounded semantics does not share any extension with the preferred and the stable semantics but seems included in the set of complete extensions.
Even if these observations only concern a particular argumentation framework, Dung has proved that they are valid for all argumentation frameworks.

Proposition 2. [Dung 1995]
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework.

- Each stable extension is a preferred extension ($\mathcal{E}_{st}(AF) \subseteq \mathcal{E}_{pr}(AF)$)
- Each preferred extension is a complete extension ($\mathcal{E}_{pr}(AF) \subseteq \mathcal{E}_{co}(AF)$)
- The grounded extension is a complete extension ($\mathcal{E}_{gr}(AF) \in \mathcal{E}_{co}(AF)$)

Figure 1.4 (page 20) gives an overview of the inclusion relations between the extension-based semantics discussed here.

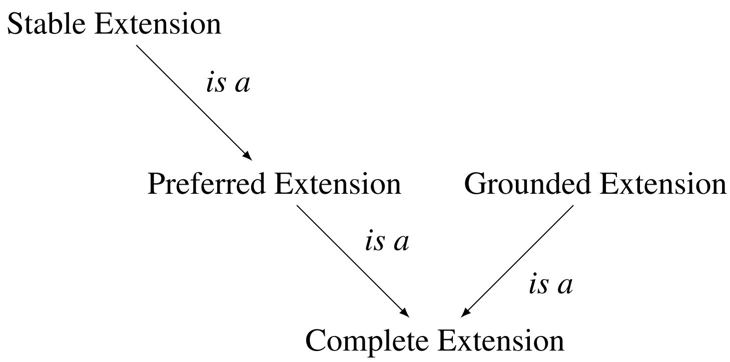


Figure 1.4 – Inclusions between Dung's semantics

Dung has also given some sufficient conditions on the argumentation framework for some semantics to coincide. The first one concerns argumentation frameworks without controversial arguments.

Definition 1.2.14 (Controversial argument)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. An argument $y \in \mathcal{A}$ is **controversial** with

respect to $x \in \mathcal{A}$ if y is an attacker and a defender of x ($y \in \mathcal{R}_-(x)$ and $y \in \mathcal{R}_+(x)$).
An argument is controversial if it is controversial with respect to at least one argument in $\mathcal{A}$.

Example 1.2.4. In the argumentation framework depicted in Figure 1.5, c is controversial with respect to a because c is an attacker of a ($c \in \mathcal{R}_1(a)$) but also a defender of a ($c \in \mathcal{R}_2(a)$).

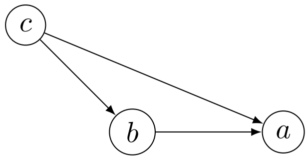


Figure 1.5 – Example of a controversial argument

Definition 1.2.15 (Uncontroversial argumentation framework)**.**
An argumentation framework AF is **uncontroversial** if none of its arguments is controversial.

Proposition 3. [DUNG 1995]
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an uncontroversial argumentation framework.

- Each preferred extension of AF is also a stable extension ($\mathcal{E}_{pr}(AF) \subseteq \mathcal{E}_{st}(AF)$)
- The grounded extension of AF coincides with the intersection of all preferred extensions ($\mathcal{E}_{gr}(AF) = \bigcap_{\varepsilon \in \mathcal{E}_{pr}(AF)} \varepsilon$)

In combining the first result from Proposition 3 with the first one from Proposition 2 (page 20), one can conclude that, in this particular case, the set of preferred extensions of AF and the set of stable extensions of AF coincide ($\mathcal{E}_{pr}(AF) = \mathcal{E}_{st}(AF)$).

The second case where some semantics coincide is when the argumentation framework is well-founded.

Definition 1.2.16 (Well-founded argumentation framework)**.**
An argumentation framework AF is **well-founded** if and only if AF does not contain any cycle.

Proposition 4. [DUNG 1995]
Every well-founded argumentation framework has exactly one complete extension which is also grounded, preferred and stable.

We know that the grounded extension can easily be computed by iterative applications of the characteristic function $\mathcal{F}_{\text{AF}}$ to the empty set. So, according to the previous proposition, we obtain, with this method, the extension of the other semantics too. Let us explain how work this function through the following algorithm;

1. accept the non-attacked arguments (the existence of at least one non-attacked argument is guarantee because the graph is acyclic);

2. reject the arguments which are directly attacked by an accepted argument and accept the arguments such that all their direct attackers are rejected;
3. iterate until each argument is either accepted or rejected.

Example 1.2.6.
Let us consider the well-founded argumentation framework AF represented in Figure 1.6.

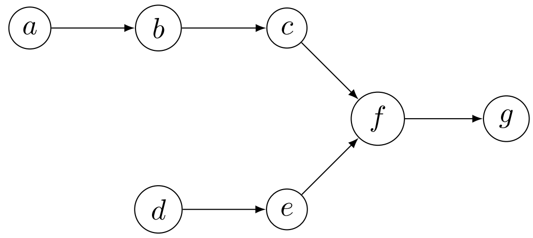


Figure 1.6 – A well-founded argumentation framework

Arguments a and d are not attacked, so they are accepted. Consequently, b and e are rejected. As b is rejected, then c is accepted. So, f has one direct attacker (c) which is accepted and one direct attacker (e) which is rejected. According to the algorithm, when an argument has at least one accepted direct attacker then this argument is rejected, so f is rejected. Consequently, g is accepted. Hence, the single extension for all Dung's semantics is $\{a, c, d, g\}$ ($\forall \sigma \in \{co, gr, pr, st\}, \mathcal{E}_\sigma(AF) = \{\{a, c, d, g\}\}$).

1.3 Extension of Dung's framework

Thanks to its simple representation, Dung's framework led many works which extend the framework with additional elements (support relation, weight on the arguments and/or on the attacks, preferences, votes, ...) in order to capture more information related to argumentation. In this section, we briefly recall some of these frameworks.

1.3.1 Bipolar argumentation frameworks

In Dung's framework, only one relation between arguments exists: the attack relation. This relation is essential in argumentation to represent a conflict between two arguments, but another relation has been introduced: the *support relation*, to represent the support, the help brought by an argument to another argument. This new relation must not be confused with the defense, represented by two successive attacks. Indeed, the support relation is totally independent of the attack relation. Thus, directly considering an explicit support relation gives the possibility to represent this positive link independently of any other argument. This positive interaction between arguments has been first introduced by [KARACAPILIDIS & PAPADIAS 2001, VERHEIJ 2002] and then included in the Dung's framework to obtain *bipolar argumentation frameworks* by [CAYROL & LAGASQUIE-SCHIEX 2005c, CAYROL & LAGASQUIE-SCHIEX 2013].

Definition 1.3.1 (Bipolar argumentation framework[CAYROL & LAGASQUIE-SCHIEX 2005c]). A **bipolar argumentation framework** (BAF) is a triplet $\langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, where $\mathcal{A}$ is a finite and non-empty set of arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is an attack relation and $\mathcal{S} \subseteq \mathcal{A} \times \mathcal{A}$ is a support relation. For two arguments $x, y \in \mathcal{A}$, the notation $(x, y) \in \mathcal{R}$ means that x attacks y and the notation $(x, y) \in \mathcal{S}$ means that x supports y.

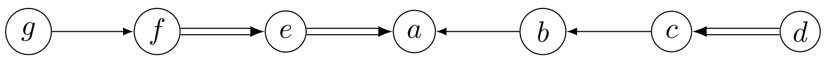


Figure 1.7 – A bipolar argumentation framework

The graph represented in Figure 1.7 is an example of bipolar argumentation framework [2] where one can find the attack relation like in Dung's framework (c attacks b, b attacks a and g attacks f) combined to the support relation (d supports c, f supports e and e supports a). When two consecutive attacks are defined as a defense in Dung's framework, additional combinations like a supported attack (*e.g.* d supports c which attacks b) or an attacked support (*e.g.* g attacks f which supports e) can be considered in a BAF. Indeed, new kinds of attack, called complex attack, have been defined [CAYROL & LAGASQUIE-SCHIEX 2005c] from the interaction between attacks and supports. For instance, these complex attacks can be defined as a supported attack (*e.g.* a supported attack from g to e) or as a secondary attack (*e.g.* a secondary attack from d to b).

An abstract bipolar framework is useful as an analytic tool for studying different notions of complex attacks and new semantics taking into account attack and support interactions between arguments. However, the drawback is the lack of guidelines for choosing the appropriate definitions and semantics depending on the application. Indeed, several interpretations of the support exist in the literature. Among them, one can find:

- Deductive support [BOELLA *et al.* 2010] says that if $(a, b) \in \mathcal{S}$ then the acceptance of a implies the acceptance of b, and its contrapositive the non-acceptance of b implies the non-acceptance of a.

- Necessary support [NOUIOUA & RISCH 2010] considers that if $(a, b) \in \mathcal{S}$ then the acceptance of a is necessary to get the acceptance of b, or equivalently the acceptance of b implies the acceptance of a. The argumentation frameworks with necessities refer to this interpretation of the support relation [NOUIOUA & RISCH 2011].

- Evidential support [OREN & NORMAN 2008] is intended to capture the notion of support by evidence: arguments can be accepted only if they are supported (directly or indirectly) by *prima-facie* arguments which are the supports of some special argument called evidence. Evidential argumentation frameworks are defined for this particular interpretation.

2. In [CAYROL & LAGASQUIE-SCHIEX 2005c] the graphical representation of the support relation is a single arrow whereas the attack relation is represented by a crossed out arrow. But, in order to follow the original notation which represents an attack by an simple arrow [DUNG 1995], we choose to represent the support relation by a double arrow.

Finally, concerning semantics, some basic notions (like conflict-freeness or acceptability), used to build Dung's semantics, have been redefined in order to take into account complex attacks and support [CAYROL & LAGASQUIE-SCHIEX 2005c].

1.3.2 Partial argumentation framework

In Dung's framework, in focusing on the attack from an argument x to another argument y, an agent can consider that the attack exists, and in this case the attack is represented in the argumentation framework, or she considers that there is no attack and then the attack is not represented in the argumentation framework. However, in multi-agent settings where each agent has its own argumentation framework, we cannot assume that one agent knows all the arguments (and consequently all the attacks) which are known by the other agents. Indeed, during an interaction between several agents, when one argument is added, this agent ignores which are the relations between this argument and all the others she knows. So, three cases must be considered:

- the agent believes that the interaction (x, y) exists (attack);
- the agent believes that the interaction (x, y) does not exist (non-attack);
- the agent does not know whether the interaction (x, y) exists (ignorance).

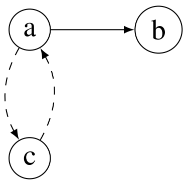
To represent these three possibilities, [COSTE-MARQUIS *et al.* 2007] introduced *partial argumentation frameworks* where an additional relation, called *ignorance relation*, is added to Dung's framework.

Definition 1.3.2 (Partial argumentation framework [COSTE-MARQUIS *et al.* 2007])**.**
A (finite) **partial argumentation framework** (PaAF) is a 4-tuple $PaAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{I}, \mathcal{N} \rangle$ where $\mathcal{A}$ is a (finite) set of arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is an attack relation, $\mathcal{I} \subseteq \mathcal{A} \times \mathcal{A}$ is an ignorance relation such that $\mathcal{R} \cap \mathcal{I} = \emptyset$ and $\mathcal{N} \subseteq \mathcal{A} \times \mathcal{A}$ is a non-attack relation such that $\mathcal{N} = (\mathcal{A} \times \mathcal{A}) \backslash (\mathcal{R} \cup \mathcal{I})$.

Each partial argumentation framework can be viewed as a compact representation of a set of argumentation frameworks, called its *completions*. Indeed, a completion of a partial argumentation framework is a classical argumentation framework where each ignorance relation between two arguments is replaced by an attack or a non-attack.

Definition 1.3.3 (Completion of a PaAF [COSTE-MARQUIS *et al.* 2007])**.**
Let $PaAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{I}, \mathcal{N} \rangle$ be a partial argumentation framework and $AF = \langle \mathcal{A}', \mathcal{R}' \rangle$ be an argumentation framework. AF is a **completion** of $PaAF$ if and only if $\mathcal{A} = \mathcal{A}'$ and $\mathcal{R} \subseteq \mathcal{R}' \subseteq \mathcal{R} \cup \mathcal{I}$.

Example 1.3.1. Let us compute all the completions of the partial argumentation framework $PaAF = \langle \{a, b, c\}, \{(a, b)\}, \{(a, c), (c, a)\}, \{(a, a), (b, b), (c, c), (b, a), (b, c), (c, b)\} \rangle$ depicted in Figure 1.8.



$PaAF$

The completions of $PaAF$ are:

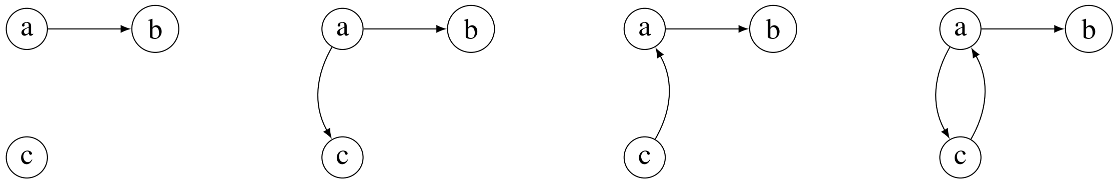


Figure 1.8 – A partial argumentation framework and its completions

This partial argumentation framework is used, for example, for the aggregation problem aiming to aggregate a set of argumentation frameworks, coming from several agents, into a single argumentation framework. Indeed, in this case, the sets of arguments reported by the agents often differ from one another. It is why, before the aggregation, a preliminary step is proposed by [COSTE-MARQUIS *et al.* 2007, CAYROL & LAGASQUIE-SCHIEX 2011a] where they expand each initial argumentation framework with its corresponding partial argumentation framework in order to all the agents shared the same set of arguments.

1.3.3 Weighted argumentation frameworks

Contrary to Dung's framework where all the arguments have the same impact during an attack, it seems quite natural that an argument could have different levels of impact on its target. Suppose, for example, that an argument attacks another one. If the attacker is given by a trusted person, it could be more natural to think that this argument will have more impact on the targeted argument than if it comes from an unknown person.
Obviously, this information cannot be captured by the classical framework, that is why *weighted argumentation frameworks* have been introduced where a positive weight is assigned to each attack [DUNNE *et al.* 2011, COSTE-MARQUIS *et al.* 2012b].

Definition 1.3.4 (Weighted argumentation framework [COSTE-MARQUIS *et al.* 2012b])**.**
A **weighted argumentation framework** (WAF) is a triplet $\langle \mathcal{A}, \mathcal{R}, w \rangle$ where $\langle \mathcal{A}, \mathcal{R} \rangle$ is a Dung abstract argumentation framework, and $w : \mathcal{A} \times \mathcal{A} \rightarrow \mathbb{N}$ is a function assigning a natural number to each couple of arguments (*i.e.* $w(x, y) > 0$ if and only if $(x, y) \in \mathcal{R}$), and a null value otherwise ($w(x, y) = 0$ if and only if $(x, y) \notin \mathcal{R}$).

Several interpretations of weights on attacks are given in [DUNNE *et al.* 2011]. Among them:

- The veracity of the attack (greater the weight is, the more viable it is) which can express the degree of trust assigned to the attack.

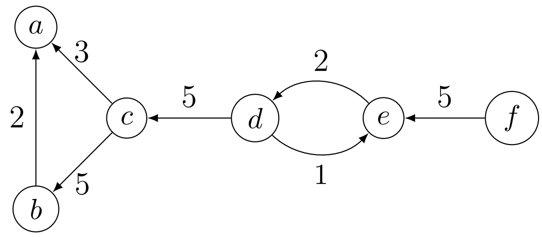


Figure 1.9 – A weighted argumentation framework

- The relative strength of the attack which can be seen as a measure of inconsistency between two arguments. Thus, a higher weight on the attack between two arguments denotes greater inconsistency between the arguments concerned than does a lower weight.
- The number of agents that support the attack used in a multi-agent setting. For example, some works [CAYROL & LAGASQUIE-SCHIEX 2011b, DELOBELLE *et al.* 2015] endorse this interpretation for the aggregation of argumentation frameworks aiming to define a suitable representation (for example a WAF) at the beliefs of the group where each agent in the group has its own argumentation framework.

Using Dung's semantics can lead to the non-existence of extensions or the existence of multiple extensions, thus some works propose to use such weights to fix these eventualities. Indeed, when an argumentation framework is trivial [3] for a given semantics σ, a way to process [DUNNE *et al.* 2011, COSTE-MARQUIS *et al.* 2012b] goes through a relaxation of the usual notion of conflict-free sets of arguments where some inconsistencies are tolerated in a set of arguments as long as the maximum (or any other aggregation function like the sum) of the weights of attacks between arguments in the set does not exceed a given inconsistency budget β. For example, if $\beta = 2$, all the attacks with a weight smaller or equal to 2 are not taken into consideration to build the conflict-free sets of arguments. Admissibility is defined in the standard way, and Dung's semantics are considered leading to various notions of so-called β-extensions.
Conversely, an argumentation framework may admit a large number of extensions for a given semantics. Within the WAF setting, it is possible to take advantage of the available weights, in order to select the "best" extensions. In [COSTE-MARQUIS *et al.* 2012a], this selection goes through a comparison of the extensions' scores, expressing intuitively how good they are.

1.3.4 Preference-based / Value-based argumentation frameworks

Another way for representing the quality of arguments consists in directly distinguishing the arguments themselves. For this, Amgoud and Cayrol [AMGOUD & CAYROL 2002b, AMGOUD & CAYROL 2002a] introduce an additional relation, called *preference relation*, defined on the set of arguments. The so-called preference based argumentation frameworks are defined as follows:

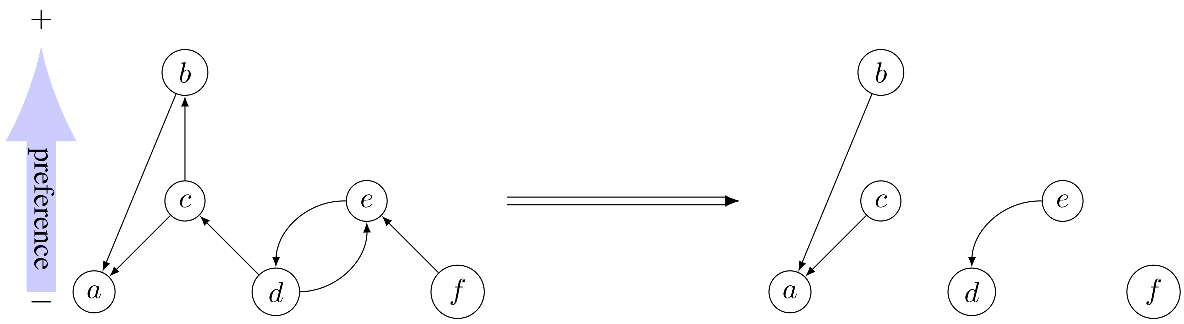
Definition 1.3.5 (Preference-based argumentation framework[AMGOUD & CAYROL 2002b])**.** A **preference-based argumentation framework** (PAF) is a triplet $\langle \mathcal{A}, \mathcal{R}, \geq_{pref} \rangle$ where $\mathcal{A}$ is

3. An argumentation framework AF is trivial for a given semantics σ if $\mathcal{E}_\sigma(AF) = \emptyset$.

a set of arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is an attack relation and $\geq_{pref}$ is a (total or partial) pre-order (preference relation) defined on $\mathcal{A}$.

While the success of an attack is always guaranteed in classical argumentation framework, it is not always the case when the preference relation is taken into account. Indeed, Amgoud and Cayrol redefine the attack relation in saying that an argument x defeats an argument y if and only if there exists an attack from x to y and y is not preferred to x with respect to the preference relation. Thus, in removing the attacks, from a preference-based argumentation framework $\langle \mathcal{A}, \mathcal{R}, \geq_{pref} \rangle$, which do not respect the second condition then we obtain a new Dung's argumentation framework $\langle \mathcal{A}, \mathcal{R}' \rangle$ where $\mathcal{R}' \subseteq \mathcal{R}$. The acceptability of the arguments are then defined in the standard way from this new argumentation framework.

Example 1.3.2.
An example of preference-based argumentation framework is illustrated in Figure 1.10 (left) where b is preferred to all the other arguments. So all the attacks from its direct attackers are cancelled (the attack from c has no effect on b). Then, as c and e are preferred to a, d and f so the attacks from d to c and e and from f to e have no effect too. Removing all these attacks allows to obtain the Dung's argumentation framework illustrated in Figure 1.10 (right) where $\{b, c, e, f\}$ is its only extension whatever the Dung's semantics used (because the framework is well-founded).



$b >_{pref} c \simeq_{pref} e >_{pref} a \simeq_{pref} d \simeq_{pref} f$

Figure 1.10 – A preference-based argumentation framework (left) and its associated Dung's argumentation framework (right)

A framework introduced by Bench-Capon [Bench-Capon 2002, Bench-Capon 2003], called *value-based argumentation framework*, is based on similar ideas. However, here it is assumed that arguments promote specific values, and the preferences are among these values rather than between the arguments themselves.

Definition 1.3.6 (Value-based argumentation framework [Bench-Capon 2002]).
A **value-based argumentation framework** (VAF) is a 5-tuple $\langle \mathcal{A}, \mathcal{R}, \mathcal{V}, v, \geq_{vpref} \rangle$ where $\mathcal{A}$ is a set of arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is an attack relation, $\mathcal{V}$ is a non-empty set of values, v is a function which maps from elements of $\mathcal{A}$ to elements of $\mathcal{V}$, and $\geq_{vpref}$ is a (total or partial) pre-order (preference relation) defined on $\mathcal{V}$.

As for the preference-based argumentation framework, the attack relation is redefined: x defeats y if and only if there exists an attack from x to y and $v(y)$ is not preferred to $v(x)$ according to $\geq_{vpref}$.

Thus, the advantage to include the preference or value information in argumentation frameworks is not only to model the problem more accurately but also to reduce the number of extensions we may obtain.

1.3.5 Probabilistic argumentation frameworks

In everyday life, the arguments, and the attacks provided between them, are often uncertain. It is the case, for example, when one uses an argument based on the weather forecasts which is not an exact science, or an argument based on a survey. Probabilistic argumentation framework aims to model this notion of uncertainty in argumentation by combining Dung's argumentation framework with probability theory.

Definition 1.3.7 (Probabilistic argumentation framework [LI *et al.* 2011])**.**
A **probabilistic argumentation framework** (PrAF) is a 4-tuple $\langle \mathcal{A}, \mathcal{R}, P_{\mathcal{A}}, P_{\mathcal{R}} \rangle$ where $\langle \mathcal{A}, \mathcal{R} \rangle$ is a Dung's argumentation framework, $P_{\mathcal{A}} : \mathcal{A} \rightarrow [0,1]$ is a probability function on $\mathcal{A}$ and $P_{\mathcal{R}} : \mathcal{R} \rightarrow [0,1]$ is a probability function on $\mathcal{R}$.

Work in the field of probabilistic argumentation can be divided [HUNTER 2013] into two different uses: the constellation approach (see e.g. [LI *et al.* 2011]) and the epistemic one (see e.g. [THIMM 2012, HUNTER & THIMM 2017]).

In the constellation approach, the uncertainty associated with each argument in the probabilistic argumentation framework is interpreted as an uncertainty over the structure of the argumentation framework. In other words, the probability on each argument or attack corresponds to its chance of existence in this framework. The meaning of a probabilistic argumentation framework is thus given in terms of possible worlds (*i.e.* Dung's argumentation framework), each of them representing a scenario that may occur in the reality. Indeed, for an argument x in a probabilistic argumentation framework $PrAF$, $P_{\mathcal{A}}(x)$ is the probability that x exists in all the classical argumentation frameworks induced from $PrAF$, and $1 - P_{\mathcal{A}}(x)$ is the probability that x does not exist in all the classical argumentation frameworks induced from $PrAF$. Thus, combining the probabilities of each argument and attack in an argumentation framework allows to obtain the probability to obtain this framework. Using all the argumentation frameworks induced associated with their probability of existing, we can then explore the notions of probability distributions over admissible sets, extensions, and inferences.

In the epistemic approach, the probability distribution over arguments allows to identify which arguments are believed or not: the higher the probability of the argument, the more it is believed. So for a probability function $P_{\mathcal{A}}$, and an argument x, $P_{\mathcal{A}}(x) > 0.5$ denotes that the argument is believed, $P_{\mathcal{A}}(x) < 0.5$ denotes that the argument is disbelieved, and $P_{\mathcal{A}}(x) = 0.5$ denotes that the argument is neither believed or disbelieved. This approach leads to the notion of an epistemic extension (or labellings) which is the subset of the arguments in the graph that are believed to be acceptable to some degree (*i.e.* the arguments such that $P_{\mathcal{A}}(x) > 0.5$). However, this definition is very general because it permits any set of arguments to be an epistemic extension. It is why a set of properties have been introduced (see [HUNTER & THIMM 2017] for an overview) to restrict the probability function which may take different aspects of the structure of the argument graph into account. For example, the probability distribution is *coherent* (COH) with the structure of the argument graph if the belief in an argument is high, then the belief in the arguments it attacks is low.

1.3.6 Abstract dialectical frameworks

Abstract dialectical frameworks have been proposed by [BREWKA & WOLTRAN 2010] as a generalization of Dung's argumentation frameworks. Their main goal is to express a wide range of relations (like the attack relation, the different kinds of support relation, ...) in order to avoid the need of introducing a new relation each time it is needed. This is achieved by adding to each argument a specific acceptance condition. More formally, an abstract dialectical framework is a directed graph whose nodes represent arguments, statements or positions which can be accepted or not. The links represent dependencies: the status of a node only depends on the status of its parents (*i.e.* the nodes with a direct link to it). In addition, an acceptance condition C_x is associated to each node x specifying the exact conditions under which x is accepted.

Definition 1.3.8 (Abstract dialectical framework [BREWKA & WOLTRAN 2010]**).**
An **abstract dialectical framework** (ADF) is a triplet $\langle S, L, C\rangle$ where S is a set of statements (positions, arguments), $L \subseteq S \times S$ is a set of links between two statements and $C = \{C_x\}_{x\in S}$ is a set of acceptance conditions with one condition for each statement. An acceptance condition is a total function $C_x : 2^{par(x)} \to \{\top, \bot\}$, where $par(x) = \{y \in S \mid (y, x) \in L\}$ is the set of parents of an argument x.

The most popular way to represent the acceptance conditions is with propositional formulas. For example, a case where an argument x can only be accepted if its parent y is not accepted (as it is the case for an attack) and its parent z is accepted (as it is the case with a necessary support) can be easily expressed with a condition $C_x = \neg y \wedge z$. Then if the formula C_x is true then x is accepted and if it is false then x is not accepted.
Dung's argumentation frameworks can be recovered as a special case where the statements are the arguments, the links between them represent the attack relation and the acceptance condition of an argument x is defined as the formula $C_x = \neg y_1 \wedge \cdots \wedge \neg y_n$ where $y_1, \ldots, y_n$ are the direct attackers of x. Indeed, recall that an argument x is accepted if its direct attackers are all rejected.

Example 1.3.3. The classical argumentation framework depicted in Figure 1.11 (right) can be represented as an abstract dialectical framework where $C_d = \top$ because d is unattacked so its acceptability does not depend on any other argument, $C_c = \neg b \wedge \neg d$ means that c is accepted only if b and d are not accepted, $C_b = \neg c$ and $C_a = \neg b$ means that the acceptability of b depends on the non-acceptability of c and the acceptability of a depends on the non-acceptability of b.

It is also possible to represent a bipolar argumentation framework with an abstract dialectical framework, as shown in the following example.

Example 1.3.4. The bipolar argumentation framework, depicted in Figure 1.12 (right), where the support is interpreted as a necessary support, can be represented as an abstract dialectical framework where $C_d = \top$ because d is unattacked, $C_c = \neg b \wedge d$ means that c is accepted only

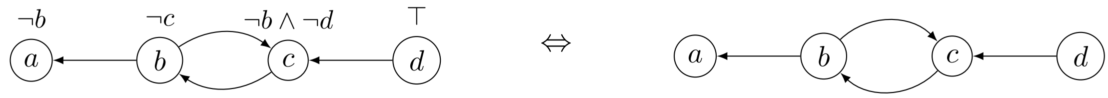


Figure 1.11 – A representation of the Dung's argumentation framework (right) in terms of an abstract dialectical framework (left)

if b is not accepted (attacker) and d is accepted (supporter), $C_b = \neg c$ because c attacks d and $C_a = b$ because b supports a so the acceptability of a depends on the acceptability of b.

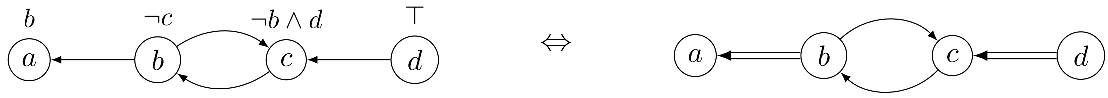


Figure 1.12 – A representation of the bipolar argumentation framework (right) in terms of an abstract dialectical framework (left)

Abstract dialectical frameworks also provide a new way to handle preferences and values [BREWKA *et al.* 2013].

The semantics of ADFs [BREWKA & WOLTRAN 2010] concern the extension-based approach (even if other approaches, like labelling-based approach [BREWKA *et al.* 2013], were subsequently studied). Indeed, a conflict–free extension can be simply seen as a set of arguments having their acceptance condition satisfied and the admissibility generalizes the original intuition from Dung by making sure that the extension can discard undesired arguments. The definition of grounded, complete, preferred and stable semantics is then derived from these two definitions.

1.3.7 Social argumentation frameworks

Recently, online debate platforms, like Debategraph (debategraph.org/home), Debatabase (idebate.org/debatabase) or Argüman (en.arguman.org), are emerging on the internet. On these debate platforms, agents argue for or against a particular topic or other existing arguments. Less expert users who prefer to take a more observational role will be provided with simple mechanisms to vote (positively if they agree or negatively if not) on individual arguments. In order to represent these debates in their abstract form, *social abstract argumentation frameworks* [LEITE & MARTINS 2011, CORREIA *et al.* 2014] (and its extended version [EGILMEZ *et al.* 2013] where votes on the attacks are also allowed) have been introduced as an extension of Dung's argumentation framework to which an assignment of votes (which is a couple of integers, one for the positive votes and one for the negative votes) is added to each argument.

Definition 1.3.9 (Social abstract argumentation frameworks [LEITE & MARTINS 2011])**.**
A **social abstract argumentation framework** (SAF) is a triplet $\langle \mathcal{A}, \mathcal{R}, v \rangle$ where $\mathcal{A}$ is a set of arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary attack relation between arguments and $v : \mathcal{A} \to \mathbb{N} \times \mathbb{N}$ represents the number of positive and negative votes for each argument.

Example 1.3.5 (Example from [LEITE & MARTINS 2011]).
A typical forum discussion about new generation phones could be of the form (argument + positive vote / negative vote) depicted in Figure 1.13 with its associated graphical representation (SAF).

(a) "The Wonder-Phone is the best new generation phone." (20/20)

(b) "The Magic-Phone is the best new generation phone." (20/20)

(c) links to a review of the M-Phone giving poor scores due to bad battery performance. (60/10)

(d) "(c) is ignorant, since subsequent reviews noted only one of the first editions had such problems: [links]." (10/40)

(e) "(d) is wrong. I found (c) knows about that but withheld the information. Here's a link to another thread proving it!" (40/10)

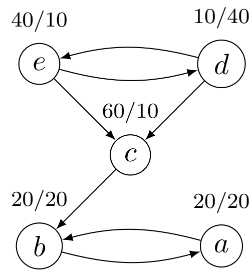


Figure 1.13 – A social argumentation framework (right) representing a debate about new generation phone (left)

Leite and Martins emphasize the limitations of classical acceptability semantics for this kind of debate. For example, to accurately represent the opinions of thousands of voting users, it could be more appropriate to evaluate arguments using degrees of acceptability or gradual acceptability. In the next chapter, we will talk about these semantics allowing to distinguish arguments using several levels of acceptability.

Chapter 2

Ranking-based Semantics

While the acceptability of an argument depends on its membership or not to a set of extensions in Dung's theory, an alternative way to evaluate arguments consists in directly reasoning on the arguments themselves by exploiting the topology of the argumentation framework. Following this idea, two kinds of semantics have been introduced: scoring semantics and ranking-based semantics. Scoring semantics assign a numerical acceptability degree to each argument, taking into account various criteria from the argumentation framework. Ranking-based semantics associate to any argumentation framework a ranking on the arguments from the most to the least acceptable ones. This allows to have a large number of levels of acceptability and not only the classical accepted/rejected (or accepted/rejected/undecided) evaluations obtained with extension-based semantics, but on the other hand the joint acceptability of arguments is no longer captured.

In this chapter, we first put forward the limits of Dung's semantics for some applications and explain why the scoring semantics and the ranking-based semantics are a better choice for those applications. Then, we formally define the scoring semantics and the ranking-based semantics in abstract argumentation and show the link between them. We list the existing ranking-based semantics in the literature in focusing on the semantics which return, for an argumentation framework, only one ranking on arguments from the most to the least acceptable ones. Finally, we present the properties introduced in the literature aiming to better understand the behavior of these ranking-based semantics in various situations.

Contents

2.1 Motivations and applications

Amgoud and Ben-Naïm [AMGOUD & BEN-NAIM 2013] give some characteristics specific to extension-based semantics (and labellings-based-semantics):

- *Killing*: The impact of an attack from an argument y to an argument x is drastic, that is, if y belongs to an extension (or is labelled in), then x is automatically excluded from that extension (or is labelled out). This is especially noticeable in the definition of the reinstatement labellings because an argument is labelled out if there exists at least one direct attacker labelled in.

- *Existence*: One successful attack against an argument x has the same effect on an argument as any number of successful attacks. Indeed, one such attack is sufficient to kill x, several attacks cannot kill x to a greater extent. Again, the definition of the reinstatement labellings says that if there exists *at least* one direct attacker labelled in then the attacked argument will be labelled out.

- *Flatness*: All the arguments with the same status have the same level of acceptability. For example, all the accepted (respectively rejected or undecided) argument cannot be distinguish, *i.e* no accepted argument are more acceptable than another accepted argument.

These considerations seem rational to be satisfied in applications like paraconsistent reasoning [BESNARD & HUNTER 2008] where arguments are represented as formulas and attacks correspond to contradictions between these formulas. Here, the killing and existence consideration seem essential to capture the fact that one attack is lethal and prevent any contradiction between arguments and thus obtain a consistent set of formulas.
However, in other applications like decision-making, online debate platforms or when additional information exist in the framework, some of these considerations are debatable.

Decision-making

Decision-making aims to help someone to select one or several options among several alternative possibilities when potential conflicts exist between them. This process can be modelized by an argumentation framework where the arguments are the options and the attacks represent the conflicts between options. To find the compatible options, one can use the classical semantics (*i.e.* Dung's semantics) and the most acceptable arguments will be the selected option(s). But, let us illustrate with the following example why the considerations defining standard semantics are not always appropriate for the decision-making paradigm. This example involves a debate between employees from an airline company who discuss to know if they will buy or not the new Airbus. The two following arguments a and b are proposed:

(a) We should buy the new Airbus A350 in order to remain competitive with our competing companies.

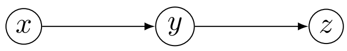
(b) A preliminary study showed that the economical impact of this purchase is maybe not guaranteed for our company.

In this case, the argument b is not strong enough to defeat/kill a because it is a preliminary study and the result of this study needs confirmation. Thus, its impact is not sufficient to conclude to avoid buying the new Airbus. However, it has certainly a negative impact on a, it is why we consider that b only weakens a.
The existence principle is also debatable because if one considers an attack as a weakening then several attacks should have more impact than just one. For example, consider the two additional arguments c and d, clearly against the purchase of the new airbus:

(c) The design of this model should be improved.

(d) The price is too expensive.

With the negative impact from the three arguments b, c and d on a, it seems obvious that a is now weaker than when b was the only direct attacker.
Finally, the flatness consideration is also debatable in decision-making. Let us consider the following argumentation framework:



Then, x and z are both accepted according Dung's semantics and have the same level of acceptability (both are accepted). But, we saw that it is reasonable to consider that an attack from an argument does not kill the targeted argument. So, if y is only weakened, then its attack against z should still have some effect, and thus the level of acceptability of z should be lower than that of x.

Thus, scoring semantics or ranking-based semantics with many levels of acceptability allowing to better distinguish arguments seem more appropriate in a decision-making context.

For more details about the link between decision-making and argumentation, we refer the reader to [AMGOUD *et al.* 2008, AMGOUD & PRADE 2009, AMGOUD & VESIC 2012].

Argumentation framework with additional information

Among Dung's framework extensions presented in the previous chapter (see section 1.3), the weighted argumentation frameworks, the value-based semantics and preference-based semantics allow to represent the strength of an attack or an argument. However, these additional information are only used to improve the extensions computed with extension-based semantics. For example, in a weighted argumentation framework[4], the weights are used to relax the extensions in removing some attacks, or to select the best extensions when a large number of extensions exists. But it could be more intuitive to use them to rank arguments. Indeed, in focusing on the weighted argumentation frameworks illustrated in Figure 2.1, without taking into account the weights on the attacks, the only extension computed with any Dung's semantics contains only the non-attacked arguments: $\{a_1, b_1, c_1\}$. The set of extensions is not trivial and contains only one extension so it is not necessary to relax the extensions or select the best one, so a, b and c are all rejected and the weights are not used.

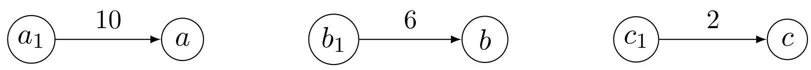


Figure 2.1 – A weighted argumentation framework

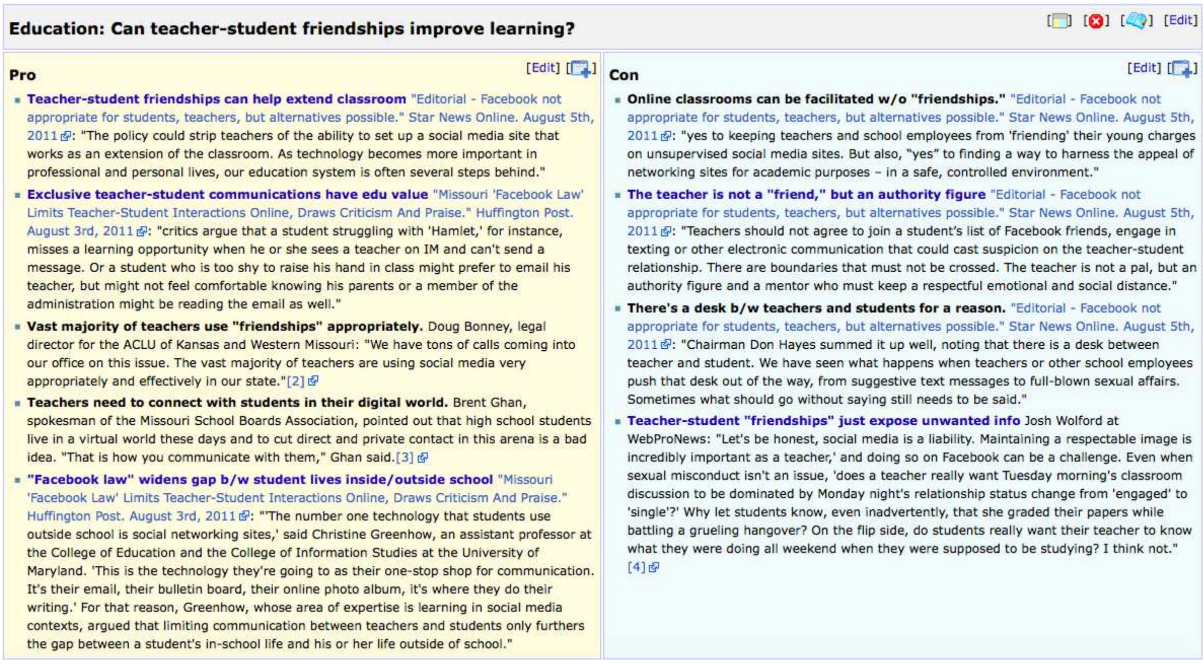
However, it is clear that the three rejected arguments receive different impact from their direct attackers. Indeed, if we adopt the interpretation such that the higher the weight on the attack, the stronger its impact, then the attack from a_1 with a weight of 10 has more impact on a than the attack from b_1 to b with a weight of 6, and the attack from c_1 to c with a weight of 2. Following this, it could be reasonable to say that even if the three arguments are rejected, the different impacts of their direct attackers allow to say that c is more acceptable than b (and a) which is more acceptable than a. This questions the flatness principle of classical semantics.

Debate-centric systems

In some online debate platforms, like *debatepedia.com*, *debate.org*, *whysaurus.com*, users contribute arguments for and against a topic, a question, etc. For example, the question of the debate from *debatepedia.com* represented in Figure 2.2 (page 37) is "Can teacher-student friendships improve learning?" and the arguments "pro" are represented in the left-hand column while the arguments "con" are listed in the right-hand column. Thus the arguments "pro" support the question while the arguments "con" attack it. So this debate can be modelized by a bipolar argumentation framework where the question, seen as an argument and called question argument, is the only argument which is supported or attacked by other arguments.
Here, the goal is not to find the arguments which can be accepted together but to evaluate how accepted is the question. Indeed, using the classical Dung's semantics, it is clear that just one attack rejects the question (existence principle) which is not an appropriate answer. A first (intuitive) solution should be to evaluate the question argument in just counting the number of its supports and its attacks. If there exists strictly more supports than attacks then the question

4. Please note that the same reasoning holds for the value-based and preference-based semantics.

argument is accepted, if the number of supports and attacks are the same so the question can be seen as undecided and the question is not accepted otherwise. [5] For example, the question argument in Figure 2.2 is accepted because it is attacked by four arguments but supported by five arguments. But one can go further in saying that there exists several levels of acceptability among these groups. Indeed, it seems natural to say that it is better for the question argument to have many supports and no attack. Thus, a ranking with many levels of acceptability would allow to capture this idea.



Education: Can teacher-student friendships improve learning? [Edit]

Pro [Edit]

- **Teacher-student friendships can help extend classroom** "Editorial - Facebook not appropriate for students, teachers, but alternatives possible." Star News Online. August 5th, 2011: "The policy could strip teachers of the ability to set up a social media site that works as an extension of the classroom. As technology becomes more important in professional and personal lives, our education system is often several steps behind."
- **Exclusive teacher-student communications have edu value** "Missouri 'Facebook Law' Limits Teacher-Student Interactions Online, Draws Criticism And Praise." Huffington Post. August 3rd, 2011: "critics argue that a student struggling with 'Hamlet,' for instance, misses a learning opportunity when he or she sees a teacher on IM and can't send a message. Or a student who is too shy to raise his hand in class might prefer to email his teacher, but might not feel comfortable knowing his parents or a member of the administration might be reading the email as well."
- **Vast majority of teachers use "friendships" appropriately.** Doug Bonney, legal director for the ACLU of Kansas and Western Missouri: "We have tons of calls coming into our office on this issue. The vast majority of teachers are using social media very appropriately and effectively in our state."[2]
- **Teachers need to connect with students in their digital world.** Brent Ghan, spokesman of the Missouri School Boards Association, pointed out that high school students live in a virtual world these days and to cut direct and private contact in this arena is a bad idea. "That is how you communicate with them," Ghan said.[3]
- **"Facebook law" widens gap b/w student lives inside/outside school** "Missouri 'Facebook Law' Limits Teacher-Student Interactions Online, Draws Criticism And Praise." Huffington Post. August 3rd, 2011: "'The number one technology that students use outside school is social networking sites,' said Christine Greenhow, an assistant professor at the College of Education and the College of Information Studies at the University of Maryland. 'This is the technology they're going to as their one-stop shop for communication. It's their email, their bulletin board, their online photo album, it's where they do their writing.' For that reason, Greenhow, whose area of expertise is learning in social media contexts, argued that limiting communication between teachers and students only furthers the gap between a student's in-school life and his or her life outside of school."

Con [Edit]

- **Online classrooms can be facilitated w/o "friendships."** "Editorial - Facebook not appropriate for students, teachers, but alternatives possible." Star News Online. August 5th, 2011: "yes to keeping teachers and school employees from 'friending' their young charges on unsupervised social media sites. But also, "yes" to finding a way to harness the appeal of networking sites for academic purposes – in a safe, controlled environment."
- **The teacher is not a "friend," but an authority figure** "Editorial - Facebook not appropriate for students, teachers, but alternatives possible." Star News Online. August 5th, 2011: "Teachers should not agree to join a student's list of Facebook friends, engage in texting or other electronic communication that could cast suspicion on the teacher-student relationship. There are boundaries that must not be crossed. The teacher is not a pal, but an authority figure and a mentor who must keep a respectful emotional and social distance."
- **There's a desk b/w teachers and students for a reason.** "Editorial - Facebook not appropriate for students, teachers, but alternatives possible." Star News Online. August 5th, 2011: "Chairman Don Hayes summed it up well, noting that there is a desk between teacher and student. We have seen what happens when teachers or other school employees push that desk out of the way, from suggestive text messages to full-blown sexual affairs. Sometimes what should go without saying still needs to be said."
- **Teacher-student "friendships" just expose unwanted info** Josh Wolford at WebProNews: "Let's be honest, social media is a liability. Maintaining a respectable image is incredibly important as a teacher,' and doing so on Facebook can be a challenge. Even when sexual misconduct isn't an issue, 'does a teacher really want Tuesday morning's classroom discussion to be dominated by Monday night's relationship status change from 'engaged' to 'single'?' Why let students know, even inadvertently, that she graded their papers while battling a grueling hangover? On the flip side, do students really want their teacher to know what they were doing all weekend when they were supposed to be studying? I think not." [4]

Figure 2.2 – Debate from the website debatepedia.com

Recent works [Leite & Martins 2011, Evripidou & Toni 2012, Baroni *et al.* 2015, Rago *et al.* 2016, Cerutti *et al.* 2016] draw a parallel between argumentation and online debates.

2.2 Formal definition

Let us formally define, in this section, the ranking-based and the scoring semantics.
First, a ranking-based semantics rank-orders a set of arguments in an argumentation framework from the most acceptable to the weakest one(s). Thus, unlike classical semantics which assign an absolute status (accepted, rejected, undecided) to each argument, this semantics compares pairs of arguments.

5. Clearly, the method used for this kind of debate is naive because only the topic argument can be supported or attacked. But, for debates where agents have the possibility to attack or support other arguments then one needs more sophisticated

Definition 2.2.1 (Ranking-based semantics)**.**
A **ranking-based semantics** σ associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{\sigma}_{\text{AF}}$ on $\mathcal{A}$, where $\succeq^{\sigma}_{\text{AF}}$ is a preorder (a reflexive and transitive relation) on $\mathcal{A}$.

- $x \succeq^{\sigma}_{\text{AF}} y$ means that x is at least as acceptable as y;
- $x \simeq^{\sigma}_{\text{AF}} y$ (shortcut for $x \succeq^{\sigma}_{\text{AF}} y$ and $y \succeq^{\sigma}_{\text{AF}} x$) means that x and y are equally acceptable;
- $x \succ^{\sigma}_{\text{AF}} y$ (shortcut for $x \succeq^{\sigma}_{\text{AF}} y$ and $y \not\succeq^{\sigma}_{\text{AF}} x$) means that x is strictly more acceptable than y;
- $x \not\succeq^{\sigma}_{\text{AF}} y$ and $y \not\succeq^{\sigma}_{\text{AF}} x$ means that x and y are incomparable.

We denote by $\sigma(AF)$ the ranking on $\mathcal{A}$ returned by σ.

A scoring semantics assigns to each argument in an argumentation framework a score depending on different criteria. This value must be selected among an ordered scale as the interval $[0, 1]$, the interval $[-1, 1]$, the set of natural numbers $\mathbb{N}$, the set of positive real numbers $\mathbb{R}^+$, etc. However, this score should not be confused with the weight assigned in weighted argumentation framework or value-based semantics coming from external sources. Indeed, the scores assigned here are only computed on the basis of the argumentation framework itself.

Definition 2.2.2 (Scoring semantics)**.**
A **scoring semantics** is a function which associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a scoring S on $\mathcal{A}$, where S is a function from $\mathcal{A}$ to $\mathbb{R}$.

It is important to note that these two families of semantics are not independent. Indeed, as illustrated in Figure 2.3, most of the time the ranking between arguments is based on the comparison of the score computed with a scoring semantics. In other words, a scoring semantics is used to assign a score to each argument and, as these score belong to an ordered scale, it is possible to compare them to obtain a ranking between arguments.

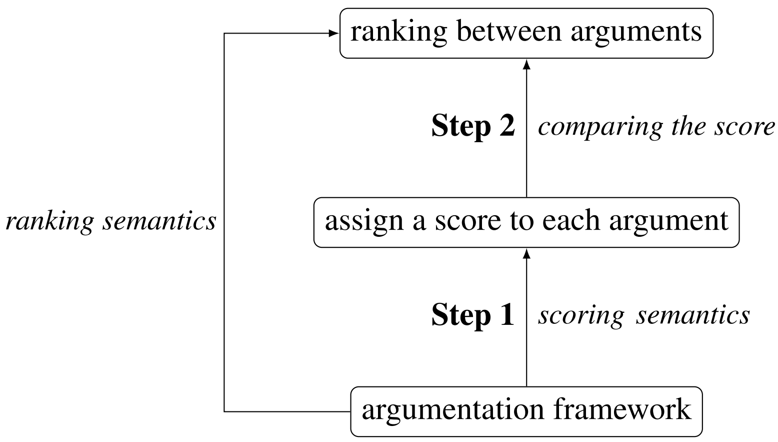


Figure 2.3 – Ranking process

However, most of the scores assigned to each argument only make sense when they are compared with each other. In addition, while it is always possible to build a ranking-based semantics using a scoring semantics, there exist other methods which do not use scoring semantics to build a ranking between arguments as we shall see in the next section. It is why we choose to focus on the ranking-based semantics in this thesis.

Finally, let us define the notion of lexicographical order which will be useful to define some ranking-based semantics.

Definition 2.2.3 (Lexicographical order).
A **lexicographical order** between two vectors of real numbers $V = \langle V_1, \ldots, V_n \rangle$ and $V' = \langle V'_1, \ldots, V'_n \rangle$ is defined as

$$V \succ_{lex} V' \text{ if and only if } \exists i \leq n \text{ such that } V_i > V'_i \text{ and } \forall j < i, V_j = V'_j$$

$V \simeq_{lex} V'$ means that $V \not\succ_{lex} V'$ and $V' \not\succ_{lex} V$; and $V \succeq_{lex} V'$ means that $V' \not\succ_{lex} V$.

2.3 Existing ranking-based semantics

In this section, we introduce ranking-based semantics from the literature in focusing on those that return, for a given argumentation framework, a unique ranking between arguments from the most to the least acceptable ones. In order to correctly illustrate each of these ranking-based semantics, we choose to apply all of them on the argumentation framework illustrated in Figure 2.4. We find this argumentation framework interesting because it contains several configurations of arguments. Indeed, in addition to the non-attacked arguments a, e and j which directly attack b, d and h once and i twice, the argument c has one defense branch while f and g have two distinct defense branches with the difference that the defender of g is the same for these two branches.

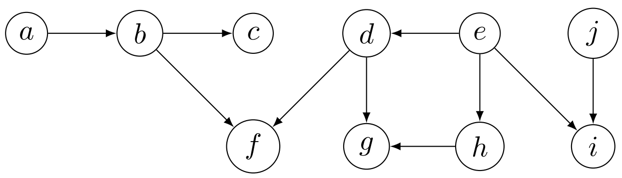


Figure 2.4 – The argumentation framework AF_c

2.3.1 Categoriser-based ranking semantics

Originally, Besnard and Hunter [BESNARD & HUNTER 2001] have proposed a *categoriser* function used for "deductive" arguments, where an argument is structured as a pair $\langle \Phi, \alpha \rangle$, where Φ is a consistent set of propositional formula, called support or premise, α is a formula, called claim or consequent of the argument such that $\Phi \vdash \alpha$. The categoriser function allows to assign a value to each argument belonging to a tree of arguments. Such a value allows to capture the relative strength of an argument taking into account the strength of its attackers, which itself takes into account the strength of its attackers, and so on. Recall that the notation $\mathcal{R}_i(x)$ selects the arguments that are bound by a path of length i to the argument x.

Definition 2.3.1 (Categoriser function)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. The **categoriser function** $Cat : \mathcal{A} \to]0, 1]$ is defined such that $\forall x \in \mathcal{A}$,

$$Cat(x) = \begin{cases} 1 & \text{if } \mathcal{R}_1(x) = \emptyset \\ \frac{1}{1+\sum_{y \in \mathcal{R}_1(x)} Cat(y)} & \text{otherwise} \end{cases}$$

The values returned by the categoriser function are called the categoriser values. Thus, $Cat(x)$ is the categoriser value of x.

To understand the idea behind the categoriser function, one can divide the formulae into two parts:

- The first part $(1 + \sum_{y \in \mathcal{R}_1(x)} Cat(y))$ allows to combine the categoriser values of all the direct attackers of an argument. Then, the smaller this value, the better it is for the argument. So, except if it has no direct attacker and in this case the value is minimal, the "better" case is when an argument has few direct attackers and many direct defenders. Indeed, even if the number of direct attackers of an argument is low, if they are not attacked then the categoriser value of this argument can be greater than a large number of direct attackers with a small categoriser value. It is why the number of direct defenders is also important to decrease the categoriser value of the direct attackers.

- The second part aims to transform the value obtained during the first step into a restricting value by using the reciprocal function $1/x$. Thus, the smaller the value obtained during the first step, the higher the final value and consequently the higher the categoriser value of this argument.

The categoriser function was initially introduced for acyclic argumentation framework, but Pu et al. [PU *et al.* 2014] proved the existence and uniqueness of such solution for any argumentation framework. In this case, the categoriser values correspond to the solution of the non-linear system of equations with one equation per argument (see Definition 2.3.1) and can be computed via a fixed point technique for any argumentation framework. The categoriser-based ranking semantics builds a ranking from the categoriser values obtained. The higher the categoriser value of an argument, the more acceptable the argument.

Definition 2.3.2 (Categoriser-based ranking semantics [PU *et al.* 2014])**.**
The **Categoriser-based ranking semantics (Cat)** associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{\text{Cat}}_{\text{AF}}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,

$$x \succeq^{\text{Cat}}_{\text{AF}} y \text{ if and only if } Cat(x) \geq Cat(y)$$

Let us compute the categoriser values of each argument in AF_c (Figure 2.4 page 39).

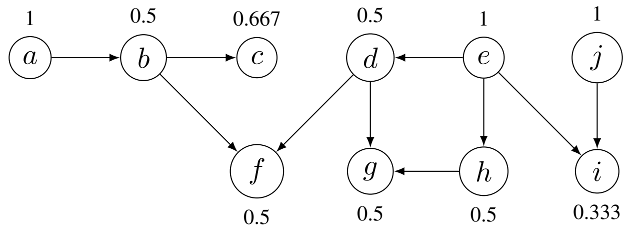


Figure 2.5 – The categoriser values of arguments of AF_c

Example 2.3.1 (cont.)**.** As illustrated in Figure 2.5, the categoriser values of each argument are $Cat(a) = Cat(e) = Cat(j) = 1$, $Cat(c) \approx 0.667$, $Cat(i) \approx 0.333$ and $Cat(b) = Cat(d) = Cat(f) = Cat(g) = Cat(h) = 0.5$.

We can now compare the categoriser values of each argument and obtain the following ranking:

$$a \simeq^{\text{Cat}} e \simeq^{\text{Cat}} j \succ^{\text{Cat}} c \succ^{\text{Cat}} b \simeq^{\text{Cat}} d \simeq^{\text{Cat}} f \simeq^{\text{Cat}} g \simeq^{\text{Cat}} h \succ^{\text{Cat}} i$$

As described above, this semantics assigns high values to arguments with low-valued attackers, a maximal value of 1 to non-attacked arguments (like a, e and j). In this way, we can see that even if an argument is always defended (like f and g) it is still attacked anyway. It is why f and g have exactly the same level of acceptability that arguments directly attacked only once but by one stronger argument (like b, d and h).

2.3.2 Discussion-based semantics

Amgoud and Ben-Naim [AMGOUD & BEN-NAIM 2013] have introduced the discussion-based semantics which compares arguments by counting the number of paths ending to them. A distinction is done concerning the polarity of the number of paths computed according to the attack relation meaning (positive for the attackers and negative for the defenders).

Definition 2.3.3 (Discussion count)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $x \in \mathcal{A}$, and $i \in \mathbb{N} \setminus \{0\}$.

$$Dis_i(x) = \begin{cases} -|\mathcal{R}_i(x)| & \text{if } i \text{ is even} \\ |\mathcal{R}_i(x)| & \text{if } i \text{ is odd} \end{cases}$$

The **discussion count of** a is denoted by $Dis(x) = \langle Dis_1(x), Dis_2(x), \ldots \rangle$.

This semantics was proposed to take into account only the number of attackers/defenders of a given argument, whatever their quality: the less attackers and the more defenders an argument has, the more acceptable the argument. The method lexicographically ranks the arguments on the basis of the number of attackers and defenders. Concretely, we start by comparing the number of direct attackers of each argument. If some arguments are still equivalent (they have the same number of direct attackers), the size of paths is recursively increased until a difference is found or the threshold [6] is reached.

Definition 2.3.4 (Discussion-based semantics)**.**
The **Discussion-based semantics (Dbs)** associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{\text{Dbs}}_{\text{AF}}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,

$$x \succeq^{\text{Dbs}}_{\text{AF}} y \text{ if and only if } Dis(y) \succeq_{lex} Dis(x)$$

Let us compute the discussion count of each argument in AF_c (Figure 2.4).

Example 2.3.1 (cont.)**.**
In Figure 2.6, we first represent the discussion count of each argument in AF_c.

step	a, e, j	c	b, d, h	f, g	i
1	0	1	1	2	2
2	0	-1	0	-2	0
3	0	0	0	0	0

$Dis(a) = Dis(e) = Dis(j) = \langle 0, 0, 0 \rangle$
$Dis(c) = \langle 1, -1, 0 \rangle$
$Dis(b) = Dis(d) = Dis(h) = \langle 1, 0, 0 \rangle$
$Dis(f) = Dis(g) = \langle 2, -2, 0 \rangle$
$Dis(i) = \langle 2, 0, 0 \rangle$

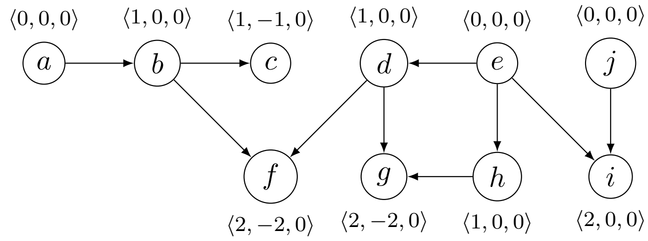


Figure 2.6 – Discussion count of arguments of AF_c

During the first step where only the direct attackers are considered, we have three groups of arguments: one contains the non-attacked arguments (a, e and j), one contains the arguments directly attacked once (b, c, d and h) and the last one contains arguments directly attacked twice (f, g and i). The first distinction between arguments in AF_c can be done at the end of the first step:

$$a \simeq e \simeq j \succ b \simeq c \simeq d \simeq h \succ f \simeq g \simeq i$$

Then, during the second step, in some group of arguments with the same level of acceptability, one can distinguish arguments in taking into account the direct defenders. Indeed, c which is defended once by a is now strictly more acceptable than b, d and h which are not defended and, with the same idea, f and g are strictly more acceptable than i.

$$a \simeq^{\text{Dbs}} e \simeq^{\text{Dbs}} j \succ^{\text{Dbs}} c \succ^{\text{Dbs}} b \simeq^{\text{Dbs}} d \simeq^{\text{Dbs}} h \succ^{\text{Dbs}} f \simeq^{\text{Dbs}} g \succ^{\text{Dbs}} i$$

There exists no path of length 3 so the process ends and the final ranking is the one obtained during the previous step.

6. If there is no cycle, the threshold is equal to the longest branch in the argumentation framework. But if cycles are permitted, the discussion count of some arguments can be infinite because $Dis_i(x)$ evolve cyclically. However, the authors strongly conjecture that there exists a threshold t after which it is no longer possible to distinguish the arguments: if $\forall i \leq t$, $Dis_i(x) = Dis_i(y)$, then $\forall i > t$, $Dis_i(x) = Dis_i(y)$.

2.3.3 Burden-based semantics

Amgoud and Ben-Naim [AMGOUD & BEN-NAIM 2013] have also introduced the burden-based semantics which follows the same idea than the discussion-based semantics in considering only direct attackers of arguments. However the approach is different. Indeed, instead of computing all the possible paths that lead to an argument, each argument receives, at each step, a *burden number* which are simultaneously computed on the basis of the burden numbers of their direct attackers at the previous step.

Definition 2.3.5 (Burden vector)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $x \in \mathcal{A}$ and $i \in \mathbb{N}$. The burden number of x at step i is computed as follow:

$$Bur_i(x) = \begin{cases} 1 & \text{if } i = 0 \\ 1 + \sum_{y \in \mathcal{R}_1(x)} \frac{1}{Bur_{i-1}(y)} & \text{otherwise} \end{cases}$$

The **burden vector of** x is denoted by $Bur(x) = \langle Bur_0(x), Bur_1(x), \ldots \rangle$.

Two arguments are then lexicographically compared on the basis of their burden vector.

Definition 2.3.6 (Burden-based semantics)**.**
The **Burden-based semantics (Bbs)** associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{\text{Bbs}}_{\text{AF}}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,

$$x \succeq^{\text{Bbs}}_{\text{AF}} y \text{ if and only if } Bur(y) \succeq_{lex} Bur(x)$$

Let us compute the burden vector of each argument in AF_c (Figure 2.4).

Example 2.3.1 (cont.)**.**

step	a, e, j	c	b, d, h	f, g	i
0	1	1	1	1	1
1	1	2	2	3	3
2	1	1.5	2	2	3
3	1	1.5	2	2	3

$Bur(a) = Bur(e) = Bur(j) = \langle 1, 1, 1, 1 \rangle$
$Bur(c) = \langle 1, 2, 1.5, 1.5 \rangle$
$Bur(b) = Bur(d) = Bur(h) = \langle 1, 2, 2, 2 \rangle$
$Bur(f) = Bur(g) = \langle 1, 3, 2, 2 \rangle$
$Bur(i) = \langle 1, 3, 3, 3 \rangle$

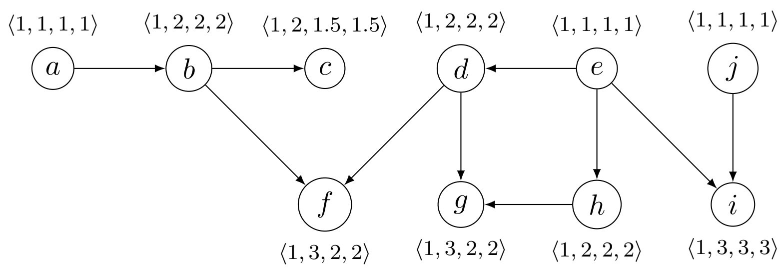


Figure 2.7 – Burden vector of arguments of AF_c

Using lexicographical order to compare the burden vector of each argument, one obtains the following ranking:

$$a \simeq^{\text{Bbs}} e \simeq^{\text{Bbs}} j \succ^{\text{Bbs}} c \succ^{\text{Bbs}} b \simeq^{\text{Bbs}} d \simeq^{\text{Bbs}} h \succ^{\text{Bbs}} f \simeq^{\text{Bbs}} g \succ^{\text{Bbs}} i$$

As in this example, Dbs and Bbs often return the same result because they only consider the number of attackers/defenders of arguments. However, we will see later that some particular cases exist where the two semantics return different rankings.

2.3.4 α-Burden-based semantics

Amgoud, Ben-Naim, Doder and Vesic [AMGOUD *et al.* 2016] have introduced the α-Burden-based semantics which is a broad class of ranking semantics that allows to choose between the importance of the quality of attacks or their quantity. This principle, called compensation, can be checked when several weak attacks (*i.e.* direct attackers of an argument are attacked) could have the same impact as one strong attack (*i.e.* direct attackers are not attacked). For example, with AF_c (see Figure 2.4 page 39), the argument f which has two weak attacks (b and d are attacked) could be more (if one prefers the quality over the quantity), less (if the quantity is preferred to the quality) or as acceptable (if both have the same importance) as the argument b which has one strong attack (a is not attacked).
Formally, the formula is quite similar to the one used by the burden-based semantics (see Definition 2.3.5 page 43) because a burden number is assigned to each argument too. But, in order to satisfy the principle of compensation, Amgoud et al. [AMGOUD *et al.* 2016] introduce a parameter α where different values of α give different behaviors (the greater the value of α, the bigger the influence of the quality of attackers).

Definition 2.3.7 (s_α)**.**
Let $\alpha \in]0, +\infty[$ and $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. The function $s_\alpha : \mathcal{A} \rightarrow [1, +\infty[$ is defined such that $\forall x \in \mathcal{A}$,

$$s_\alpha(x) = 1 + \left(\sum_{y \in \mathcal{R}_1(x)} \frac{1}{(s_\alpha(y))^\alpha} \right)^{1/\alpha}$$

The parameter α is both used for the compensation and to ensure the uniqueness of the solution of a system of equations, with one equation per argument. Indeed, contrary to the burden-based semantics where the lexicographical order is used, the α-burden-based semantics uses a fixed-point form to compute the burden number of each argument. Thus, the higher the score s_α of an argument, the less acceptable the argument.

Definition 2.3.8 (α-Burden-based semantics)**.**
Let $\alpha \in]0, +\infty[$. The **α-Burden-based semantics (α-Bbs)** associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq_{\text{AF}}^{\alpha\text{-Bbs}}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,

$$x \succeq_{\text{AF}}^{\alpha\text{-Bbs}} y \textit{ if and only if } s_\alpha(x) \leq s_\alpha(y)$$

As mentioned when we introduced this semantics, the greater the value of α, the bigger the influence of the quality of attackers. It is why, for different values of α, the computed ranking can vary as shown in the following example which computes the function s_α of each argument in AF_c (Figure 2.4 page 39).

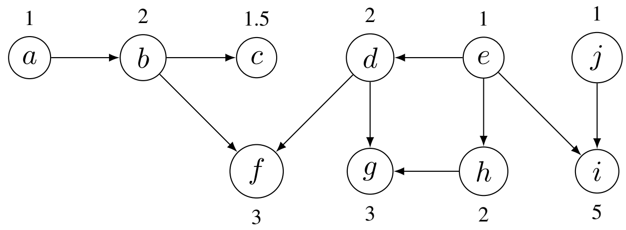


Figure 2.8 – Values returned by the function s_α when $\alpha = 0.5$ of arguments of AF_c

Example 2.3.1 (cont.)**.** As illustrated in Figure 2.8, if $\alpha = 0.5$, we have $s_\alpha(a) = s_\alpha(e) = s_\alpha(j) = 1$, $s_\alpha(c) = 1.5$, $s_\alpha(b) = s_\alpha(d) = s_\alpha(h) = 2$, $s_\alpha(f) = s_\alpha(g) = 3$ and $s_\alpha(i) = 5$. Thus, according the α-burden-based semantics, we obtain the following ranking:

$$\alpha = 0.5 \qquad a \simeq^{\alpha\text{-Bbs}} e \simeq^{\alpha\text{-Bbs}} j \succ^{\alpha\text{-Bbs}} c \succ^{\alpha\text{-Bbs}} \mathbf{b} \simeq^{\alpha\text{-Bbs}} \mathbf{d} \simeq^{\alpha\text{-Bbs}} \mathbf{h} \succ^{\alpha\text{-Bbs}} \mathbf{f} \simeq^{\alpha\text{-Bbs}} \mathbf{g} \succ^{\alpha\text{-Bbs}} i$$

However, if we increase the value of α, the quality becomes more important than the quantity and we obtain different preorders:

$$\alpha = 1 \qquad a \simeq^{\alpha\text{-Bbs}} e \simeq^{\alpha\text{-Bbs}} j \succ^{\alpha\text{-Bbs}} c \succ^{\alpha\text{-Bbs}} \mathbf{b} \simeq^{\alpha\text{-Bbs}} \mathbf{d} \simeq^{\alpha\text{-Bbs}} \mathbf{f} \simeq^{\alpha\text{-Bbs}} \mathbf{g} \simeq^{\alpha\text{-Bbs}} \mathbf{h} \succ^{\alpha\text{-Bbs}} i$$

$$\alpha = 5 \qquad a \simeq^{\alpha\text{-Bbs}} e \simeq^{\alpha\text{-Bbs}} j \succ^{\alpha\text{-Bbs}} c \succ^{\alpha\text{-Bbs}} \mathbf{f} \simeq^{\alpha\text{-Bbs}} \mathbf{g} \succ^{\alpha\text{-Bbs}} \mathbf{b} \simeq^{\alpha\text{-Bbs}} \mathbf{d} \simeq^{\alpha\text{-Bbs}} \mathbf{h} \succ^{\alpha\text{-Bbs}} i$$

2.3.5 Tuples-based semantics

Cayrol and Lagasquie-Schiex [CAYROL & LAGASQUIE-SCHIEX 2005b] have introduced the tuples-based semantics defined as a "global" approach where only the defense and attack branches of an argument are taken into consideration to compare arguments. The quantity and the quality of the branches are stored in tupled values:

Definition 2.3.9 (Tupled value)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x \in \mathcal{A}$.

- Let $v_p(x)$ be the (ordered) tuple of even integers representing the lengths of all the defense branches of x, i.e. $v_p(x)$ is the smallest ordered tuple such that

  $$\forall n \in 2\mathbb{N}, |\mathcal{B}_n(x)| = k \Rightarrow n \in_k v_p(x)$$

  where $\in_k$ means "appears at least k times".

- Let $v_i(x)$ be the (ordered) tuple of odd integers representing the lengths of all the attack branches of x, i.e. $v_i(x)$ is the smallest ordered tuple such that

  $$\forall n \in 2\mathbb{N} + 1, |\mathcal{B}_n(x)| = k \Rightarrow n \in_k v_i(x)$$

If x is not attacked then $v_p(x) = (0, 0, \dots) = 0^\infty$ and $v_i(x) = ()$.
A **tupled value** for x is the pair $v(x) = [v_p(x), v_i(x)]$.

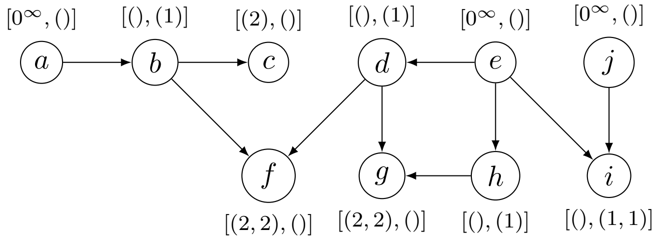


Figure 2.9 – Tupled values of each argument in AF_c

Let us compute the tupled value of each argument in AF_c (Figure 2.4 page 39).

Example 2.3.1 (cont.)**.**

- $v(a) = v(e) = v(j) = [0^\infty, ()]$ because they are non-attacked;
- $v(c) = [(2), ()]$ because it has one defense branch with a length of 2 from a and no attack branch;
- $v(b) = v(d) = v(h) = [(), (1)]$ because they have one attack branch with a length of 1 and no defense branch;
- $v(f) = v(g) = [(2, 2), ()]$ because they have two defense branches with a length of 2 (from a and e for f and twice e for g);
- $v(i) = [(), (1, 1)]$ because it has two attack branches with a length of 1 from e and j.

When cycles are admitted, there may have no non-attacked argument in the graph and thus no branch. The solution proposed in [Cayrol & Lagasquie-Schiex 2005b] is to consider that a cycle is like an infinity of branches which gives an infinite acyclic graph. This process is called the rewriting process of a cycle and is illustrated in Figure 2.10. From a simple cycle argumentation framework where a and b attack each others, there exists two representations of infinite graphs which are acyclic (one with a as a leaf and the other one with b instead).

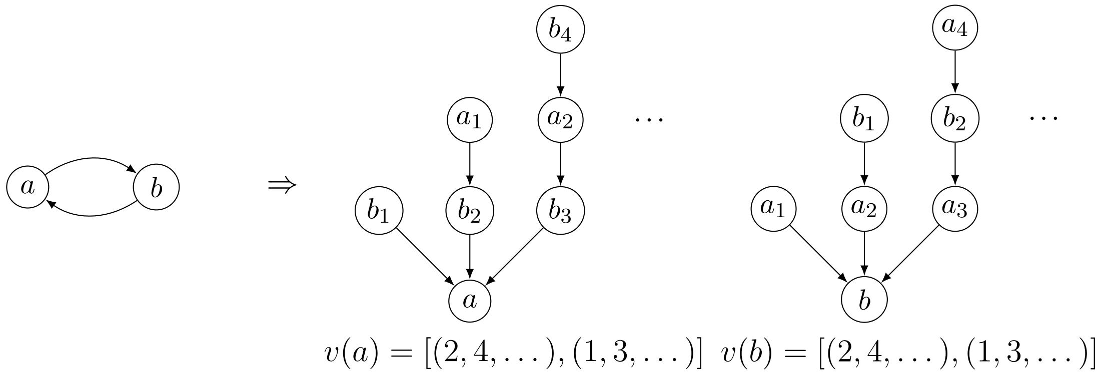


Figure 2.10 – The rewriting process of a cycle

Consequently, the existence of cycles implies that some tuples can be now infinite as shown the Figure 2.10 where $v(a) = v(b) = [(2, 4, \dots), (1, 3, \dots)]$.

Once the tupled values have been computed for each argument, the next step consists in comparing them. To do so, the number of attack and defense branches of two arguments (*i.e.* the length of v_p and v_i) are first compared and, in case of a tie (*i.e.* both arguments have the same number of attack and defense branches), the values inside each tuples (so the length of each branches) are lexicographically compared (see Algorithm 1). Thus, the priority is given to the quantity, and the quality is taken into consideration only if the quantity cannot allow to decide between two arguments.

Algorithm 1 Tuples comparaison [Cayrol & Lagasquie-Schiex 2005b]

Require: $v(a), v(b)$ two tupled values of arguments a and b
Ensure: A ranking $\succeq^{\mathrm{T}}$ between a and b

1: **if** $v_i(a) = v_i(b)$ **and** $v_p(a) = v_p(b)$ **then** $a \succeq^{\mathrm{T}} b$ and $b \succeq^{\mathrm{T}} a$
2: **else**
3:     **if** $|v_i(a)| = |v_i(b)|$ **and** $|v_p(a)| = |v_p(b)|$ **then**
4:         **if** $v_p(a) \preceq_{lex} v_p(b)$ **and** $v_i(a) \succeq_{lex} v_i(b)$ **then** $a \succ^{\mathrm{T}} b$
5:         **else**
6:             **if** $v_p(a) \succeq_{lex} v_p(b)$ **and** $v_i(a) \preceq_{lex} v_i(b)$ **then** $a \prec^{\mathrm{T}} b$
7:             **else** $a \not\succeq^{\mathrm{T}} b$ and $a \not\preceq^{\mathrm{T}} b$
8:             **end if**
9:         **end if**
10:     **else**
11:         **if** $|v_i(a)| \geq |v_i(b)|$ **and** $|v_p(a)| \leq |v_p(b)|$ **then** $a \prec^{\mathrm{T}} b$
12:         **else**
13:             **if** $|v_i(a)| \leq |v_i(b)|$ **and** $|v_p(a)| \geq |v_p(b)|$ **then** $a \succ^{\mathrm{T}} b$
14:             **else** $a \not\succeq^{\mathrm{T}} b$ and $a \not\preceq^{\mathrm{T}} b$
15:             **end if**
16:         **end if**
17:     **end if**
18: **end if**

Example 2.3.1 (cont.)**.** Let us apply Algorithm 1 to compare the previously computed tuples. We obtain the following ranking:

$$a \simeq^{\mathrm{T}} e \simeq^{\mathrm{T}} j \succ^{\mathrm{T}} f \simeq^{\mathrm{T}} g \succ^{\mathrm{T}} c \succ^{\mathrm{T}} b \simeq^{\mathrm{T}} d \simeq^{\mathrm{T}} h \succ^{\mathrm{T}} i$$

Let us remark that two arguments can be incomparable. It is the case, for example, if an argument has strictly more attack branches and strictly more defense branches than another one (see line 14 in Algorithm 1). Thus, an argument included in a cycle with a infinite number of attack and defense branches will be always incomparable with an argument (except the non-attacked arguments) in an acyclic argumentation framework. For instance the argument a in Figure 2.10 (page 46) is incomparable with all the attacked arguments in AF_c in Figure 2.4 (page 39). Consequently, this semantics returns a partial preorder between arguments.

2.3.6 Matt & Toni

Matt and Toni [MATT & TONI 2008] compute the strength of an argument using a two-person zero-sum strategic game. This game confronts two players, a proponent and an opponent for a given argument, where the strategies of the players are sets of arguments. For an argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and an argument $x \in \mathcal{A}$, the set of strategies for the proponent is all the subsets of arguments that contain x: $S_P(x) = \{P \mid P \subseteq \mathcal{A}, x \in P\}$ and for the opponent it is all the subsets of arguments: $S_O = \{O \mid O \subseteq \mathcal{A}\}$. The goal of the game is to evaluate the interactions between the strategies chosen by the two players. In a classical argumentation framework, the only interaction is the attack relation between arguments, so let us define how a strategy (i.e. a set of arguments) can attack another one.

Definition 2.3.10 (Attacks from a set of arguments to another one)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $X, Y \subseteq \mathcal{A}$. The **set of attacks from** X **to** Y is defined by $Y_{\text{AF}}^{\leftarrow X} = \{(x, y) \in X \times Y \mid (x, y) \in \mathcal{R}\}$.

Thus, the set of attacks from a set of arguments to another one is composed of all the attacks in AF such that an argument from the first set directly attacks an argument from the targeted set. Matt and Toni ensure that, in a dispute, it is better for the proponent of an argument to have more attacks against opponents to the argument and fewer attacks from them. To capture this idea, they introduced the notion of degree of acceptability of a set of arguments with respect to another one.

Definition 2.3.11 (Degree of acceptability)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $X, Y \subseteq \mathcal{A}$. The **degree of acceptability** of X with respect to Y is given by the following formula:

$$\phi(X, Y) = \frac{1}{2}\left[1 + f(|Y_{\text{AF}}^{\leftarrow X}|) - f(|X_{\text{AF}}^{\leftarrow Y}|)\right] \textit{ with } f(n) = \frac{n}{n+1}$$

Please note that another mapping f can be used as long as f is a monotonic increasing mapping $f : \mathbb{N} \to [0, 1[$ such that $f(0) = 0$ and $\lim_{n \to \infty} f(n) = 1$.

To defend her argument properly, the proponent should avoid to contradict herself, i.e. her opinions should always correspond to sets of arguments that are at least conflict-free. Also, since the opponent's role in the game is to criticize the proponent, the opponent should get a maximal penalty whenever her opinion fails to attack the proponent's one. Finally, the game should provide an incentive for the proponent to attack the opponent's opinion with as many attacks as possible and at the same time force her to avoid the opponent's attacks. To implement these principles, Matt and Toni choose to use a reward function to represent the relative degree of acceptability of the players opinions.

Definition 2.3.12 (Rewards of the game)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. Given an argument $x \in \mathcal{A}$, $P \in S_P(x)$ (respectively $O \in S_O$) represents a strategy chosen by the proponent (respectively opponent). The **rewards** of P over O, denoted by $r_{\text{AF}}(P, O)$, are defined by :

$$r_{\text{AF}}(P, O) = \begin{cases} 0 & \text{if and only if } \exists x, y \in P, (x, y) \in \mathcal{R} \\ 1 & \text{if and only if } |P_{\text{AF}}^{\leftarrow O}| = 0 \\ \phi(P, O) & \text{otherwise} \end{cases}$$

Recall that each player has to change her strategy (if needed) in order to prevent her adversary to adapt her own strategy, and thus get a better reward. Thus, proponent and opponent have the possibility to use a strategy according to some probability distributions, respectively $p = (p_1, p_2, \ldots, p_m)$ and $q = (q_1, q_2, \ldots, q_n)$, with $m = |S_P|$ and $n = |S_O|$. Thus, for the proponent (respectively opponent), the probability of choosing her i^{th} strategy is equal to p_i (respectively q_i). For each argument $x \in \mathcal{A}$, the proponent's expected payoff $E(x, p, q)$ is then given by $E(x, p, q) = \sum_{j=1}^{n} \sum_{i=1}^{m} p_i q_j r_{i,j}$ with $r_{i,j} = r_{AF}(P_i, O_j)$ where P_i (respectively O_j) represents the i^{th} (respectively j^{th}) strategy of in $S_P(x)$ (respectively S_O). The proponent can therefore expect to get at least $\min_q E(x, p, q)$, where the minimum is taken over all the strategies available to the opponent. Hence the proponent can choose a strategy which will guarantee her a reward of $\max_p \min_q E(x, p, q)$. The opposite is also true with $\min_q \max_p E(x, p, q)$. The value of the zero-sum game for an argument x is:

$$s(x) = \max_p \min_q E(x, p, q) = \min_q \max_p E(x, p, q)$$

Definition 2.3.13 (Ranking-based semantics M&T)**.**
The **ranking-based semantics M&T** associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{\text{MT}}_{\text{AF}}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,

$$x \succeq^{\text{MT}}_{\text{AF}} y \text{ if and only if } s(x) \geq s(y)$$

Let us compute the value of the zero-sum game for each argument in AF_c (Figure 2.4).

Example 2.3.1 (cont.)**.** As illustrated in Figure 2.11, the strength of each argument is: $s(a) = s(e) = s(j) = 1$, $s(c) = s(f) = s(g) = 0.5$, $s(b) = s(d) = s(h) = 0.25$ and $s(i) \simeq 0.16$.

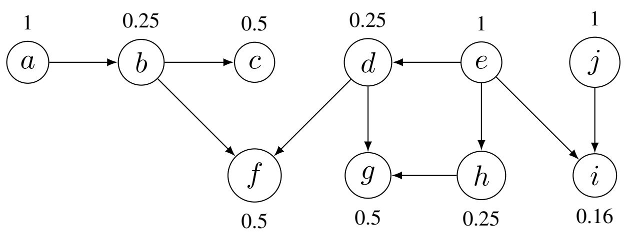


Figure 2.11 – The value of the zero-sum game for each argument in AF_c

Thus, we obtain the following ranking:

$$a \simeq^{\text{MT}} e \simeq^{\text{MT}} j \succ^{\text{MT}} c \simeq^{\text{MT}} f \simeq^{\text{MT}} g \succ^{\text{MT}} b \simeq^{\text{MT}} d \simeq^{\text{MT}} h \succ^{\text{MT}} i$$

2.3.7 Fuzzy labelling

Da Costa Pereira, Tettamanzi and Villata [DA COSTA PEREIRA *et al.* 2011] study how an agent changes her mind in response to new information/arguments. For this, they combine belief revision and argumentation in a single framework close to the Dung's framework, called fuzzy argumentation framework, where a degree of trust is first assigned to each argument.

Indeed, an argument could come from different sources with a more or less important trustworthiness. Thus, when a new argument is proposed, it has more or less influence on the evaluation of existing arguments according to its degree of trust. Then, to compute the influence of an argument on the others, it is necessary to solve a system of non-linear equations (with an equation for each argument). The obtained values express how much the agent tends to accept an argument coming from not fully trusted agents.
Even if this work does not directly propose a ranking-based semantics, the score obtained by each argument after computation could be used to rank the arguments. In order to compare this semantics with the existing ranking-based semantics in the classical framework, we consider that all arguments are totally trusted.

Definition 2.3.14 (Fuzzy reinstatement labelling)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $i \in \mathbb{N}$. The function $f : \mathcal{A} \rightarrow [0,1]$ is defined such that $\forall x \in \mathcal{A}$,

$$f_i(x) = \begin{cases} 1 & \text{if } i = 0 \\ \frac{1}{2}(f_{i-1}(x) + (1 - \max\limits_{y \in \mathcal{R}_1(x)} f_{i-1}(y))) & \text{otherwise} \end{cases}$$

A **fuzzy reinstatement labelling** for AF is, $\forall x \in \mathcal{A}$, $f(x) = \lim\limits_{i \to \infty} f_i(x)$.

According to the formula, the score of an argument during the step i depends both on its score at the previous step ($f_{i-1}(a)$) and on the score of its direct attacker with the highest score at the previous step ($1 - max(f_{i-1}(b))$). Indeed, its score (and so its acceptability) should not be greater than the degree to which its direct attackers are unacceptable: $f(a) \leq 1 - \max\limits_{b \in \mathcal{R}_1(a)} f(b)$. For instance, an argument with a score of 0 is necessarily attacked by at least one argument with a score of 1.

Definition 2.3.15 (Fuzzy labelling)**.**
The **Fuzzy labelling (FL)** associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{\text{FL}}_{\text{AF}}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,

$$x \succeq^{\text{FL}}_{\text{AF}} y \text{ if and only if } f(x) \geq f(y)$$

Let us compute the value of each argument in AF_c (Figure 2.4).

Example 2.3.1 (cont.)**.** As illustrated in Figure 2.12, the fuzzy reinstatement labeling returns the following values : $f(a) = f(e) = f(j) = f(c) = f(f) = f(g) = 1$ and $f(b) = f(d) = f(h) = f(i) = 0$.
Thus, we obtain the following ranking:

$$a \simeq^{\text{FL}} e \simeq^{\text{FL}} j \simeq^{\text{FL}} c \simeq^{\text{FL}} f \simeq^{\text{FL}} g \succ^{\text{FL}} b \simeq^{\text{FL}} d \simeq^{\text{FL}} h \simeq^{\text{FL}} i$$

2.3.8 Iterated graded defense

The next semantics, introduced by Grossi and Modgil [Grossi & Modgil 2015], proposes a generalisation of Dung's notion of acceptability. The theory is based on two assumptions:

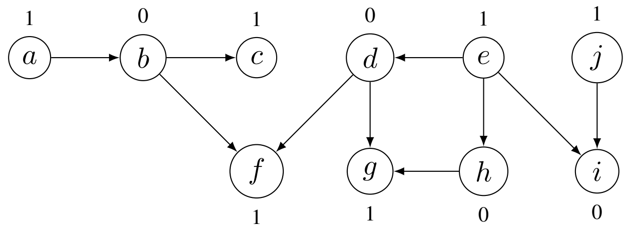


Figure 2.12 – The values returned by the fuzzy reinstatement labelling for each argument in AF_c

(A1) having fewer direct attackers is better than having more; and (A2) having more direct defenders is better than having fewer. To catch these two principles, Grossi and Modgil define a generalisation of the notion of defense initially defined by Dung.
Let x be an argument among a set of arguments $\mathcal{X} \subseteq \mathcal{A}$. Let m be the number of direct attackers of x ($\mathcal{R}_1(x) = \{y_1, ..., y_m\}$) and, for each y_i, let n_i be the number of direct attackers of y_i in $\mathcal{X}$ with $n \leq n_i, \forall i$ (all direct attackers have at least n counter-attackers: $\forall y_i \in \mathcal{R}_1(x)$, $|\mathcal{R}_1(y_i)| \geq n$).

Definition 2.3.16 (Graded defense)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $m, n \in \mathbb{N}$. The **graded defense** of a subset of arguments $\mathcal{X} \subseteq \mathcal{A}$ is :

$$d_{\frac{m}{n}}(\mathcal{X}) = \{x \in \mathcal{A} \mid \nexists_{\geq m} y \in \mathcal{A} : [(y, x) \in \mathcal{R} \textit{ and } \nexists_{\geq n} z \in \mathcal{A}, (z, y) \in \mathcal{R} \textit{ and } z \in \mathcal{X}]\}$$

where $\nexists_{\geq n}$ means "it does not exist at least n".

Thus, $d_{\frac{m}{n}}(\mathcal{X})$ contains the arguments which do not have at least m direct attackers (*i.e.*, which have at most $m-1$ direct attackers) that are not counter-attacked by at least n arguments in $\mathcal{X}$. For example, $d_{\frac{1}{2}}(\mathcal{X})$ selects the arguments such that none of their direct attackers are directly attacked at most once. To better understand the definition of the graded defense, one can use the graded neutrality function:

Definition 2.3.17 (Graded neutrality function)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $m \in \mathbb{N}$. The **graded neutrality function** $\mathfrak{N}_m : 2^{\mathcal{A}} \rightarrow 2^{\mathcal{A}}$ is defined such that for any $\mathcal{X} \subseteq \mathcal{A}$, $\mathfrak{N}_m(\mathcal{X}) = \{x \in \mathcal{A} \mid \nexists_{\geq m} y \in \mathcal{X}$ such that $(y, x) \in \mathcal{R}\}$.

For example, $\mathfrak{N}_1(\mathcal{X})$ is the set of arguments which are not attacked by the arguments in $\mathcal{X}$ while $\mathfrak{N}_2(\mathcal{X})$ contains the arguments which are attacked at most once by the arguments in $\mathcal{X}$ (*i.e.* the arguments belonging to $\mathfrak{N}_1(\mathcal{X})$ and the arguments attacked exactly once by $\mathcal{X}$), and so on. Grossi and Modgil showed that it is possible to define the graded defense from the graded neutrality function: $d_{\frac{m}{n}}(\mathcal{X}) = \mathfrak{N}_m(\mathfrak{N}_n(\mathcal{X}))$. Thus, $d_{\frac{1}{2}}(\mathcal{X})$ can be rewritten $\mathfrak{N}_1(\mathfrak{N}_2(\mathcal{X}))$ which selects the arguments which are not attacked ($\mathfrak{N}_1$) among the arguments directly attacked at most once by $\mathcal{X}$ ($\mathfrak{N}_2(\mathcal{X})$).

Example 2.3.1 (cont.)**.** Given the argumentation framework AF_c in Figure 2.4, let us compute some graded defenses of the set $\mathcal{X} = \{a, e, j\}$ which contains the non-attacked arguments of AF_c:

$$d_{\substack{1\\1}}(\mathcal{X}) = \mathfrak{N}_1(\mathfrak{N}_1(\mathcal{X})) = \mathfrak{N}_1(\{a, c, e, f, g, j\}) = \{a, c, e, f, g, j\}$$

$$d_{\substack{1\\2}}(\mathcal{X}) = \mathfrak{N}_1(\mathfrak{N}_2(\mathcal{X})) = \mathfrak{N}_1(\{a, b, c, d, e, f, g, h, j\}) = \{a, e, j\}$$

$$d_{\substack{2\\1}}(\mathcal{X}) = \mathfrak{N}_2(\mathfrak{N}_1(\mathcal{X})) = \mathfrak{N}_2(\{a, c, e, f, g, j\}) = \{a, b, c, d, e, f, g, h, j\}$$

$$d_{\substack{2\\2}}(\mathcal{X}) = \mathfrak{N}_2(\mathfrak{N}_2(\mathcal{X})) = \mathfrak{N}_2(\{a, b, c, d, e, f, g, h, j\}) = \{a, b, c, d, e, h, j\}$$

Thus, in agreement with the assumptions (A1) and (A2) (defined on the previous page), the arguments belonging to $d_{\substack{m\\n}}(\mathcal{X})$ are at least as strong as the arguments belonging to $d_{\substack{s\\t}}(\mathcal{X})$ when $m \leq s$ (less direct attackers) and $n \geq t$ (more direct defenders). However, this method can be insufficient on its own to compare arguments: the set of arguments computed that way may be not strong enough to defend itself. It is why this method must be recursively applied until obtaining a "stabilized" set of arguments. Thus, for an ordinal α, the α-fold iteration of $d_{\substack{m\\n}}$ is denoted by $d^{\alpha}_{\substack{m\\n}}$ (with $d^{0}_{\substack{m\\n}}(\mathcal{X}) = \mathcal{X}$, $d^{1}_{\substack{m\\n}}(\mathcal{X}) = d_{\substack{m\\n}}(\mathcal{X})$, $d^{2}_{\substack{m\\n}}(\mathcal{X}) = d_{\substack{m\\n}}(d_{\substack{m\\n}}(\mathcal{X}))$, ...). A set of arguments is stabilized if and only if there exists an ordinal α such that $d^{\alpha}_{\substack{m\\n}}(\mathcal{X}) = d^{\alpha+1}_{\substack{m\\n}}(\mathcal{X})$. Grossi and Modgil explain that since $d_{\substack{m\\n}}$ is monotonic the existence of such α is always guaranteed according to the Knaster-Tarski theorem.[7] Thus, the *indefinite iteration* of $d_{\substack{m\\n}}(\mathcal{X})$ is defined as

$$d^{*}_{\substack{m\\n}}(\mathcal{X}) = \bigcup_{0 \leq i \leq \alpha} d^{i}_{\substack{m\\n}}(\mathcal{X})$$

Take two arguments x and y, and some fixed set $\mathcal{X}$. Is it the case that every time y is defended through the iteration of some graded defense function of $\mathcal{X}$, x also is? If it is the case then every kind of defense met by y (with respect to $\mathcal{X}$) is also met by x and consequently x is at least more acceptable than y (because x may belong to a more graded defense).

Definition 2.3.18 (Iterated Graded Defense semantics)**.**
The **Iterated Graded Defense semantics (IGD)** associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{\text{IGD}}_{\text{AF}}$ on $\mathcal{A}$ with respect to $\mathcal{X} \subseteq \mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,

$$x \succeq^{\text{IGD}}_{\text{AF}} y \text{ if and only if } \forall m, n \geq 0 \text{ if } y \in d^{*}_{\substack{m\\n}}(\mathcal{X}) \text{ then } x \in d^{*}_{\substack{m\\n}}(\mathcal{X})$$

Please note that two arguments can be incomparable. Indeed, this occurs when, for two arguments a and b and a subset of arguments $\mathcal{X}$, $a \in d^{*}_{\substack{m\\n}}(\mathcal{X})$ and $a \notin d^{*}_{\substack{s\\t}}(\mathcal{X})$ but $b \notin d^{*}_{\substack{m\\n}}(\mathcal{X})$ and $b \in d^{*}_{\substack{s\\t}}(\mathcal{X})$.

Example 2.3.1 (cont.)**.** Given the argumentation framework in Figure 2.4, let us compute the indefinite iteration of the graded defense of the empty set ($\mathcal{X} = \emptyset$) for all the values of $1 \geq m, n \geq 3$.[8] The results are given in Table 2.1 (page 53).

7. also called Tarski's fixed point theorem [TARSKI 1955]
8. It is useless to check the values greater than 3 because the maximal number of direct attackers in AF_c is 2.

$d^*_{\substack{m\\n}}(\emptyset)$		m		
		1	2	3
n	1	$\{a, c, e, f, g, j\}$	$\{a, b, c, d, e, f, g, h, j\}$	$\mathcal{A}$
	2	$\{a, e, j\}$	$\{a, b, c, d, e, h, j\}$	$\mathcal{A}$
	3	$\{a, e, j\}$	$\{a, b, c, d, e, h, j\}$	$\mathcal{A}$

Table 2.1 – The indefinite iteration of the graded defense of the empty set for all the values of $m, n \in \{1, 2, 3\}$

One can remark that f (respectively g) and b (respectively d and h) are incomparable because $f \in d^*_{\substack{1\\1}}(\emptyset)$ but $b \notin d^*_{\substack{1\\1}}(\emptyset)$ and $b \in d^*_{\substack{2\\2}}(\emptyset)$ but $f \notin d^*_{\substack{2\\2}}(\emptyset)$. Thus, we obtain the partial preorder represented in Figure 2.13 by a Hasse diagram [9] ranking arguments where each argument in $\{b, d, h\}$ are incomparable with each argument in $\{f, g\}$.

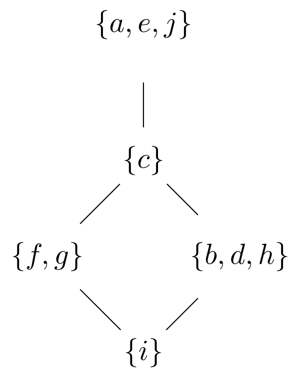


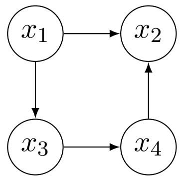
Figure 2.13 – Partial preorder, between arguments of AF_c, returned by IGD semantics and represented by a Hasse diagram

2.3.9 Counting semantics

Pu et al. [Pu *et al.* 2015c] introduced the counting semantics which allows to rank arguments by counting the number of their respective attackers and defenders. However, contrary to the tuples-based semantics which only focuses on the branches, the counting semantics takes into account a large part of paths that leads to a given argument (and which continues the process even if a difference is found contrary to the Discussion-based semantics). In order to assign a value to each argument, they consider an argumentation framework as a dialogue game between proponents of a given argument x (*i.e.* the defenders of x) and opponents of x (*i.e.* the attackers of x). The idea is that an argument is more acceptable if it has many arguments from proponents and few arguments from opponents.

9. Concretely, for a partially ordered set $(\mathcal{A}, \succ)$ one represents each argument of $\mathcal{A}$ as a vertex in the diagram and draws a line segment that goes upward from x to y whenever $y \succ x$

Formally, they first convert a given AF into a matrix $M_{n \times n}$ (where n is the number of arguments in AF) which corresponds to the adjacency matrix of AF (as an AF is a directed graph). Thus, the matrix M, depicted in Figure 2.14, is the adjacency matrix of AF_m where $M_{ij} = 1$ if $(x_i, x_j) \in \mathcal{R}$. As shown in Figure 2.14, the particularity of this matrix is that the matrix product of k copies of M, denoted by M^k, represents, for all the arguments in AF, their number of defenders (if k is even) or attackers (if k is odd) situated at the beginning of a path of length k.



$$M = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad M^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad M^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad M^4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure 2.14 – An argumentation framework and its adjacency matrix M

Finally, a normalization factor N (*e.g.* the matrix infinite norm [Horn & Johnson 2012]) is applied to M in order to guarantee the convergence, and a damping factor α is used to have a more refined treatment on different length of attacker and defenders (*i.e.* shorter attacker/defender lines are preferred).

Definition 2.3.19 (Counting model)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $\mathcal{A} = \{x_1, \ldots, x_n\}$, $\alpha \in]0, 1[$ be a damping factor and $k \in \mathbb{N}$. The n-dimensional column vector w over $\mathcal{A}$ at step k is defined by,

$$w_\alpha^k = \sum_{i=0}^{k} (-1)^i \alpha^i \tilde{M}^i I$$

where $\tilde{M}$ is the normalized matrix such that $\tilde{M} = M/N$ with N as normalization factor and I the n-dimensional column vector containing only 1s.
The **counting model** of AF is $w_\alpha = \lim_{k \to +\infty} w_\alpha^k$. The strength value of $x_i \in \mathcal{A}$ is the i^{th} component of w_α, denoted by $w(x_i)$.

In [Pu *et al.* 2015a], they deepen their work by presenting some complements about how the damping factor α allows to control the convergence speed of the computation for the counting semantics.

Following the previous definition, for any argumentation framework, the counting model can range the strength value of each argument into the interval $[0, 1]$. Thus, an argument is at least as acceptable as another argument if and only if its strength value is equal or higher than the strength value of the other argument.

Definition 2.3.20 (Counting semantics)**.**
The **Counting semantics (CS)** associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{\text{CS}}_{\text{AF}}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,

$$x \succeq^{\text{CS}}_{\text{AF}} y \text{ if and only if } w(x) \geq w(y)$$

Example 2.3.1 (cont.)**.** M is the adjacency matrix associated to AF_c illustrated in Figure 2.4 (page 39).
We first compute the normalization factor which is the matrix infinite norm:

$$N = ||M||_\infty = \max_{1 \leq i \leq n} \sum_{j=0}^{n} |m_{ij}| = 2$$

Then, we compute the adjacency matrix of AF divided by the normalization factor N previously computed:

$$\tilde{M} = \frac{M}{N} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Assume $\alpha = 0.9$, the different steps to compute the counting values of each argument, representing by its transpose [10] form, are summarized below:

$$w^0_\alpha = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1)^T$$
$$w^1_\alpha = w^0_\alpha - 0.9\tilde{M}I = (1, 0.55, 0.55, 0.55, 1, 0.1, 0.1, 0.55, 0.1, 1)^T$$
$$w^2_\alpha = w^1_\alpha + 0.9^2\tilde{M}^2I = (1, 0.55, 0.7525, 0.55, 1, 0.505, 0.505, 0.55, 0.1, 1)^T$$

So, we obtain the following values for each argument:

$$w_\alpha = (1, 0.55, 0.7525, 0.55, 1, 0.505, 0.505, 0.55, 0.1, 1)^T$$

And so, when one compares these values between them (see Figure 2.15 for the graphical representation), we obtain the following ranking:

$$a \simeq^{\text{cs}} e \simeq^{\text{cs}} j \succ^{\text{cs}} c \succ^{\text{cs}} b \simeq^{\text{cs}} d \simeq^{\text{cs}} h \succ^{\text{cs}} f \simeq^{\text{cs}} g \succ^{\text{cs}} i$$

10. The transpose of a matrix A is an operator which switches the row and column indices of the matrix by producing another matrix denoted as A^T.

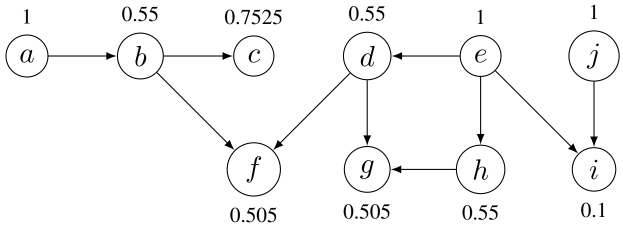


Figure 2.15 – The values returned by the counting model when $\alpha = 0.9$ for AF_c

2.4 Alternative semantics

In the previous section, we focused on ranking-based semantics which return only one ranking for a given argumentation framework. These rankings have the particularity to return a (total or partial) preorder between arguments from the most to the least acceptable ones. However, there also exists other kinds of semantics which return several results (several rankings for example) for a given argumentation framework (or several values are computed for a same argument) or propose alternative rankings.

2.4.1 Several results

Social model

In addition to have introduced the social argumentation framework (see Section 1.3.7), Leite and Martins [LEITE & MARTINS 2011] also proposed a class of semantics where the social value assigned to arguments is computed from their social support based on the votes assigned to each arguments and the values of their direct attackers. Let us first define the main components used to determine a social model corresponding to a social argumentation framework.

Definition 2.4.1 (Social abstract argumentation semantics)**.**
Let $SAF = \langle \mathcal{A}, \mathcal{R}, v \rangle$ be a social argumentation framework. A **social abstract argumentation semantics** is a 5-tuple $\langle \mathcal{L}, \tau, \curlywedge, \curlyvee, \neg \rangle$ where:

- $\mathcal{L}$ is a totally ordered set with top $\top$ and bottom $\bot$ elements, containing all possible valuations of an argument;
- $\tau : \mathbb{N} \times \mathbb{N} \rightarrow \mathcal{L}$ is a vote aggregation function which produces the social support of an argument based on its positive and negative votes.
- $\curlywedge : \mathcal{L} \times \mathcal{L} \rightarrow \mathcal{L}$ is a binary algebraic operation on argument valuations used to determine the valuation of an argument based on its social support and the score of its direct attackers.
- $\curlyvee : \mathcal{L} \times \mathcal{L} \rightarrow \mathcal{L}$ is a binary algebraic operation on argument valuations used to aggregate the score of direct attackers.
- $\neg : \mathcal{L} \rightarrow \mathcal{L}$ is a unary algebraic operation on argument valuations used to restrict the value of the attacked arguments.

Please note that $v^+(x)$ and $v^-(x)$ represent the number of positive votes of x and the number of negative votes of x respectively: $v(x) = (v^+(x), v^-(x))$.

Definition 2.4.2 (Social model)**.**
Let $SAF = \langle \mathcal{A}, \mathcal{R}, v \rangle$ be a social argumentation framework and $S = \langle \mathcal{L}, \tau, \curlywedge, \curlyvee, \neg \rangle$ be a social argumentation framework semantics. The total mapping $M : \mathcal{A} \to \mathcal{L}$ is a **social model** of SAF under the semantics S such that $\forall x \in \mathcal{A}$,

$$M_S(x) = \tau(v^+(x), v^-(x)) \curlywedge \neg \curlyvee \{M_S(y) \mid y \in \mathcal{R}_1(x)\}$$

A model is then a solution to the equation system with one equation for each argument.

The above definitions are intentionally general to be able to accommodate semantics with many distinct features. But Leite and Martins focused on a particular class of semantics following some desired properties for social debates, and defined well-behaved social argumentation framework semantics.

Definition 2.4.3 (Well-behaved social argumentation framework semantics)**.**
A social abstract argumentation semantics $\langle \mathcal{L}, \tau, \curlywedge, \curlyvee, \neg \rangle$ is **well-behaved** if:

- τ is monotonic with respect to the first argument and antimonotonic with respect to the second argument;
- $\curlywedge$ is continuous, commutative, associative, monotonic with respect to both values and $\top$ is its identity element;
- $\curlyvee$ is continuous, commutative, associative, monotonic with respect to both values and $\perp$ is its identity element;
- $\neg$ is antimonotonic, continuous, $\neg\perp = \top$, $\neg\top = \perp$ and $\neg\neg a = a$.

The *simple product semantics* is one possible choice of well-behaved social argumentation framework semantics proposed in [LEITE & MARTINS 2011] such that $SP_\epsilon = \langle [0,1], \tau_\epsilon, \curlywedge, \curlyvee, \neg \rangle$ where:

- $\tau_\epsilon(x, y) = \frac{x}{x+y+\epsilon}$ (with $\epsilon > 0$);
- $x_1 \curlywedge x_2 = x_1 \times x_2$ (Product T-Norm);
- $x_1 \curlyvee x_2 = x_1 + x_2 - x_1 \times x_2$ (Probabilistic Sum T-CoNorm);
- $\neg x_1 = 1 - x_1$.

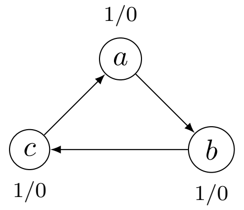
In [CORREIA *et al.* 2014], the authors present an iterative algorithm to efficiently approximate one model of the equation system established from the Definition 2.4.2 with the simple product semantics.

Leite and Martins proved that the existence of at least one social model is guaranteed when the semantics is well-behaved for any $\mathcal{L}$. However, if it is true when $\mathcal{L}$ is an infinite set, it is not the case when $\mathcal{L}$ is a finite discrete set, as shown by the following example:

Example 2.4.1. Let us define another well-behaved semantics, similar to the simple product semantics, except that $\mathcal{L}$ contains only the value 0 and 1; and ϵ does not belong to τ anymore. This semantics remains well-behaved according to the Definition 2.4.3. So, we have *SP* $= \langle\{0,1\}, \tau, \curlywedge, \curlyvee, \neg\rangle$ where:

- $\tau = \frac{x}{x+y}$;
- $x_1 \curlywedge x_2 = x_1 \times x_2$;
- $x_1 \curlyvee x_2 = x_1 + x_2 - x_1 \times x_2$;
- $\neg x_1 = 1 - x_1$.

Let us now compute the social model of the following social argumentation framework:



The social supports are $\tau(a) = \tau(b) = \tau(c) = 1$ and the equation system is:

$$\begin{cases} M_{SP}(a) &=& 1 - M_{SP}(c) \\ M_{SP}(b) &=& 1 - M_{SP}(a) \\ M_{SP}(c) &=& 1 - M_{SP}(b) \end{cases}$$

Thus if $M_{SP}(a) = 1$ then $M_{SP}(a) = 1 \Rightarrow M_{SP}(b) = 0 \Rightarrow M_{SP}(c) = 1 \Rightarrow M_{SP}(a) = 0$ and if $M_{SP}(a) = 0$ then $M_{SP}(a) = 0 \Rightarrow M_{SP}(b) = 1 \Rightarrow M_{SP}(c) = 0 \Rightarrow M_{SP}(a) = 1$. In both cases, we find a contradiction about the value of a (the same reasoning holds for b and c). So there exists no solution for the equation system with the well-behaved semantics *SP*.

Another important property that could be required is the uniqueness of the model which means that there exists only one social model satisfying the equation from definition 2.4.2. Leite and Martins showed that when $|\mathcal{R}_1(x)| \times \tau(v^+(x), v^-(x)) < 1$ for all arguments x in the social argumentation framework, the uniqueness of the model can be proven [LEITE & MARTINS 2011, Theorem 13]. They also conjecture [LEITE & MARTINS 2011, Conjoncture 14] that the simple product semantics satisfies this property for social abstract argumentation frameworks *in general*. However, we proved in [AMGOUD *et al.* 2017b] that this conjecture only holds up to 3 arguments in the social argumentation framework (*i.e.* from 4 arguments, several social models may exist).

Example 2.4.2 (Non-uniqueness of models [AMGOUD *et al.* 2017b]**).**
The example contains four arguments involved in pairwise reciprocal attacks, as illustrated (in Figure 2.16).

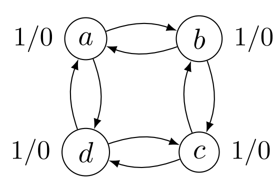


Figure 2.16 – Example of SAF with multiple valid models

Thus, by taking $\epsilon = 0.1$, the social support of all the arguments is: $\tau_\epsilon(a) = \tau_\epsilon(b) = \tau_\epsilon(c) = \tau_\epsilon(d) = \frac{1}{1+0+\epsilon} = \frac{1}{1.1} \approx 0.909$.

Let us now write the equation system (with one equation for each argument) from the AF illustrated in Figure 2.16.

$$\begin{cases} M_{SP_\epsilon}(a) = M_{SP_\epsilon}(c) = \frac{1}{1.1} \times (1 - (M_{SP_\epsilon}(b) + M_{SP_\epsilon}(d) - M_{SP_\epsilon}(b) \times M_{SP_\epsilon}(d))) \\ M_{SP_\epsilon}(b) = M_{SP_\epsilon}(d) = \frac{1}{1.1} \times (1 - (M_{SP_\epsilon}(a) + M_{SP_\epsilon}(c) - M_{SP_\epsilon}(a) \times M_{SP_\epsilon}(c))) \end{cases}$$

It can be checked, in Table 2.2, that this equation system has three distinct valid models:

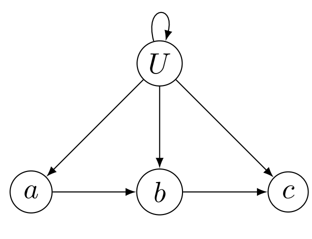
	$M_{SP_\epsilon}(a)$	$M_{SP_\epsilon}(b)$	$M_{SP_\epsilon}(c)$	$M_{SP_\epsilon}(d)$
model 1	0.36573	0.36573	0.36573	0.36573
model 2	0.01125	0.88875	0.01125	0.88875
model 3	0.88875	0.01125	0.88875	0.01125

Table 2.2 – The three distinct valid models from the SAF illustrated in Figure 2.16

The U-approach using $Eq_{inverse}$

In [GABBAY & RODRIGUES 2016], Gabbay and Rodrigues explain that the equational approach (see Section 1.2.3 page 17) cannot distinguish the arguments which are labelled in (respectively out). Indeed, all the arguments labelled in (respectively out) have a score of 1 (respectively 0) and cannot be compared. Thus, the previous approach is only able to distinguish the undecided arguments with a value between 0 and 1. The proposed solution, called U-approach, is to make all nodes undecided with an external additional node U which attacks every node in the argumentation framework (including itself). Solving the new system of equations using $Eq_{inverse}$ for this "augmented" network will give the degree of in, out as well as $undec$.

Example 2.4.1 (cont.)**.**
Let us build the "augmented" network of AF_1 where the new argument U attacks itself and all the arguments in AF_1:

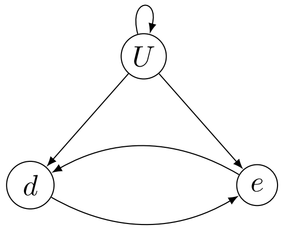


Using $Eq_{inverse}$, the new system of equations is:

$$\begin{cases} f(U) = 0.5 \\ f(a) = 1 - f(U) \\ f(b) = (1 - f(a))(1 - f(U)) \\ f(c) = (1 - f(b))(1 - f(U)) \end{cases}$$

These equations have only one solution: $f(U) = f(a) = 0.5$, $f(b) = 0.25$ and $f(c) = 0.375$. From this solution, we have $f(a) > f(c) > f(b)$. Consequently, a and c are both labelled in but a is more in than c.

Example 2.4.2 (cont.)**.**
Let us build the "augmented" network of AF_2:



Using $Eq_{inverse}$, the new system of equations is:

$$\begin{cases} f(U) = 0.5 \\ f(d) = (1 - f(e))(1 - f(U)) \\ f(e) = (1 - f(d))(1 - f(U)) \end{cases}$$

These equations have only one solution: $f(U) = 0.5$ and $f(d) = f(e) = \frac{1}{3}$. From this solution, we have $f(d) = f(e)$, so d and e are both labelled $undec$ at the same level.

2.4.2 Alternative ranking

The works studied until now rank arguments from the most to the least acceptable one. Intuitively, for a particular labelling, which can be seen as a particular ranking with three levels of acceptability, the arguments labelled in are more acceptable than the arguments labelled $undec$ which are more acceptable than the arguments labelled out. Another possibility suggested in [THIMM & KERN-ISBERNER 2014] consists in assigning a measure of controversiality to the arguments, *i.e.* the larger the value of an argument the more controversial the argument can be seen. Thus, given a semantics, a ranking is associated to an argumentation framework where the arguments are ranked from the most to the least controversial one. The idea behind this ranking is that the arguments labelled in are less controversial than the arguments labelled out (although they are not accepted they are uncontroversially classified as out) which are less controversial than the arguments labelled $undec$ (because we do not know if this argument is in or out). They justify this new way to rank arguments in comparing the definition of controversiality order with the dynamics of argumentation frameworks and, specifically, the notion of enforcement [BAUMANN 2012]: how much must an argumentation framework be changed in order to accept a given argument? Arguments uncontroversially classified as out are (basically) more easily enforced.
Thus, stratified labelings are introduced to assign to each argument of an argumentation framework some natural number (or infinity) representing its degree of controversiality.

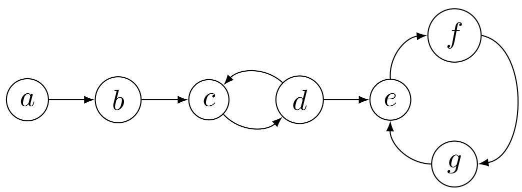
Definition 2.4.4 (Stratified labellings)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and let σ be a classical semantics. A **stratified labelling** $\mathcal{S}$ with respect to σ for AF is a function $\mathcal{S} : \mathcal{A} \to \mathbb{N} \cup \{\infty\}$ such that there is a labelling $\mathcal{L}$ with respect to σ for AF and

- if $in(\mathcal{L}) = \emptyset$ then for all $x \in \mathcal{A}$, $\mathcal{S}(x) = \infty$
- if $in(\mathcal{L}) \neq \emptyset$ then there is a stratified labelling $\mathcal{S}'$ with respect to σ for $AF' = (\mathcal{A}', \mathcal{R}')$ with $\mathcal{A}' = \mathcal{A} \backslash in(\mathcal{L})$ and $\mathcal{R}' = \{(x, y) \mid x \in \mathcal{A}' \text{ and } y \in \mathcal{A}'\}$ such that
  - $\mathcal{S}(x) = 0$ for all $x \in in(\mathcal{L})$ and
  - $\mathcal{S}(x) = 1 + \mathcal{S}'(x)$ for all $x \in \mathcal{A}'$.

A ranking between arguments can be computed based on the value returned by the function $\mathcal{S}$: the larger the value the more controversial the argument. The idea of the previous definition is to build the rank by successively taking the arguments labelled in (so the accepted arguments) of an argumentation framework according to a given semantics, assigning them a rank, then considering the argumentation framework resulting from the removal of these accepted arguments and then re-calculating the arguments in the next rank until the framework is empty or the absence of accepted arguments.

Let us illustrate the previous definition with the following example.

Example 2.4.3. Consider again the argumentation framework AF illustrated in Figure 1.2 and recalled below:



Under the preferred semantics, there exists two reinstatement labellings $Labs_{pr}(AF) = \{\mathcal{L}_2, \mathcal{L}_3\}$ for AF (see Section 1.2.2 page 14) where

$$in(\mathcal{L}_2) = \{a, c\} \quad, out(\mathcal{L}_2) = \{b, d\} \quad, undec(\mathcal{L}_2) = \{e, f, g\}$$
$$in(\mathcal{L}_3) = \{a, d, f\}, out(\mathcal{L}_3) = \{b, c, e, g\}, undec(\mathcal{L}_3) = \emptyset$$

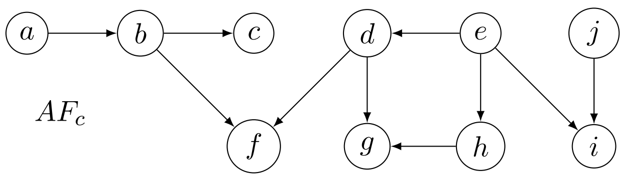
Let us first compute the stratified labelling of $\mathcal{L}_2$.

In $\mathcal{L}_2$, a and c are labelled in so according to the definition $\mathcal{S}(a) = \mathcal{S}(c) = 0$. In removing a and c from AF, we obtain AF_1 where $Labs_{pr}(AF_1) = \mathcal{L}_4$ where $in(\mathcal{L}_4) = \{b, d, f\}$, $out(\mathcal{L}_4) = \{e, g\}$ and $undec(\mathcal{L}_4) = \emptyset$, so $\mathcal{S}(b) = \mathcal{S}(d) = \mathcal{S}(f) = 1$. In removing b, d and f from AF_1, we obtain AF_2 where $Labs_{pr}(AF_2) = \mathcal{L}_5$ where $in(\mathcal{L}_5) = \{g\}$, $out(\mathcal{L}_5) = \{e\}$ and $undec(\mathcal{L}_5) = \emptyset$. So $\mathcal{S}(g) = 2$. Finally, we obtain AF_3 where e is the only argument so e is

labelled in which implies $\mathcal{S}(e) = 3$.
Following the same reasoning, for $\mathcal{L}_3$, one obtains that $\mathcal{S}(a) = \mathcal{S}(d) = \mathcal{S}(f) = 0$, $\mathcal{S}(b) = \mathcal{S}(g) = 1$ and $\mathcal{S}(c) = \mathcal{S}(e) = 2$.

2.5 Properties for ranking-based semantics

Figure 2.17 recalls the argumentation AF_c and all the rankings returned by the ranking-based semantics introduced in Section 2.3. One can remark that they are rarely identical de-



<table>
<tr><th>Semantics</th><th>Ranking between arguments</th></tr>
<tr><td>M&T</td><td>$a \simeq e \simeq j \succ c \simeq f \simeq g \succ b \simeq d \simeq h \simeq i$</td></tr>
<tr><td>FL</td><td>$a \simeq e \simeq j \simeq c \simeq f \simeq g \succ b \simeq d \simeq h \simeq i$</td></tr>
<tr><td>Cat</td><td rowspan="2">$a \simeq e \simeq j \succ c \succ b \simeq d \simeq f \simeq g \simeq h \succ i$</td></tr>
<tr><td>1-Bbs</td></tr>
<tr><td>Dbs</td><td rowspan="4">$a \simeq e \simeq j \succ c \succ b \simeq d \simeq h \succ f \simeq g \succ i$</td></tr>
<tr><td>Bbs</td></tr>
<tr><td>0.5-Bbs</td></tr>
<tr><td>CS</td></tr>
<tr><td>5-Bbs</td><td rowspan="2">$a \simeq e \simeq j \succ c \succ f \simeq g \succ b \simeq d \simeq h \succ i$</td></tr>
<tr><td>IGD</td></tr>
<tr><td>Tuples</td><td>$a \simeq e \simeq j \succ f \simeq g \succ c \succ b \simeq d \simeq h \succ i$</td></tr>
</table>

Figure 2.17 – Rankings on the arguments in AF_c computed by the different ranking-based semantics

spite the small number of arguments because there exists nine distinct rankings. However the differences between these semantics are not always obvious which makes difficult the choice of a particular ranking-based semantics for a user. Indeed, it seems important to answer to some questions like: which arguments should be the most acceptable? Is an argument more acceptable if it has less direct attackers than another one? Following this idea, some works [CAYROL & LAGASQUIE-SCHIEX 2005b, MATT & TONI 2008, AMGOUD & BEN-NAIM 2013] propose a set of properties (postulates), each of which represents a specific criterion, allowing a better understanding of these semantics. In this section, we recall the intuition and the formal definition of these existing properties and, for completeness, we also give the basic idea (the formal definition and some examples will be given in Chapter 4, Section 4.2) of the additional properties that we introduced.

2.5.1 Existing properties

We choose to categorise the existing properties into four groups focusing on different aspects of an argumentation framework: the general properties, the properties aiming to select the most acceptable argument(s) and the least acceptable argument(s), the properties focusing on the direct attackers of arguments and the ones focusing on their direct defenders.

General properties

It seems natural that the ranking on the set of abstract arguments should be defined only on the basis of the attacks between arguments and should not depend on the identity of the arguments.

Definition 2.5.1 (Isomorphism)**.**
An isomorphism γ between two argumentation frameworks $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ is a bijective function $\gamma : \mathcal{A} \to \mathcal{A}'$ such that $\forall x, y \in \mathcal{A}$, $(x, y) \in \mathcal{R}$ if and only if $(\gamma(x), \gamma(y)) \in \mathcal{R}'$. With a slight abuse of notation, we will note $AF' = \gamma(AF)$.

The property Abstraction (Abs) states that if there exists an isomorphism between two argumentation frameworks, then they should have the same ranking for a given ranking-based semantics.

Property 1 (Abstraction (Abs))**.** [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Abstraction if and only if for any $AF, AF' \in \mathbb{AF}$, for every isomorphism γ such that $AF' = \gamma(AF)$, we have $x \succeq^{\sigma}_{\text{AF}} y$ if and only if $\gamma(x) \succeq^{\sigma}_{\text{AF}'} \gamma(y)$.

Example 2.5.1. Consider the two argumentation frameworks depicted in Figure 2.18.

Figure 2.18 – Abstraction

The property Abstraction ensures that the ranking between a and b is the same as the one between c and d.

The next property considers that the ranking between two arguments x and y should be independent of any argument that is neither connected to x nor to y. In other words, if the graph which represents the argumentation framework is composed of several disconnected subgraphs then the arguments in a subgraph have no influence on the arguments in another subgraph. The set of such subgraphs is called connected components.

Definition 2.5.2 (Connected components)**.**
The connected components of an argumentation framework AF are the set of largest subgraphs of AF, denoted by $cc(AF)$, where two arguments are in the same component of AF if and only if there exists some path (ignoring the direction of the edges) between them.

Property 2 (Independence (In))**.** [Matt & Toni 2008, Amgoud & Ben-Naim 2013]
A ranking-based semantics σ satisfies Independence if and only if for any argumentation framework AF such that $\forall AF' \in cc(AF)$, $\forall x, y \in Arg(AF')$, $x \succeq^{\sigma}_{\text{AF}'} y$ if and only if $x \succeq^{\sigma}_{\text{AF}} y$.

Example 2.5.1 (cont.)**.** Consider again the two argumentation frameworks depicted in Figure 2.18. The property Independence ensures that the ranking between a and b (and the ranking between c and d) remains the same after the union of the two frameworks (which contains the four arguments).

Best and worst arguments

We may have expectations regarding the best and worst arguments that we may find in an argumentation framework.

If one considers that an attack always weaken its target, then it seems natural to consider the non-attacked arguments as the best arguments in an argumentation framework. Following this idea, the property Void Precedence (VP) states that a non-attacked argument should be strictly more acceptable than an attacked argument.

Property 3 (Void Precedence (VP))**.** [Matt & Toni 2008, Amgoud & Ben-Naim 2013]
A ranking-based semantics σ satisfies Void Precedence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $\mathcal{R}_1(x) = \emptyset$ and $\mathcal{R}_1(y) \neq \emptyset$ then $x \succ^{\sigma}_{\text{AF}} y$.

Example 2.5.2. Consider the argumentation framework depicted in Figure 2.19.

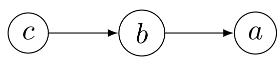


Figure 2.19 – Void Precedence

The property Void Precedence ensures that c which is not attacked ($\mathcal{R}_1(c) = \emptyset$) is strictly more acceptable than a and b which are attacked ($\mathcal{R}_1(b) = \{c\}$ and $\mathcal{R}_1(a) = \{b\}$).

Conversely, the worst arguments can be those which attacks themselves as proposed in [Matt & Toni 2008]. Thus, the property Self-Contradiction (SC) states that an argument that attack itself should be strictly less acceptable than an argument that does not.

Property 4 (Self-Contradiction (SC))**.** [Matt & Toni 2008]
A ranking-based semantics σ satisfies Self-Contradiction if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $(x, x) \notin \mathcal{R}$ and $(y, y) \in \mathcal{R}$ then $x \succ^{\sigma}_{\text{AF}} y$.

Example 2.5.3. Consider the argumentation framework depicted in Figure 2.20.

The property Self-Contradiction ensures that d which attacks itself ($\mathcal{R}_1(d) = \{d\}$) is strictly less acceptable than c and b which are not attacked and a which is attacked twice but by other arguments.

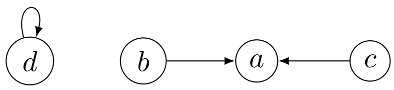


Figure 2.20 – Self-Contradiction

Direct attackers

During the presentation of existing ranking-based semantics in section 2.3, the cardinality and the quality (*i.e.* level of acceptability) of the direct attackers are often considered as a major criteria to evaluate the arguments. Amgoud and Ben-Naïm [AMGOUD & BEN-NAIM 2013] showed that these two principles can be opposed in using the following example.

Example 2.5.4. Consider the argumentation framework depicted in Figure 2.21.

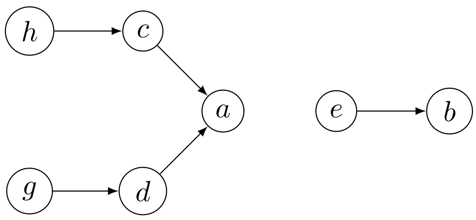


Figure 2.21 – Cardinality Precedence and Quality Precedence

In this example, a is directly attacked by c and d which are also directly attacked, while b is directly attacked by e which is not attacked. So, one needs to make a choice between giving precedence to cardinality over quality (*i.e.* two attacked attackers are worse for the target than one non-attacked attacker) and considers b as more acceptable than a, or giving precedence to quality over cardinality (*i.e.* one non-attacked attacker is worse for the target than two attacked attackers) and considers a as more more acceptable than b.

In order to know if a ranking-based semantics gives precedence to cardinality or quality, Amgoud and Ben-Naïm [AMGOUD & BEN-NAIM 2013] defined two properties: Cardinality precedence (CP) and Quality precedence (QP).

The first one focuses on the cardinality of direct attackers in saying that the greater the number of direct attackers for an argument, the weaker the level of acceptability of this argument.

Property 5 (Cardinality Precedence (CP))**.** [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Cardinality Precedence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $|\mathcal{R}_1(x)| < |\mathcal{R}_1(y)|$ then $x \succ^{\sigma}_{\text{AF}} y$.

Example 2.5.4 (cont.)**.** Consider the argumentation framework depicted in Figure 2.21 (page 65). The property Cardinality Precedence ensures that b is strictly more acceptable than a because $|\mathcal{R}_1(b)| = 1 < 2 = |\mathcal{R}_1(a)|$.

The second property focuses on the quality of direct attackers in saying that the greater the acceptability of one direct attacker for an argument, the weaker the level of acceptability of this argument.

Property 6 (Quality Precedence (QP))**.** [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Quality Precedence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $\exists y' \in \mathcal{R}_1(y)$ such that $\forall x' \in \mathcal{R}_1(x)$, $y' \succ^\sigma_{\text{AF}} x'$ then $x \succ^\sigma_{\text{AF}} y$.

Example 2.5.4 (cont.)**.** Consider the argumentation framework depicted in Figure 2.21 (page 65). If we suppose that e is strictly more acceptable than c and d, then the property Quality Precedence ensures that a is strictly more acceptable than b.

We saw that the "problem" of choosing between the cardinality precedence and the quality precedence occurs when an argument x has less direct attackers than another argument y and the direct attackers of x are less acceptable than the direct attackers of x. But one can wonder what happens when x has less direct attackers (or the same number) than y and that all the direct attackers of y are at least as acceptable as the direct attackers of x. This is this idea which is caught by the two following properties.
To compare the direct attackers of two arguments, let us introduce a relation that compares sets of arguments on the basis of their rankings.

Definition 2.5.3 ((Strict) group comparison [AMGOUD & BEN-NAIM 2013])**.**
Let $\succeq^\sigma_{\text{AF}}$ be a ranking on $\mathcal{A}$. For any $S_1, S_2 \subseteq \mathcal{A}$, $S_1 \geq^\sigma_S S_2$ if and only if there exists an injective mapping f from S_2 to S_1 such that $\forall a \in S_2, f(a) \succeq^\sigma_{\text{AF}} a$. And $S_1 >^\sigma_S S_2$ if and only if $S_1 \geq^\sigma_S S_2$ and $(|S_2| < |S_1|$ or $\exists a \in S_2, f(a) \succ^\sigma_{\text{AF}} a)$.

Example 2.5.5. Consider the two argumentation frameworks depicted in Figure 2.22.
If the ranking-based semantics σ considers that $b_1 \simeq^\sigma b_2 \simeq^\sigma a_2 \simeq^\sigma a_3 \succ^\sigma a_1$ then it exists a

Figure 2.22 – (Strict) Counter-Transitivity and Defense Precedence

injective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(b)$ such that $\forall a' \in \mathcal{R}_1(a)$, $f(a') \succeq^\sigma a'$ because $b_1 \succeq^\sigma a_1$ and $b_2 \succeq^\sigma a_3$. So we have $\mathcal{R}_1(b) \geq^\sigma_S \mathcal{R}_1(a)$. In addition, b_1 (or b_2) is strictly more acceptable than a_1 so $\mathcal{R}_1(b) >^\sigma_S \mathcal{R}_1(a)$.

The first property Counter-Transitivity (CT) states that if the direct attackers of y are at least as numerous and acceptable as those of x with respect to a ranking-based semantics σ, then x should be at least as acceptable as y.

Property 7 (Counter-Transitivity (CT))**.** [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Counter-Transitivity if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $\mathcal{R}_1(y) \geq^\sigma_S \mathcal{R}_1(x)$ then $x \succeq^\sigma_{\text{AF}} y$.

Its strict version, called Strict Counter-Transitivity (SCT), states that if counter-transitivity is satisfied and either the direct attackers of y are strictly more numerous or acceptable than those of x, then x should be strictly more acceptable than y.

Property 8 (Strict Counter-Transitivity (SCT))**.** [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Strict Counter-Transitivity if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $\mathcal{R}_1(y) >_S^g \mathcal{R}_1(x)$ then $x \succ^\sigma y$.

Example 2.5.5 (cont.)**.** Consider again the two argumentation frameworks depicted in Figure 2.22. As $\mathcal{R}_1(b) \geq_S^g \mathcal{R}_1(a)$, then the counter-transitivity property ensures that a is at least as acceptable as b. But, we also have $\mathcal{R}_1(b) >_S^g \mathcal{R}_1(a)$ so the strict counter-transitivity property ensures that a is strictly more acceptable than b.

Direct defenders

The property Defense Precedence (DP) states that, when two arguments have the same number of direct attackers, an argument with at least one direct defender should be strictly more acceptable than an argument without any direct defender.

Property 9 (Defense Precedence (DP))**.** [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Defense Precedence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$ such that $|\mathcal{R}_1(x)| = |\mathcal{R}_1(y)|$, if $\mathcal{R}_2(x) \neq \emptyset$ and $\mathcal{R}_2(y) = \emptyset$ then $x \succ_{\text{AF}}^\sigma y$.

Example 2.5.5. Consider the two argumentation frameworks depicted in Figure 2.22. The two arguments a and b have the same number of attackers ($\mathcal{R}_1(a) = \{a_1, a_3\}$ and $\mathcal{R}_1(b) = \{b_1, b_2\}$) but a is defended ($\mathcal{R}_2(a) = \{a_2\}$) whereas b is not ($\mathcal{R}_2(b) = \emptyset$). Thus, the property DP ensures that a is strictly more acceptable than b.

When DP says that it is better for an argument to be defended, it could be interesting to know if different kinds of defense of an argument have the same impact on it. Two kinds of defense of an argument are then defined: the simple defense and the distributed defense.

Definition 2.5.4 (Simple defense and distributed defense [AMGOUD & BEN-NAIM 2013])**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x \in \mathcal{A}$.

- The defense of x is simple if and only if every direct defender of x directly attacks exactly one direct attacker of x (*i.e.* $\bigcap_{y \in \mathcal{R}_1(x)} \mathcal{R}_1(y) = \emptyset$).
- The defense of x is distributed if and only if every direct attacker of x is attacked by at most one argument (*i.e.* $\forall y \in \mathcal{R}_1(x)$, $\mathcal{R}_1(y) = \emptyset$ or $\exists! z \in \mathcal{R}_2(x)$ such that $(z, y) \in \mathcal{R}$).

The idea is to compare two arguments having the same number of direct attackers and the same number of direct defenders with the condition that all the direct defender attacks exactly one direct attacker. The property states that, in this case, it is preferable for an argument that each of its defender attacks a distinct direct attacker in order to weaken all of them instead of focusing on a specific direct attacker so as to greatly weaken it (but at the price of leaving its others attackers unaffected).

Property 10 (Distributed-Defense Precedence (DDP))**.** [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Distributed-Defense Precedence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$ such that $|\mathcal{R}_1(x)| = |\mathcal{R}_1(y)|$ and $|\mathcal{R}_2(x)| = |\mathcal{R}_2(y)|$, if the defense of x is simple and distributed and the defense of y is simple but not distributed, then $x \succ^{\sigma}_{\text{AF}} y$.

Example 2.5.6. Consider the two argumentation frameworks depicted in Figure 2.23.

Figure 2.23 – Distributed-Defense Precedence

The two arguments a and b have the same number of attackers ($\mathcal{R}_1(a) = \{a_1, a_3\}$ and $\mathcal{R}_1(b) = \{b_1, b_4\}$) and the same number of defenders ($\mathcal{R}_2(a) = \{a_2, a_4\}$ and $\mathcal{R}_2(b) = \{b_2, b_3\}$). The defense of a is simple because a_2 directly attacks a_1 and a_4 directly attacks a_3 and distributed because a_1 and a_3 are attacked by exactly one argument (a_2 and a_4 respectively). Conversely, the defense of b is also simple but not distributed because b_1 is directly attacked by two arguments (b_2 and b_3). Thus, the property DDP ensures that a is more acceptable than b.

2.5.2 Additional properties

Finally, for completeness, let us list the new properties that we introduced in this thesis. Their formal definition illustrated by examples can be found in Chapter 4, Section 4.2.
The five following properties check if some change in an argumentation framework can improve or degrade the ranking of an argument. These properties have been proposed informally by Cayrol and Lagasquie-Schiex [CAYROL & LAGASQUIE-SCHIEX 2005b], in the context of their semantics but we propose to generalize them for any argumentation framework.

Strict addition of a Defense Branch ($\oplus$DB). Adding a defense branch to any argument increases its level of acceptability.

Addition of a Defense Branch (+DB). Adding a defense branch to any attacked argument increases its level of acceptability.

Increase of an Attack branch ($\uparrow$AB). Increasing the length of an attack branch of an argument increases its level of acceptability.

Addition of an Attack Branch (+AB). Adding an attack branch to any argument decreases its level of acceptability.

Increase of a Defense branch (↑DB). Increasing the length of a defense branch of an argument decreases its level of acceptability.

The last five properties catch additional parameters that are important for a better understanding of the behavior of ranking-based semantics.
Total (Tot). All pairs of arguments can be compared.

Non-attacked Equivalence (NaE). All the non-attacked arguments should have the same rank.

Argument Equivalence (AE). If there exists an isomorphism between the ancestors' graph of two arguments, then they are equally acceptable.

Ordinal Equivalence (OE). If two arguments x and y have the same number of direct attackers and that, for each direct attacker of x, there exists a direct attacker of y such that the two attackers are equally acceptable, then x and y are equally acceptable too.

Attack vs Full Defense (AvsFD). An argument without any attack branch is ranked higher than an argument only attacked by one non-attacked argument.

2.6 Conclusion

In this chapter, we introduced ranking-based semantics which is an alternative way to the classical semantics (extension-based semantics and labelling-based semantics), aiming to compare arguments between them. Such semantics allow to rank-order the arguments from the most to the least acceptable one and thus provide many levels of acceptability for the arguments more appropriate for applications like decision-making, online debate platforms, etc. The study of these semantics is still quite recent: a large majority of ranking-based semantics, among those studied in this chapter, have been introduced after 2013. Scoring semantics have been introduced earlier but keep the same goal which is to compare arguments between them (as shown by the categoriser function, originally introduced as a scoring semantics [BESNARD & HUNTER 2001] and then defined as a ranking-based semantics [PU *et al.* 2014]). However, all these semantics have never been compared between them, making it difficult for a potential user to choose between them. It is why, we regrouped the postulates for ranking-based semantics, from different papers, having for objective to better understand the behavior of these semantics. Thus a natural contribution will be to study the existing ranking-based semantics in the light of the proposed properties. This allows us to propose a better reading of the different choices one has on this matter but also to provide additional semantics not yet introduced but which would be compatible for specific applications.

Part II

Contributions

Chapter 3

Ranking-based Semantics based on Propagation

In this chapter, we will introduce a new family of ranking-based semantics for abstract argumentation framework *à la* Dung. While existing ranking-based semantics compare the arguments only focusing on the quality and the quantity of paths towards arguments, we propose another principle which takes specifically into account the role and impact of non-attacked arguments. Indeed, non-attacked arguments play a key role in the classical semantics (extension-based semantics and labelling-based semantics) to select accepted arguments. So, it could be interesting to observe what is happening when more importance are given to them in the ranking-based semantics. In order to take into account this new principle while preserving those concerning the quality and the quantity of attackers or defenders, we introduce the propagation semantics where an initial value is assigned to each argument and then propagated into the graph. The values received by each argument are then aggregated to be compared allowing to rank-order the arguments.
We first motivate and formally define the propagation principle used by our propagation semantics. Three propagation semantics are then introduced with different levels of impact given to the non-attacked arguments. Finally, we compare these propagation semantics between them showing that, in some cases, they return the same ranking. Interestingly, we also prove that for some instance of our semantics, the result is the same as existing ranking-based semantics.

This chapter develops the results published in [BONZON *et al.* 2016b].

Contents

3.1 Motivations

It is undeniable that, in addition to belonging to all the extensions, non-attacked arguments play a key role (when they exist) in extension-based semantics and labelling-based semantics. Indeed, take the example of the grounded semantics which selects the unquestionable arguments. A basic algorithm [MODGIL & CAMINADA 2009, Algorithm 6.1] to find the grounded labellings consists in assigning first the label in to all non-attacked arguments, and then iteratively: the label out is assigned to any argument attacked by at least one argument just labelled in, and then labelled in the arguments which have all their direct attackers labelled out. The iteration continues until no more new arguments are labelled in or out. All the non-labelled arguments are labelled $undec$. It is clear that the non-attacked arguments are the basis of this algorithm because without them, the only grounded labelling is the one containing only arguments labelled $undec$.

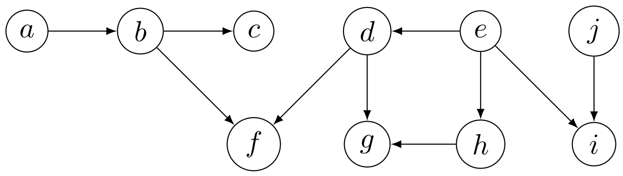


Figure 3.1 – Recall of the argumentation framework AF_c

For the existing ranking-based semantics listed in Section 2.3 (page 39), the results computed from AF_c suggest that non-attacked arguments are always among the most acceptable arguments. Indeed, the rankings computed on the argumentation framework AF_c (recall in Table 2.17 page 62) show that the non-attacked arguments a, e and j are more acceptable than all the other arguments. However, the impact they have on the other arguments are not always clear: should the arguments directly attacked by them be less acceptable than the other arguments? Should the impact of a non-attacked argument be the same as an attacked argument? So, in Dung's semantics, non-attacked arguments have too much impact while, in existing ranking-based semantics, they have no special impact. Thus, a median approach which allows to control their impact in the argumentation framework, could be interesting to define. For this purpose, we introduce three ranking-based semantics for which the influence of non-attacked arguments are more or less important.

3.2 The propagation principle

In order to control the impact of non-attacked arguments while taking into account as best as possible the quality and the quantity of attackers or defenders, we propose to use methods based on the propagation principle. This method has already proved its effectiveness in graph theory, for example to compute the centrality measure of the nodes [FREEMAN 1978, BONACICH 1987]. Such indicators of centrality provide a ranking which identifies the most "important" nodes within a graph. Applications of these centrality measures are multiple, like selecting the most influential person(s) in a social network. They are also used in the PageRank algorithm, developed by Google, to rank websites in their search engine results [PAGE *et al.* 1999]. The idea is to propagate a value from each argument and select those such that a maximum number of values go through them. In argumentation, the propagation principle is also used to compute the grounded extension or labelling or to compute the only existing extension in a well-founded argumentation framework.

The propagation method that we define for our ranking-based semantics has two steps:

1. During the first step, a positive initial weight is assigned to each argument. The score of 1 attached to non-attacked arguments is set to be higher (or equal) than the score of attacked arguments, which is an ϵ between 0 and 1. The value of this ϵ is chosen accordingly to the degree of influence of the non-attacked arguments that we want: the smaller the value of ϵ is, the more important the influence of non-attacked arguments on the order prevails. But it is also possible to assign the same initial weight to all the arguments in the argumentation framework if one considers that all the arguments should have the same influence.

2. Then, during the second step, each argument propagates step by step its value into the argumentation framework in changing their polarities in order to comply with the attack relation meaning (attack or defense). For each argument, the weights from its attackers and defenders are then accumulated and stored.

Before formally defining the propagation principle, we want to pay close attention to a particular case concerning the selection of attackers and defenders of an argument. It could happen that an argument attacks or defends another argument through several paths with the same length. For example, on the argumentation framework AF_c described in Figure 3.1, two paths of length 2 exist from e to g: $\langle e, d, g\rangle$ and $\langle e, h, g\rangle$. So there are two possibilities for e to propagate its value to g (or equally g receives the value from e): either g receives one value from e because it is its only defender or g receives two values from e because there exists two paths between both arguments. We consider that none of these options is better than the other, it is why we include a new parameter $\oplus$ aiming to select the set (S) of arguments at the end of the path without taking into account the number of possible paths, or the multiset (M) which encodes the fact that there are several possible paths.

Definition 3.2.1 (Attacker, Defender according to $\oplus$)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R}\rangle$ be an argumentation framework and $x, y \in \mathcal{A}$ be two arguments. Let $\oplus \in \{M, S\}$, where M (respectively S) stands for multiset (respectively set). The (multi)set

of arguments such that there exists a path to x with a length of n is denoted by $\mathcal{R}_n^{\oplus}(x) = \{y \mid \exists P(y, x) \text{ with } l_P = n\}$.

Obviously there is no change for the direct attackers because an argument cannot be directly attacked several times by the same argument but only concerns the path with a length greater than one. To return to AF_c, with the set, the result is $\mathcal{R}_2^S(g) = \{e\}$, whereas with the multiset, the result is $\mathcal{R}_2^M(g) = \{e, e\}$. Until now, the multiset was used to select the attackers and defenders (see Definition 1.1.5 page 10). Indeed, for the discussion-based semantics which counts the number of paths of arguments and for the tuple semantics which only considers the branches of arguments, the multiset is the best choice. It is also the case during the matrix process used in the counting semantics. But surprisingly, none of these existing ranking-based semantics used the set to select the attackers and defenders of arguments.

Let us now formally define the propagation principle and more precisely the propagation vector of an argument which contains all the values received by its attackers and defenders.

Definition 3.2.2 (Propagation vector)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $\oplus \in \{M, S\}$. The valuation P of $x \in \mathcal{A}$, at step i, is given by:

$$P_i^{\epsilon,\oplus}(x) = \begin{cases} v_\epsilon(x) & \text{if } i = 0 \\ P_{i-1}^{\epsilon,\oplus}(x) + (-1)^i \sum\limits_{y \in \mathcal{R}_i^{\oplus}(x)} v_\epsilon(y) & \text{otherwise} \end{cases}$$

where $v_\epsilon : \mathcal{A} \to \mathbb{R}^+$ is a valuation function giving an initial weight to each argument, with $\epsilon \in [0, 1]$ such that $\forall y \in \mathcal{A}$,

$$v_\epsilon(y) = \begin{cases} 1 & \text{if } \mathcal{R}_1^{\oplus}(y) = \emptyset \\ \epsilon & \text{otherwise} \end{cases}$$

The **propagation vector** of x is denoted $P^{\epsilon,\oplus}(x) = \langle P_0^{\epsilon,\oplus}(x), P_1^{\epsilon,\oplus}(x), \ldots \rangle$.

The first step of the propagation principle is ensured by the valuation function v_ϵ where 1 is assigned to the non-attacked arguments and ϵ for the attacked arguments. The propagation is then defined step by step: at step i, we add or remove (according the value of $(-1)^i$) the accumulated score of x until the previous step ($P_{i-1}^{\epsilon,\oplus}(x)$) and the initial score (v_ϵ) received from arguments at the beginning of a path with a length of i ($\mathcal{R}_i^{\oplus}$).

Example 3.2.1. Let us compute the valuations P with $\epsilon = 0.75$ for each argument in AF_c (Figure 3.1 page 74). These results are given in Table 3.1. If no distinction exists between the set and multiset then the value is put in the same cell. Otherwise, the cell is divided into two parts (valuation for set at left and for multiset at right). For instance, when $i = 2$, $P_2^{0.75,S}(c) = P_2^{0.75,M}(c) = 1$ but $P_2^{0.75,S}(g) = 0.25$ whereas $P_2^{0.75,M}(g) = 1.25$.

In focusing on the argument f, its initial weight is 0.75 because it is attacked by at least one argument,

$$P_0^{0.75,\oplus}(f) = 0.75$$

Then, during the step $i = 1$, the direct attackers (b and d which are also attacked) propagate negatively their weights of 0.75 to f,

$$P_1^{0.75,\oplus}(f) = P_0^{0.75,\oplus}(f) - (v_{0.75}(d) + v_{0.75}(b)) = -0.75$$

$P_i^{0.75,\oplus}$	a,e,j		b,d,h		c		f		g		i	
	S	M	S	M	S	M	S	M	S	M	S	M
0	1		0.75		0.75		0.75		0.75		0.75	
1	1		-0.25		0		-0.75		-0.75		-1.25	
2	1		-0.25		1		1.25		0.25	1.25	-1.25	

Table 3.1 – Valuation P when $\epsilon = 0.75$ for each argument in AF_c

Finally, during the step $i = 2$, f receives positively the weights of 1 from a and e which are non-attacked,

$$P_2^{0.75,\oplus}(f) = P_1^{0.75,\oplus}(f) + (v_{0.75}(a) + v_{0.75}(e)) = 1.25$$

As there exists no path to f with a length higher than 2, this score remains the same, and $P^{0.75,\oplus}(f) = \langle 0.75, -0.75, 1.25 \rangle$.
Figure 3.2 represents AF_c with the propagation vector associated to each argument.

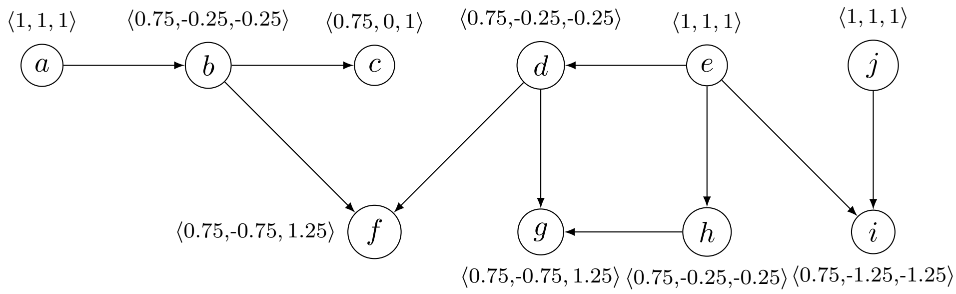


Figure 3.2 – The propagation vectors of each argument belonging to AF_c when $\epsilon = 0.75$ and $\oplus = M$

We deliberately focused on one argument in this example to explain the propagation principle, but it is important to keep in mind that all the propagations are done in parallel at each step.

It is important to note that $P^{\epsilon,\oplus}(x)$ may be infinite (this may occur when an argument is involved in at least one cycle). Moreover, the valuation $P_n^{\epsilon,\oplus}(x)$ of an argument x is not even necessarily bounded as $n \to \infty$ (see definition 3.2.2 page 76). For example, for a simple argumentation framework where x attacks y and y attacks x, then their propagation number are $P^{\epsilon,\oplus}(x) = P^{\epsilon,\oplus}(y) = \langle \epsilon, 0, \epsilon, 0, \ldots \rangle$. But, after a finite number of steps though, an argument is bound to receive the influence of exactly the same arguments than in a previous step of the vector which means that the vector can be finitely encoded (this cyclic aspect can be observed on the example because the propagation vector of x and y is composed by ϵ followed by 0 repeated again and again). More precisely, this number of steps is in the order of the least common multiplier of the cycle lengths occurring in the argumentation graph. As a ranking-based semantics is not concerned with the exact values of arguments, but only in their relative ordering, this is sufficient for our purpose.

3.3 Propagation semantics

Once the propagation vector is computed for each argument in the argumentation framework, the goal is now to compare these vectors in order to provide a ranking between all the arguments. For this purpose, we introduce three ranking-based semantics (more exactly six ranking-based semantics since we have the set/multiset version of each semantics which can return different rankings between arguments for a same argumentation framework) giving more and more importance to non-attacked arguments: $Propa_\epsilon$, $Propa_{1+\epsilon}$ and $Propa_{1\to\epsilon}$.

3.3.1 Propa$_\epsilon$

Our first ranking-based semantics, called $Propa_\epsilon$, compares the propagation vectors of each argument in using the lexicographical order like it is the case with the discussion-based semantics or the burden-based semantics. We want the influence of arguments to quickly decrease with the length of a path, and the lexicographical comparison is a good option to capture this idea. Thus, an argument x is at least as acceptable as an argument y if the propagation vector of x is at least as large as the propagation vector of y for the lexicographical order.

Definition 3.3.1 ($Propa_\epsilon$)**.**
Let $\oplus \in \{M, S\}$ and $\epsilon \in]0,1]$. The ranking-based semantics $Propa_\epsilon^{\epsilon,\oplus}$ associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^P_{\text{AF}}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,

$$x \succeq^P_{\text{AF}} y \text{ if and only if } P^{\epsilon,\oplus}(x) \succeq_{lex} P^{\epsilon,\oplus}(y)$$

Example 3.3.1 (cont.)**.** Let us compute the ranking returned by $Propa_\epsilon^{0.75,S}$ step by step (see Table 3.2), and lexicographically compare the propagation vectors of each argument in AF_c.

$P_i^{0.75,S}$	a,e,j	b,d,h	c	f	g	i	
							$a \simeq b \simeq c \simeq d \simeq e \simeq f \simeq g \simeq h \simeq i \simeq j$
0	1	0.75	0.75	0.75	0.75	0.75	$a \simeq e \simeq j \succ b \simeq c \simeq d \simeq f \simeq g \simeq h \simeq i$
1	1	-0.25	0	-0.75	-0.75	-1.25	$a \simeq e \simeq j \succ c \succ b \simeq d \simeq h \succ f \simeq g \succ i$
2	1	-0.25	1	1.25	0.25	-1.25	$a \simeq^P e \simeq^P j \succ^P c \succ^P b \simeq^P d \simeq^P h \succ^P f \succ^P g \succ^P i$

Table 3.2 – The evolution of the ranking step by step using $Propa_\epsilon^{0.75,S}$ on AF_c

So, the ranking returned by $Propa_\epsilon^{0.75,S}$ is:

$$\oplus = S \qquad a \simeq^P e \simeq^P j \succ^P c \succ^P b \simeq^P d \simeq^P h \succ^P f \succ^P g \succ^P i$$

With the same reasoning, when the multiset ($\oplus = M$) is used, $Propa_\epsilon^{0.75,M}$ provides the following ranking:

$$\oplus = M \qquad a \simeq^P e \simeq^P j \succ^P c \succ^P b \simeq^P d \simeq^P h \succ^P f \simeq^P g \succ^P i$$

This semantics mainly focuses on the attackers and defenders, like existing semantics, with the difference that the non-attacked arguments begin with a greater initial value (except if $\epsilon = 1$ which implies that all the arguments begin with the same initial value 1). Thus, the non-attacked arguments can have more influence when they propagate their value than the attacked arguments (or similarly the attacked argument have less influence when they propagate their value). But this influence is variable and depends on the value of ϵ. Indeed, if the value of ϵ is close to 1, then the value propagated by the non-attacked arguments and the value propagated by the attacked arguments are almost the same. This implies, in this case, that the difference for an argument between to be attacked (or defended) by a non-attacked argument and to be attacked (or defended) by an attacked argument, is weak. And conversely, if the value of ϵ is close to 0, then the influence of the non-attacked arguments will be high. Consequently, as shown in the following example, two values of ϵ can lead to different rankings.

Example 3.3.1 (cont.). Let us recall the ranking returned by $Propa_\epsilon^{0.75,S}$ on AF_c:

$$\oplus = S \qquad a \simeq^P e \simeq^P j \succ^P c \succ^P b \simeq^P d \simeq^P \mathbf{h} \succ^P \mathbf{f} \succ^P g \succ^P i$$

If we focus on f, which is directly attacked twice but defended twice, and h, which is attacked once but not defended, one can see that h is strictly more acceptable than f because $P^{0.75,\oplus}(f) < P^{0.75,\oplus}(h)$. However, let us show that if $\epsilon = 0.3$, then the result is different.
Let us compute the ranking returned by $Propa_\epsilon^{0.3,S}$ step by step (see Table 3.3), and lexicographically compare the propagation vectors of each argument in AF_c.

$P_i^{0.3,S}$	a, e, j	b, d, h	c	f	g	i	$a \simeq b \simeq c \simeq d \simeq e \simeq f \simeq g \simeq h \simeq i \simeq j$
0	1	0.3	0.3	0.3	0.3	0.3	$a \simeq e \simeq j \succ b \simeq c \simeq d \simeq f \simeq g \simeq h \simeq i$
1	1	-0.7	0	-0.3	-0.3	-1.7	$a \simeq e \simeq j \succ c \succ f \simeq g \succ b \simeq d \simeq h \succ i$
2	1	-0.7	1	1.7	0.7	-1.7	$a \simeq^P e \simeq^P j \succ^P c \succ^P f \succ^P g \succ^P b \simeq^P d \simeq^P h \succ^P i$

Table 3.3 – The evolution of the ranking step by step using $Propa_\epsilon^{0.3,S}$ on AF_c

So, the ranking returned by $Propa_\epsilon^{0.3,S}$ is:

$$\oplus = S \qquad a \simeq^P e \simeq^P j \succ^P c \succ^P \mathbf{f} \succ^P g \succ^P b \simeq^P d \simeq^P \mathbf{h} \succ^P i$$

Thus, one can remark that, when $\epsilon = 0.3$, we have $P^{0.3,\oplus}(f) = \langle 0.3, -0.3, 1.7 \rangle$ and $P^{0.3,\oplus}(h) = \langle 0.3, -0.7, -0.7 \rangle$. So, in using the lexicographical order to compare these two propagation vectors, we have $P_1^{0.3,\oplus}(f) > P_1^{0.3,\oplus}(h)$ which implies that f is now more acceptable than h.

So, with the semantics $Propa_\epsilon$, an argument with only (but numerous) defense branches can be worse than an argument only attacked by one non-attacked argument. It is a possible point of view to focus only on the attackers in saying that the more an argument is directly attacked, the less acceptable the argument. This idea is found with the discussion-based semantics or the burden-based semantics [AMGOUD & BEN-NAIM 2013]. But other approaches are possible, as we shall see now.

3.3.2 Propa$_{1+\epsilon}$

With $Propa_\epsilon$, the impact of non-attacked arguments is related to the value of ϵ: the smaller the value of ϵ, the more important the influence of non-attacked arguments (and similarly the less important the influence of attacked arguments). Especially, when ϵ is null, all the attacked argument propagate the value 0 which has no impact on the other arguments. So, in this case, the non-attacked arguments are the only ones to propagate their values in the argumentation framework. However, for several reasons developed further in section 3.4 (page 83), taking into account only this particular case is not always appropriate (for example when all arguments are attacked at least once). Thus, we chose to combine this particular case giving an absolute priority to non-attacked arguments, with the case where ϵ is non-null in order to still take into account the attacked arguments. To combine these two propagation vectors (one for a null value of ϵ and one for a non-null value of ϵ), we need to define the shuffle operation.

Definition 3.3.2 (Shuffle)**.**
The **shuffle** $\cup_s$ between two vectors of real numbers $V = \langle V_1, \dots, V_n\rangle$ and $V' = \langle V'_1, \dots, V'_n\rangle$ is defined as

$$V \cup_s V' = \langle V_1, V'_1, V_2, V'_2, \dots, V_n, V'_n\rangle$$

The goal of the ranking-based semantics $Propa_{1+\epsilon}$ is to simultaneously look at the result of the two propagation vectors $P^{0,\oplus}$ and $P^{\epsilon,\oplus}$ step by step, using the shuffle operation, starting with the first value of the propagation vector $P^{0,\oplus}$ (*i.e.* the one that only takes into account non-attacked arguments). In the case where two arguments are still equally acceptable, we compare the first value of the propagation vector $P^{\epsilon,\oplus}$. Then, in case of equality, we move to the second step and so on.

Definition 3.3.3 ($Propa_{1+\epsilon}$)**.**
Let $\oplus \in \{M, S\}$ and $\epsilon \in]0,1]$. The ranking-based semantics $Propa^{\epsilon,\oplus}_{1+\epsilon}$ associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R}\rangle$ a ranking $\succeq^{\widehat{P}}_{\text{AF}}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,

$$x \succeq^{\widehat{P}}_{\text{AF}} y \text{ if and only if } P^{0,\oplus}(x) \cup_s P^{\epsilon,\oplus}(x) \succeq_{lex} P^{0,\oplus}(y) \cup_s P^{\epsilon,\oplus}(y)$$

Example 3.3.2 (cont.)**.** Let us first compute the propagation number of each argument in AF_c when $\epsilon = 0$ and when $\epsilon = 0.75$ (see Figure 3.3).

<table>
<tr><th rowspan="2">$P_i^{0,\oplus}$</th><th colspan="2">a,e,j</th><th colspan="2">b,d,h</th><th colspan="2">c</th><th colspan="2">f</th><th colspan="2">g</th><th colspan="2">i</th></tr>
<tr><th>S</th><th>M</th><th>S</th><th>M</th><th>S</th><th>M</th><th>S</th><th>M</th><th>S</th><th>M</th><th>S</th><th>M</th></tr>
<tr><td>0</td><td colspan="2">1</td><td colspan="2">0</td><td colspan="2">0</td><td colspan="2">0</td><td colspan="2">0</td><td colspan="2">0</td></tr>
<tr><td>1</td><td colspan="2">1</td><td colspan="2">-1</td><td colspan="2">0</td><td colspan="2">0</td><td colspan="2">0</td><td colspan="2">-2</td></tr>
<tr><td>2</td><td colspan="2">1</td><td colspan="2">-1</td><td colspan="2">1</td><td colspan="2">2</td><td>1</td><td>2</td><td colspan="2">-2</td></tr>
</table>

<table>
<tr><th rowspan="2">$P_i^{0.75,\oplus}$</th><th colspan="2">a,e,j</th><th colspan="2">b,d,h</th><th colspan="2">c</th><th colspan="2">f</th><th colspan="2">g</th><th colspan="2">i</th></tr>
<tr><th>S</th><th>M</th><th>S</th><th>M</th><th>S</th><th>M</th><th>S</th><th>M</th><th>S</th><th>M</th><th>S</th><th>M</th></tr>
<tr><td>0</td><td colspan="2">1</td><td colspan="2">0.75</td><td colspan="2">0.75</td><td colspan="2">0.75</td><td colspan="2">0.75</td><td colspan="2">0.75</td></tr>
<tr><td>1</td><td colspan="2">1</td><td colspan="2">-0.25</td><td colspan="2">0</td><td colspan="2">-0.75</td><td colspan="2">-0.75</td><td colspan="2">-1.25</td></tr>
<tr><td>2</td><td colspan="2">1</td><td colspan="2">-0.25</td><td colspan="2">1</td><td colspan="2">1.25</td><td>0.25</td><td>1.25</td><td colspan="2">-1.25</td></tr>
</table>

Figure 3.3 – Valuation P for each argument in AF_c when $\epsilon = 0$ (left) and when $\epsilon = 0.75$ (right)

Let us focus on the argument f. According to Figure 3.3, its propagation vector when $\epsilon = 0$ is $P^{0,\oplus}(f) = \langle 0, 0, 2\rangle$ and $P^{0.75,\oplus}(f) = \langle 0.75, -0.75, 1.25\rangle$ when $\epsilon = 0.75$. Let us now use the shuffle $\cup_s$ to combine these two propagation vectors:

$$P^{0,\oplus}(f) \cup_s P^{0.75,\oplus}(f) = \langle 0, 0, 2\rangle \cup_s \langle 0.75, -0.75, 1.25\rangle = \langle 0, 0.75, 0, -0.75, 2, 1.25\rangle$$

Of course, the same method is used for all the others arguments. Comparing them with the lexicographical order gives the following ranking:

$$\oplus = S \qquad a \simeq^{\widehat{P}} e \simeq^{\widehat{P}} j \succ^{\widehat{P}} c \succ^{\widehat{P}} f \succ^{\widehat{P}} g \succ^{\widehat{P}} b \simeq^{\widehat{P}} d \simeq^{\widehat{P}} h \succ^{\widehat{P}} i$$

$$\oplus = M \qquad a \simeq^{\widehat{P}} e \simeq^{\widehat{P}} j \succ^{\widehat{P}} c \succ^{\widehat{P}} f \simeq^{\widehat{P}} g \succ^{\widehat{P}} b \simeq^{\widehat{P}} d \simeq^{\widehat{P}} h \succ^{\widehat{P}} i$$

Recall that when $Propa_{\epsilon}^{0.75,S}$ is used, h is strictly more acceptable than f. But, in taking into account first the values propagated by the non-attacked argument, one can remark that f is now strictly more acceptable that h. Indeed, f is directly attacked twice but by attacked arguments while h is attacked only once but by one non-attacked argument. So, during the step $i = 1$, h first receives the negative value from e while f does not receive any value which implies that $P_1^{0,\oplus}(h) = -1 < 0 = P_1^{0,\oplus}(f)$.

It is also important to notice that $Propa_{1+\epsilon}$, conversely to $Propa_{\epsilon}$, returns the same ranking whatever the value of ϵ, that removes the problem of choosing "a good" ϵ.

Proposition 5. Let $\oplus \in \{M, S\}$. For any argumentation framework AF, for any $\epsilon, \epsilon' \in]0, 1]$, it holds that

$$Propa_{1+\epsilon}^{\epsilon,\oplus}(AF) = Propa_{1+\epsilon}^{\epsilon',\oplus}(AF)$$

The ranking obtained in the example 3.3.2 is thus the same for any value of $\epsilon \in]0, 1]$. However, it is necessary to keep the parameter ϵ in the process in order to make a distinction between non-attacked and attacked arguments.

Thus, while with $Propa_{\epsilon}$, an argument with many attacked direct attackers can be more acceptable than an argument directly attacked by one non-attacked argument, it is not possible when $Propa_{1+\epsilon}$ is used. This shows that being directly attacked by non-attacked argument is bad for the acceptability of such arguments.

3.3.3 Propa$_{1\rightarrow\epsilon}$

A last possibility is to give a higher priority to the non-attacked arguments, by propagating only their weights in the graph. In other words, the acceptability of an argument depends only on its attack and defense roots. But if two arguments are still equivalent for this comparison (*i.e.* they have the same number of roots at each step), they are compared using the $Propa_{\epsilon}$ method. Technically, the priority to the non-attacked arguments is given by using $\epsilon = 0$. So, we first compare the propagation vectors $P^{0,\oplus}$. And if the two propagation vectors are identical, we restart with a non-zero ϵ and compare the propagation vectors $P^{\epsilon,\oplus}$.

Definition 3.3.4 ($Propa_{1\rightarrow\epsilon}$).
Let $\oplus \in \{M, S\}$ and $\epsilon \in]0, 1]$. The ranking-based semantics $Propa_{1\rightarrow\epsilon}^{\epsilon,\oplus}$ associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq_{\text{AF}}^{\overline{P}}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,

$x \succeq_{\text{AF}}^{\overline{P}} y$ if and only if $P^{0,\oplus}(x) \succ_{lex} P^{0,\oplus}(y)$ *or* $(P^{0,\oplus}(x) \simeq_{lex} P^{0,\oplus}(y)$ *and* $P^{\epsilon,\oplus}(x) \succeq_{lex} P^{\epsilon,\oplus}(y))$

The fact to first focus on the non-attacked argument allows an argument with a lot of defense branches to receive a lot of positive weights, and conversely, an argument with a lot of attack branches, to receive a lot of negative weights. Let us explain, with the following example, the importance of the non-attacked arguments in this semantics.

Example 3.3.3 (cont.)**.** Let us compute the ranking returned by $Propa_{1\rightarrow\epsilon}^{0.75,S}$, step by step, for AF_c (see Table 3.4 page 82) in beginning by the case $\epsilon = 0$ and then the case $\epsilon = 0.75$.

$P_i^{0,S}$	a,e,j	b,d,h	c	f	g	i	$a \simeq b \simeq c \simeq d \simeq e \simeq f \simeq g \simeq h \simeq i \simeq j$
0	1	0	0	0	0	0	$a \simeq e \simeq j \succ b \simeq c \simeq d \simeq f \simeq g \simeq h \simeq i$
1	1	-1	0	0	0	-2	$a \simeq e \simeq j \succ c \simeq f \simeq g \succ b \simeq d \simeq h \succ i$
2	1	-1	1	2	1	-2	$a \simeq e \simeq j \succ f \succ c \simeq g \succ b \simeq d \simeq h \succ i$
$P_i^{0.75,S}$	a,e,j	b,d,h	c	f	g	i	
0	1	0.75	0.75	0.75	0.75	0.75	$a \simeq e \simeq j \succ f \succ c \simeq g \succ b \simeq d \simeq h \succ i$
1	1	-0.25	0	-0.75	-0.75	-1.25	$a \simeq e \simeq j \succ f \succ c \succ g \succ b \simeq d \simeq h \succ i$
2	1	-0.25	1	1.25	0.25	-1.25	$a \simeq^P e \simeq^P j \succ^P f \succ^P c \succ^P g \succ^P b \simeq^P d \simeq^P h \succ^P i$

Table 3.4 – The evolution of the ranking step by step using $Propa_{1\rightarrow\epsilon}^{0.75,S}$ on AF_c

One can remark that f has one more defense branch than c and g (when $\oplus = S$), which have also one more defense branch than b, d and h. This difference has a direct impact on the ranking between these arguments because one can see that at the end of the step $i = 2$ (when $\epsilon = 0$), f is strictly more acceptable than g and c which are strictly more acceptable than b, d and h.
As some argument are still equally acceptable (in particular c and g which have both only one defense root), we restart the process with $\epsilon = 0.75$. Thank to this second process, c, which is directly attacked only once, is now strictly more acceptable than g which is directly attacked twice ($P_1^{0.75,S}(c) = 0 > -0.75 = P_1^{0.75,S}(g)$). So we obtain the following ranking:

$$\oplus = S \qquad a \simeq^P e \simeq^P j \succ^P f \succ^P c \succ^P g \succ^P b \simeq^P d \simeq^P h \succ^P i$$

Following the same reasoning, we obtain the following ranking when the multiset is used:

$$\oplus = M \qquad a \simeq^P e \simeq^P j \succ^P f \simeq^P g \succ^P c \succ^P b \simeq^P d \simeq^P h \succ^P i$$

However, as shown by Example 3.3.4, focusing only on $\epsilon = 0$ cannot distinguish the arguments with the same number of defense and attack branches at each step. In this case, those are the attacked argument that will be used to distinguish arguments.

Example 3.3.4. Let us consider the two argumentation frameworks described in Figure 3.4.

Clearly, a and a' cannot be distinguished only by comparing their propagation vectors when $\epsilon = 0$: $P^{0,\oplus}(a) = P^{0,\oplus}(a') = \langle 0, 0, 2\rangle$. However, with a non-zero ϵ, a, directly attacked once, becomes strictly more acceptable than a', directly attacked twice, because $P^{\epsilon,\oplus}(a) = \langle \epsilon, 0, 2\rangle \succ_{lex} \langle \epsilon, \text{-}\epsilon, 2-\epsilon\rangle = P^{\epsilon,\oplus}(a')$.

Without surprise, $Propa_{1\rightarrow\epsilon}$ returns the same ranking whatever the value of ϵ.

Figure 3.4 – Two distinct argumentation frameworks where a and a' have the same number of defense branches

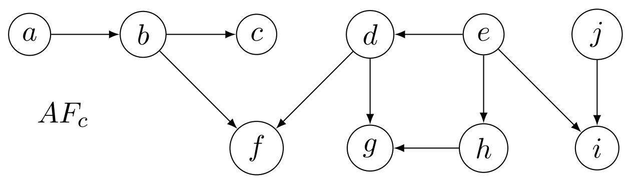
Proposition 6. Let $\oplus \in \{M, S\}$. For any argumentation framework AF, for any $\epsilon, \epsilon' \in]0, 1]$, it holds that

$$Propa^{\epsilon,\oplus}_{1\to\epsilon}(AF) = Propa^{\epsilon',\oplus}_{1\to\epsilon}(AF)$$

This result confirms that the influence of the attacked arguments is only taken into consideration if non-attacked arguments fail to distinguish the arguments.

3.4 Relation between semantics

Table 3.5 recalls the different rankings on the arguments of AF_c, returned by our propagation semantics. One can clearly see, in this recall, the different degrees of the influence given to



$\text{Propa}^{0.75,M}_{\epsilon}$	$a \simeq e \simeq j \succ c \succ b \simeq d \simeq h \succ f \simeq g \succ i$
$\text{Propa}^{0.75,S}_{\epsilon}$	$a \simeq e \simeq j \succ c \succ b \simeq d \simeq h \succ f \succ g \succ i$
$\text{Propa}^{0.3,M}_{\epsilon}$, $\text{Propa}^{\epsilon,M}_{1+\epsilon}$	$a \simeq e \simeq j \succ c \succ f \simeq g \succ b \simeq d \simeq h \succ i$
$\text{Propa}^{0.3,S}_{\epsilon}$, $\text{Propa}^{\epsilon,S}_{1+\epsilon}$	$a \simeq e \simeq j \succ c \succ f \succ g \succ b \simeq d \simeq h \succ i$
$\text{Propa}^{\epsilon,S}_{1\to\epsilon}$	$a \simeq e \simeq j \succ f \succ c \succ g \succ b \simeq d \simeq h \succ i$
$\text{Propa}^{\epsilon,M}_{1\to\epsilon}$	$a \simeq e \simeq j \succ f \simeq g \succ c \succ b \simeq d \simeq h \succ i$

Table 3.5 – Rankings on the arguments of AF_c computed by the different propagation semantics

the non-attacked arguments by the three propagation semantics. Indeed, if we focus on the argument f, one can remark that when $Propa_\epsilon$ is used with a high value of ϵ (which corresponds to the case where the non-attacked and the attacked argument have the lowest difference), f, which is directly attacked twice, is among the less acceptable arguments. When $Propa_{1+\epsilon}$ is used (or for $Propa_\epsilon$ with a small value of ϵ), f is now more acceptable than the arguments (b,

d and h) directly attacked by the non-attacked arguments showing their strong negative impact. Finally, when $Propa_{1\to\epsilon}$ is used, f, which is directly defended by two non-attacked arguments, is strictly more acceptable than c directly defended only once.

Our propagation semantics are all based on the propagation principle. Thus, it could be natural to think that there exists particular cases where these semantics return the same ranking between arguments. In this section, we present the particular cases where the ranking returned by our propagation semantics coincide but also with existing semantics.

3.4.1 Links between propagation semantics

One can remark that in the definitions of the three propagation semantics, the value of ϵ cannot be 0. Recall that, in this case, only the weights propagated by the non-attacked arguments are taken into account, which means that the acceptability of an argument only depends on its attack and defense roots. Thus, the influence given to the non-attacked argument, initially different according to the propagation semantics used, is at its maximum. So, if we suppose that the value of ϵ can be null for the propagation semantics, the ranking returned by these semantics will be identical.

Proposition 7. Let $\oplus \in \{M, S\}$. For any argumentation framework AF,

$$Propa_{\epsilon}^{0,\oplus}(AF) = Propa_{1+\epsilon}^{0,\oplus}(AF) = Propa_{1\to\epsilon}^{0,\oplus}(AF)$$

The idea to compute the score of an argument only on the basis of its attack and defense roots, is close to the one proposed by [Cayrol & Lagasquie-Schiex 2005b] with the tuples-based semantics. Recall that, with this semantics, the arguments are first compared on the basis of their number of attack and defense roots and then of their quality in case of tie. However, as illustrated with the two argumentation frameworks depicted in Figure 3.4 (page 83), some argument can have the "same" number of attack roots and of defense roots with the same length, but with different configuration. In this case, the tuples-based semantics considers that a and a' are equally acceptable without ever take into account the attacked arguments. We choose to use the attacked arguments to distinguish the arguments in this case. Thus, our propagation semantics, $Propa_{\epsilon}$, $Propa_{1+\epsilon}$ and $Propa_{1\to\epsilon}$ can be seen as a kind of "improvement" of the tuples-based semantics in this sense.

In addition to the case where $\epsilon = 0$, there is another particular situation where all the propagation semantics return the same ranking whatever the value of ϵ: when there exists no non-attacked argument in the argumentation framework.

Proposition 8. Let $\oplus \in \{M, S\}$ and $\epsilon \in]0, 1]$. For any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ such that $\nexists x \in \mathcal{A}$, $\mathcal{R}_1^{\oplus}(x) = \emptyset$, $Propa_{\epsilon}^{\epsilon,\oplus}(AF) = Propa_{1+\epsilon}^{\epsilon,\oplus}(AF) = Propa_{1\to\epsilon}^{\epsilon,\oplus}(AF)$.

Indeed, if there is no non-attacked argument, for $Propa_{1+\epsilon}$ and $Propa_{1\to\epsilon}$, the first case where $\epsilon = 0$ returns the same propagation vector for all the arguments (for all argument x, $P^{0,\oplus}(x) = \langle 0, 0, \ldots \rangle$). Consequently the only way to make a difference between arguments is with an $\epsilon \neq 0$ exactly like $Propa_{\epsilon}$. In other words, when there is no non-attacked argument, the semantics compare the arguments only on the number of attackers and defenders.

Finally, the last link only concerns $Propa_\epsilon$ and $Propa_{1+\epsilon}$. We saw that different values of ϵ can lead to different rankings with $Propa_\epsilon$. This is due to several attacked arguments can have more influence than a non-attacked arguments with a value of ϵ high enough. However, there exists a maximal value of ϵ based on the maximal indegree of an argumentation framework which guarantees that this situation does not occur when the semantics $Propa_\epsilon$ is used.

Definition 3.4.1 (Maximal indegree)**.**
Given an argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$, the **maximal indegree** of AF, denoted by $maxdeg(AF)$, is defined as $maxdeg(AF) = \max_{x \in \mathcal{A}} |\mathcal{R}_1^{\oplus}(x)|$.

In other words, we compute the number of direct attackers of each argument in an argumentation framework and the maximum value corresponds to the maximal indegree. For example, in the argumentation framework AF_c (see Figure 3.1 page 74), the maximal indegree is 2 which corresponds to the number of direct attackers of f, g and i, while all the other arguments have less direct attackers.
The following proposition shows that if ϵ is lower than the multiplicative inverse of the maximal indegree then several attacked arguments cannot have more influence than a non-attacked argument. In this case, $Propa_\epsilon$ and $Propa_{1+\epsilon}$ return the same ranking between arguments.

Proposition 9. Let $\oplus \in \{M, S\}$. For any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ such that $\exists x \in \mathcal{A}$, $|\mathcal{R}_1^{\oplus}(x)| > 0$,

$$\text{if } \epsilon < \frac{1}{maxdeg(AF)}, \text{ then } Propa_\epsilon^{\epsilon,\oplus}(AF) = Propa_{1+\epsilon}^{\epsilon,\oplus}(AF)$$

Example 3.4.1. Let us consider the argumentation framework AF_c, depicted in Figure 3.1 page 74, where $maxdeg(AF_c) = 2$. Following the previous proposition, if $\epsilon < 0.5$ then $Propa_\epsilon^{\epsilon,\oplus}(AF_c) = Propa_{1+\epsilon}^{\epsilon,\oplus}(AF_c)$.

Just a word on the case $maxdeg(AF) = 0$ which means that there exists only non-attacked arguments in the argumentation framework. Both ranking-based semantics obviously return the same ranking in this case because all the arguments are equally acceptable.

3.4.2 Link with existing ranking-based semantics

In this part, we list the links between our propagation semantics and ranking-based semantics from the literature. Indeed, the discussion-based semantics (Dbs) and the propagation semantics ($Propa_\epsilon$, $Propa_{1+\epsilon}$ and $Propa_{1\rightarrow\epsilon}$) share similar principles regarding the way paths are counted and the use of the lexicographical comparison. However, let us recall that, in the general case, our semantics also take into account the role of the non-attacked arguments which has consequences on the order between arguments. But, in the case where there is no non-attacked argument, the ranking returned by these semantics is the same.

Proposition 10. Let $\epsilon \in]0, 1]$. For any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ such that $\nexists x \in \mathcal{A}$, $\mathcal{R}_1^M(x) = \emptyset$, $Propa_\epsilon^{\epsilon,M}(AF) = Propa_{1+\epsilon}^{\epsilon,M}(AF) = Propa_{1\rightarrow\epsilon}^{\epsilon,M}(AF) = Dbs(AF)$.

This proposition confirms that when there is no non-attacked argument, our propagation semantics focus on the number of attackers and defenders at each step, like it is done by the discussion-based semantics. Note that this result is obtained with the multiset version of the three kinds of semantics. The set versions are not equivalent.

A last link can be established between $Propa_\epsilon$ and the discussion-based semantics (Dbs). Let us recall that the only way for $Propa_\epsilon$ to give more impact to the non-attacked arguments is through the value of ϵ. But the previous proposition shows that when all the arguments begin with the same initial weight (it is the case when there is no non-attacked argument because all the arguments begin with an initial score of ϵ) then the result is similar to the one returned by the discussion-based semantics. But, according to the definition of $Propa_\epsilon$, it is possible to assign an initial value of 1 to all the arguments (included the attacked argument) with $\epsilon = 1$. Thus, in this case, all the arguments also begin with the same initial score and the following proposition states that the ranking returned by both semantics is similar too.

Proposition 11. For any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$,

$$Propa_\epsilon^{1,M}(AF) = Dbs(AF)$$

Again, only the multiset version allows to obtain this result.

3.4.3 Link with Dung's semantics

Recall that the goal of the propagation semantics we introduced is to give more impact to the non-attacked arguments than existing ranking-based semantics but also less influence than Dung's semantics. If the first part, consisting to give more impact to the non-attacked argument, can be easily observed (see Table 3.5 page 83), one can wonder if our propagation semantics refine the Dung's semantics. In other words, for all the argumentation frameworks, if an argument x is accepted and that an argument y is not accepted with respect to a Dung's semantics σ (*i.e.* x is strictly more acceptable than y), does x is more acceptable than y when our propagation semantics are used? The argumentation framework AF depicted in Figure 3.5, shows that it is not the case.

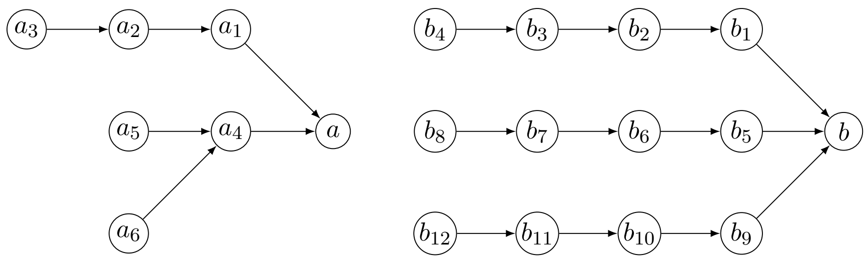


Figure 3.5 – Argumentation framework AF showing the incompatibility between the propagation semantics and the Dung's semantics

As AF is well-founded, its extension is unique and shared by the complete, grounded, stable and preferred semantics:

$$\forall \sigma \in \{co, gr, pr, st\}, \mathcal{E}_\sigma(AF) = \{a_1, a_3, a_5, a_6, b_2, b_4, b_6, b_8, b_{10}, b_{12}, b\}$$

One can remark that $\forall\sigma \in \{co, gr, pr, st\}, b \in \mathcal{E}_\sigma(AF)$ and $a \notin \mathcal{E}_\sigma(AF)$. So, if our propagation semantics refine the Dung's semantics, then b should be strictly more acceptable than a with $Propa_\epsilon$, $Propa_{1+\epsilon}$ and $Propa_{1\to\epsilon}$. Let us now compute the propagation vectors of a and b.

$P_i^{0,\oplus}$	a	b
0	0	0
1	0	0
2	2	0
3	1	0
4	1	3

$P_i^{\epsilon,\oplus}$	a	b
0	ϵ	ϵ
1	-ϵ	-2ϵ
2	2	ϵ
3	1	-2ϵ
4	1	3-2ϵ

Table 3.6 – Valuation P for a and b in AF (Figure 3.5 page 86) when $\epsilon = 0$ (left) and when $\epsilon = 0.75$ (right)

From the propagation vectors of a and b (see Table 3.6), let us now see which one is more acceptable than the other one according to the propagation semantics used.

($Propa_\epsilon$) $a \succ^{P}_{\text{AF}} b$ because $\forall\epsilon \in]0,1], P^{\epsilon,\oplus}(a) \succ_{lex} P^{\epsilon,\oplus}(b)$
$$\Rightarrow P_1^{\epsilon,\oplus}(a) = -\epsilon > -2\epsilon = P_1^{\epsilon,\oplus}(b)$$

($Propa_{1+\epsilon}$) $a \succ^{\widehat{P}}_{\text{AF}} b$ because $P^{\epsilon,\oplus}(a) \cup_s P^{0,\oplus}(a) \succ_{lex} P^{\epsilon,\oplus}(b) \cup_s P^{0,\oplus}(b)$
$$\Rightarrow P_1^{\epsilon,\oplus}(a) = -\epsilon > -2\epsilon = P_1^{\epsilon,\oplus}(b))$$

($Propa_{1\to\epsilon}$) $a \succ^{\overline{P}}_{\text{AF}} b$ because $P^{0,\oplus}(a) \succ_{lex} P^{0,\oplus}(b)$
$$\Rightarrow P_2^{\epsilon,\oplus}(a) = 2 > 0 = P_2^{\epsilon,\oplus}(b)$$

Clearly, for all the propagation semantics, a is strictly more acceptable than b, in contradiction with the result returned by the Dung's semantics.

However, this is not surprising because, for $Propa_\epsilon$ and $Propa_{1+\epsilon}$, the attacked arguments still play a role in the acceptability of the argument. It is why, b, which is directly attacked three times, is less acceptable than a which is directly attacked twice. Dung's semantics are, by definition, blind to the number of attackers/defenders, which explains this difference.
For $Propa_{1\to\epsilon}$, recall that the non-attacked arguments are the only arguments to propagate their value, so as the defense branches of b are shorter than the defense branches of a then b is the first to receive its positive value. Dung's semantics are blind to the length of the defense or attack paths, which explains this difference.

3.5 Conclusion

In this chapter we proposed new ranking-based semantics based on the propagation of the weights of arguments, with a higher initial weight given to non-attacked arguments. The differences between our semantics lie in the choice of the interaction between attacked and non-attacked arguments (i.e. how much priority do we give to non-attacked arguments), and in the choice of sets or multisets to select the attackers and the defenders of the arguments.

Some relationships exist between our semantics and existing ones: the ranking returned by all the propagation semantics based on multisets coincide with the one returned by the semantics Discussion-based semantics when there is no non-attacked arguments in the argumentation framework. Our ranking-based semantics $Propa_\epsilon$ goes further in the relation with the discussion-based semantics because when all the arguments begin with the same weight then the ranking returned by both semantics (when multiset version is used) are similar. So they can be viewed as improvement of the discussion-based semantics allowing to take into account the impact of non-attacked arguments.
By many respects, $Propa_{1\to\epsilon}$, the semantics which gives the more influence to the non-attacked arguments, is close to the tuples-based semantics [CAYROL & LAGASQUIE-SCHIEX 2005b]. Indeed, both focus on the attack and defense roots of arguments to compare them. However, the tuples-based semantics does not necessarily provide a total preorder between arguments, and it cannot be applied (easily) if there is a cycle in the argumentation framework. So, in a sense, $Propa_{1\to\epsilon}$ could be seen as a refinement of the ideas of the tuples-based semantics that allows to overcome these limitations.

However, based solely on the formal definition, the comparison with the other existing ranking-based semantics is still difficult. It is why, in the next chapter, we provide a full comparison study between all the ranking-based semantics detailed in this thesis in order to better understand their behavior. This will also make it possible to put forward the differences of our semantics with the existing ones.

Chapter 4

Comparative Study of Ranking-based Semantics

Recently, a number of ranking-based semantics have been proposed independently (see Chapter 2, Section 2.3), each with more or less different behaviors, and often associated with some desirable properties (see Chapter 2, Section 2.5). However, all these semantics have never been thoroughly compared between them, making the choice of a particular ranking-based semantics difficult for a potential user. In this chapter we propose such a comparative study.

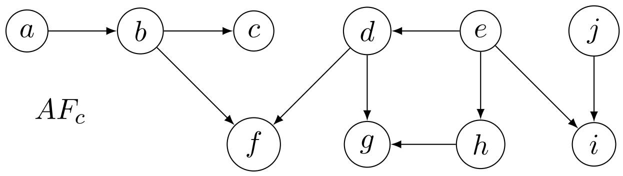
We propose two ways to compare the ranking-based semantics. The first one is an experimental comparison where we examine the rankings returned by these semantics on a benchmark of argumentation framework, in order to evaluate the degree of similarity between each pair of semantics. The second comparison allows to understand where the similarity and the differences between the rankings come from. For this, we study the ranking-based semantics in the light of a set of proposed properties. We also generalize some existing properties only defined in the context of a particular semantics and propose new ones which allow to capture other aspects playing on the diversity between the rankings. Finally, we analyse the obtained results and discuss the different properties.

This chapter develops the results obtained in [BONZON *et al.* 2016a, BONZON *et al.* 2016c].

Contents

4.1 Concordance of ranking-based semantics

As it can be easily checked with the running-example on AF_c (the argumentation framework and the associated rankings are recalled below in Table 4.1) used to illustrate the ranking-based semantics studied in this thesis (see Chapter 2, Section 2.3), these ranking-based semantics mostly return distinct rankings between arguments even though AF_c contains "only" ten arguments.



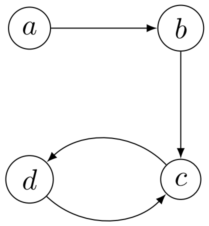
Semantics	*Ranking between arguments*
M&T	$a \simeq e \simeq j \succ c \simeq f \simeq g \succ b \simeq d \simeq h \simeq i$
FL	$a \simeq e \simeq j \simeq c \simeq f \simeq g \succ b \simeq d \simeq h \simeq i$
Cat	$a \simeq e \simeq j \succ c \succ b \simeq d \simeq f \simeq g \simeq h \succ i$
1-Bbs	
Dbs	$a \simeq e \simeq j \succ c \succ b \simeq d \simeq h \succ f \simeq g \succ i$
Bbs	
0.5-Bbs	
CS	
Propa$_\epsilon^{0.75,M}$	
Propa$_\epsilon^{0.75,S}$	$a \simeq e \simeq j \succ c \succ b \simeq d \simeq h \succ f \succ g \succ i$
5-Bbs	$a \simeq e \simeq j \succ c \succ f \simeq g \succ b \simeq d \simeq h \succ i$
IGD	
Propa$_\epsilon^{0.3,M}$	
Propa$_{1+\epsilon}^{\epsilon,M}$	
Propa$_\epsilon^{0.3,S}$	$a \simeq e \simeq j \succ c \succ f \succ g \succ b \simeq d \simeq h \succ i$
Propa$_{1+\epsilon}^{\epsilon,S}$	
Propa$_{1\to\epsilon}^{\epsilon,S}$	$a \simeq e \simeq j \succ f \succ c \succ g \succ b \simeq d \simeq h \succ i$
Tuples	$a \simeq e \simeq j \succ f \simeq g \succ c \succ b \simeq d \simeq h \succ i$
Propa$_{1\to\epsilon}^{\epsilon,M}$	

Table 4.1 – Rankings on the arguments in AF_c computed by the different ranking-based semantics

One can first remark that the differences between these rankings only concerns a subset of arguments (here b, c, d, f, g, h) and not all the arguments. Conversely, some common behaviors seem to appear between the semantics like the fact that a, e and j are always equally acceptable and more acceptable than all the other arguments (except for the semantics FL) or that i is always ranked last (even if it can be a tie). Thus, it could be interesting to know if such observations can be generalized to all argumentation frameworks or if they only concern this particular argumentation framework. The goal of this section consists in evaluating how different or similar are these ranking-based semantics. To this purpose, we choose to compute and to compare the ranking of each ranking-based semantics on several randomly generated argumentation frameworks. But before, we explain how to compute and compare the different rankings, we choose to exclude some ranking-based semantics from this study. Indeed, it is difficult to compare total and partial preorders (because some arguments could be incomparable), it is why the semantics that return a partial preorder, like IGD [Grossi & Modgil 2015] and Tuples [Cayrol & Lagasquie-Schiex 2005b], are excluded. The semantics M&T is also excluded from this study because, according to the authors, this semantics can only be used for argumentation frameworks with less than a dozen of arguments (see [Matt & Toni 2008]). Indeed, the size of the players strategy spaces grows exponentially fast with the total number of arguments in the argumentation framework considered so when it contains more than twelve arguments, the computation becomes almost impossible.

4.1.1 Computation process

The number of softwares aiming to evaluate argumentation frameworks under different usual acceptability semantics and to perform the usual inference tasks, continues to increase these last years (*e.g.* [Egly *et al.* 2008, Cerutti *et al.* 2014, Lagniez *et al.* 2015]). This interest to create solvers more and more effective even led to the creation of a first competition in argumentation [Thimm & Villata 2015]. However there exists no available software aiming to compute the ranking of a given argumentation framework with respect to a given ranking-based semantics. So we first started by implementing all the ranking-based semantics used for this experimental study (*i.e.* Cat, Dbs, Bbs, α-Bbs, FL, CS and the propagation semantics). We chose the aspartix format (apx) to represent an argumentation framework (see Figure 4.1 for an example): each argument of this argumentation framework is defined by *arg*(‹name of the argument›) and each attack is defined by *att*(‹name of the attacking argument›,‹name of the attacked argument›), one by line. Each line finishes with a dot.



arg(a).
arg(b).
arg(c).
arg(d).
att(a, b).
att(b, c).
att(c, d).
att(d, c).

aspartix format

Figure 4.1 – An argumentation framework and its representation in the aspartix format

Thus, our software accepts as input an argumentation framework in aspartix format and a ranking-based semantics, sometimes together with a parameter, and returns the ranking between arguments.

Let us now create a set of randomly generated argumentation frameworks which will be used to compare the rankings computed by each ranking-based semantics thanks to our software. Our generation algorithm (see Algorithm 2) is based on one of the three algorithms used to create the benchmarks of the competition [11] ICCMA'15 [THIMM & VILLATA 2015]. The algorithm that we implemented, first randomly chooses the number of arguments (with a given minimum and maximum value). Then, attacks between arguments are added with a given probability. And finally, random attacks are added between the not-yet connected arguments (*i.e.* the arguments without direct attacker and which attacks no argument) and the other arguments in order to avoid isolated arguments.

Algorithm 2 Argumentation Framework Generator

Require: max_arg is the maximum number of arguments, min_arg is the minimum number of arguments, $propa\_attack$ is the probability of an attack (between 0 and 1)

Ensure: One randomly generated argumentation framework AF which contains between min_arg and max_arg arguments

function RANDOM(i, j)
  return an integer between i and j
end function
$nb\_arg \leftarrow$ RANDOM(min_arg, max_arg) ▷ Number of arguments in AF
for $i \leftarrow min\_arg$ **to** nb_arg **do**
  for $j \leftarrow min\_arg$ **to** nb_arg **do**
    if (RANDOM$(0, 100)/100$) < $propa\_attack$ **then** "a_i" attacks "a_j"
    end if
  end for
end for
for all argument a_k unconnected **do**
$l \leftarrow$ RANDOM(min_arg, nb_arg)
"a_k" attacks "a_l" or "a_l" attacks "a_k"
end for

Thus, in using Algorithm 2, we create 1000 randomly generated argumentation frameworks [12] which contain between 5 and 100 arguments ($5 \leq |\mathcal{A}| \leq 100$). [13] For each of these argumentation frameworks, we used our software to compute the ranking of each ranking-based semantics. Let us now detail how to compare these rankings in order to represent the concordance of the ranking-based semantics.

11. http://argumentationcompetition.org/2015/results.html
12. This set of argumentation frameworks can be found on the following address: http://www.cril.univ-artois.fr/~delobelle/bench.zip
13. A lot of differences between the rankings obtained from AF_c (see Table 4.1 page 90) are already evident despite the "low" number of arguments, so studying argumentation frameworks with less that 100 arguments is sufficient enough to have a good approximation of the concordance of the ranking-based semantics.

4.1.2 Experimental comparison

A way to compare the ranking-based semantics on the basis of the rankings previously computed consists in using the Kendall's tau coefficient [KENDALL 1938]. This value corresponds to the total number of rank disagreements over all unordered pairs of arguments between two rankings from distinct semantics. It therefore allows us to obtain a dissimilarity degree between two rankings.

Definition 4.1.1 (Kendall's tau coefficient)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and σ_1 and σ_2 be two ranking-based semantics. The **Kendall's tau coefficient** between $\sigma_1(AF)$ and $\sigma_2(AF)$ is calculated as follow:

$$K(\sigma_1(AF), \sigma_2(AF)) = \frac{\sum_{\{i,j\} \in \mathcal{A}} \overline{K_{i,j}}(\sigma_1(AF), \sigma_2(AF))}{0.5 \times |\mathcal{A}| \times (|\mathcal{A}| - 1)}$$

with :

- $\overline{K_{i,j}}(\sigma_1(AF), \sigma_2(AF)) = 0$ if $i \succ^{\sigma_1}_{\text{AF}} j$ and $i \succ^{\sigma_2}_{\text{AF}} j$, or $i \prec^{\sigma_1}_{\text{AF}} j$ and $i \prec^{\sigma_2}_{\text{AF}} j$, or $i \simeq^{\sigma_1}_{\text{AF}} j$ and $i \simeq^{\sigma_2}_{\text{AF}} j$,
- $\overline{K_{i,j}}(\sigma_1(AF), \sigma_2(AF)) = 1$ if $i \succ^{\sigma_1}_{\text{AF}} j$ and $i \prec^{\sigma_2}_{\text{AF}} j$ or vice versa,
- $\overline{K_{i,j}}(\sigma_1(AF), \sigma_2(AF)) = 0.5$ if $i \succ^{\sigma_1}_{\text{AF}} j$ or $i \prec^{\sigma_1}_{\text{AF}} j$ and $i \simeq^{\sigma_2}_{\text{AF}} j$ or vice versa.

A Kendall's tau coefficient of 1 ($K(\sigma_1(AF), \sigma_2(AF)) = 1$) between two rankings means that both rankings are opposite (*i.e.* for all arguments $x, y \in \mathcal{A}$, if $x \succ^{\sigma_1} y$ then $y \succ^{\sigma_2} x$) while a Kendall's tau coefficient of 0 ($K(\sigma_1(AF), \sigma_2(AF)) = 0$) means that both rankings are identical. So, the smaller the Kendall's tau coefficient between two rankings, the higher their similarity.

Example 4.1.1. Given an argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$, let us compute the Kendall's tau coefficient between the two following rankings $\sigma_1(AF)$ and $\sigma_2(AF)$:

$$\sigma_1(AF) = a \succ^{\sigma_1} b \simeq^{\sigma_1} c \succ^{\sigma_1} d \succ^{\sigma_1} e$$
$$\sigma_2(AF) = a \succ^{\sigma_2} c \succ^{\sigma_2} d \succ^{\sigma_2} b \succ^{\sigma_2} e$$

Both rankings clearly disagree on pair $\{b, d\}$ ($b \succ^{\sigma 1} d$ and $d \succ^{\sigma 2} b$) and, the semantics $\sigma 1$ considers that b and c are equally acceptable while σ_2 considers that c is strictly more acceptable than b ($b \simeq^{\sigma 1} c$ and $c \succ^{\sigma 2} b$). Thus, as $\sum_{\{i,j\} \in \mathcal{A}} \overline{K_{i,j}}(\sigma_1(AF), \sigma_2(AF)) = 1 + 0.5 = 1.5$ and the number of arguments in AF is 5 ($|\mathcal{A}| = 5$) then the Kendall's tau coefficient is 0.15 ($K(\sigma_1(AF), \sigma_2(AF)) = 0.15$).

The goal is to measure the dissimilarity degree between the ranking-based semantics using the Kendall's tau coefficient. For this purpose, from the rankings computed for each the argumentation framework in input, we compute the Kendall's tau coefficient between all pairs of rankings. Finally, for each pair of ranking-based semantics, we average the Kendall's tau coefficients computed from rankings for each argumentation frameworks and multiply the result by 100 to obtain a percentage of dissimilarity. All the process to compute these values is sum up in the diagram depicted in Figure 4.2.

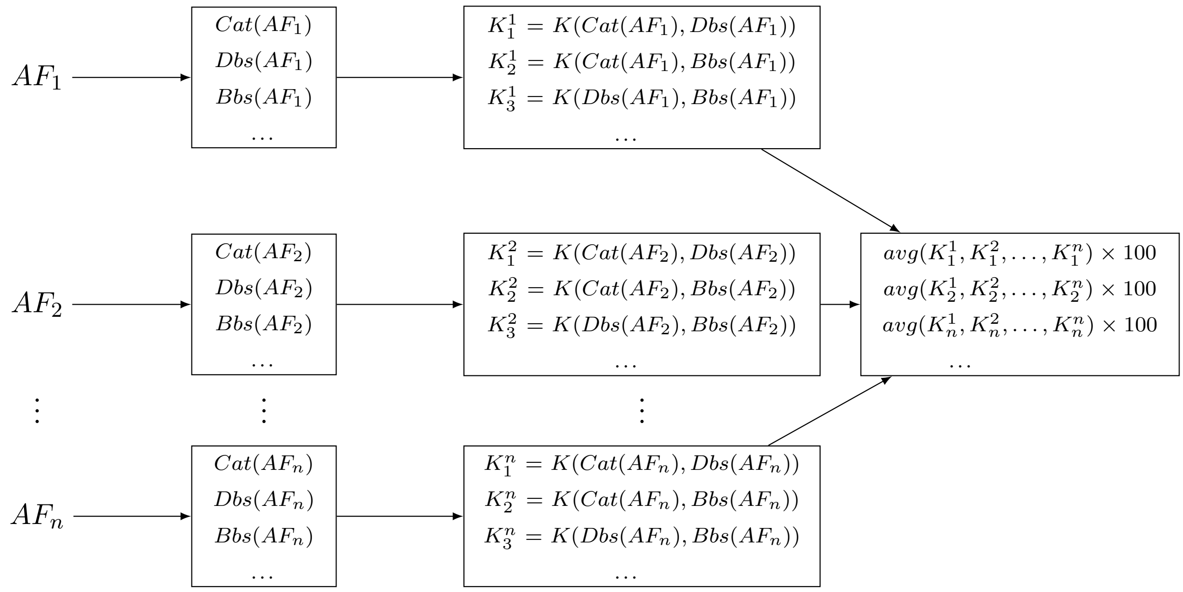


Figure 4.2 – Diagram showing how to compute the percentage of dissimilarity between ranking-based semantics from a set of n argumentation frameworks in input

	Cat	**Bbs**	**Dbs**	0.3-**Bbs**	1-**Bbs**	10-**Bbs**	**FL**	**CS**	**Propa**$_\epsilon^{0.5,S}$	**Propa**$_\epsilon^{0.5,M}$	**Propa**$_{1+\epsilon}^{\epsilon,S}$	**Propa**$_{1+\epsilon}^{\epsilon,M}$	**Propa**$_{1\to\epsilon}^{\epsilon,S}$	**Propa**$_{1\to\epsilon}^{\epsilon,M}$
Cat	0	8.49	8.57	0.72	0	1.38	21.86	5.91	5.50	4.45	1.58	1.23	3.82	8.32
Bbs	8.49	0	0.26	8.17	8.49	9.73	22.38	3.11	3.95	4.30	9.27	9.08	11.68	16.75
Dbs	8.57	0.26	0	8.18	8.57	9.81	22.37	2.92	4.03	4.35	9.35	9.14	11.76	16.82
0.3-**Bbs**	0.72	8.17	8.18	0	0.72	1.98	22.01	5.47	5.53	4.54	2.28	1.95	4.42	8.79
1-**Bbs**	0	8.49	8.57	0.72	0	1.38	21.86	5.91	5.50	4.45	1.58	1.23	3.82	8.32
10-**Bbs**	1.38	9.73	9.81	1.98	1.38	0	21.56	7.15	6.45	5.47	1.13	0.76	3.01	7.15
FL	21.86	22.38	22.37	22.01	21.86	21.56	0	22.16	21.11	21.90	21.40	21.63	21.39	21.66
CS	5.91	3.11	2.92	5.47	5.91	7.15	22.16	0	2.28	2.04	6.96	6.69	9.20	14.04
Propa$_\epsilon^{0.5,S}$	5.50	3.95	4.03	5.53	5.50	6.45	21.11	2.28	0	1.17	5.32	5.80	7.76	13.39
Propa$_\epsilon^{0.5,M}$	4.45	4.30	4.35	4.54	4.45	5.47	21.90	2.04	1.17	0	5.10	4.79	7.52	12.47
Propa$_{1+\epsilon}^{\epsilon,S}$	1.58	9.27	9.35	2.28	1.58	1.13	21.40	6.96	5.32	5.10	0	0.48	2.43	8.06
Propa$_{1+\epsilon}^{\epsilon,M}$	1.23	9.08	9.14	1.95	1.23	0.76	21.63	6.69	5.80	4.79	0.48	0	2.90	7.68
Propa$_{1\to\epsilon}^{\epsilon,S}$	3.82	11.68	11.76	4.42	3.82	3.01	21.39	9.20	7.76	7.52	2.43	2.90	0	5.73
Propa$_{1\to\epsilon}^{\epsilon,M}$	8.32	16.75	16.82	8.79	8.32	7.15	21.66	14.04	13.39	12.47	8.06	7.68	5.73	0

Table 4.2 – Percentage of dissimilarity between the ranking-based semantics obtained from the rankings computed on the 1000 randomly generated argumentation frameworks ($5 \leq |\mathcal{A}| \leq 100$)

All the dissimilarity degrees are given in a symmetric matrix represented by Table 4.2. Thus, the biggest dissimilarity degree between two ranking-based semantics is observed between the burden-based semantics (Bbs) and the fuzzy labellings (FL) with a value of 22.38%. FL clearly stands out from the other semantics with a degree of dissimilarity always greater than 21%.

This is not surprising since FL extends the complete semantics by considering varying degrees of acceptability (rather than the three classical ones: in, outand undec) and thus does not take into account the number of attackers, while all the other semantics do. But, globally, the others ranking-based semantics seems to share a solid common basis with a dissimilarity degree often smaller than 10%.

In order to better represent the "closeness" between these ranking-based semantics, from the previous matrix, we compute a dendrogram, which is a graphical representation of the results of hierarchical cluster analysis. In our case, the method used is a stepwise algorithm for n semantics which merges two semantics or clusters with the least dissimilarities at each step until obtaining a unique cluster. Several operators exist [TAN *et al.* 2006] to compute the distance between the new cluster and the other clusters like the single link (minimum), complete link (maximum), group average, median, etc. However, a few number of inputs make the differences negligible between these methods, so we choose the average method to compute the dendrogram illustrated in Figure 4.3.

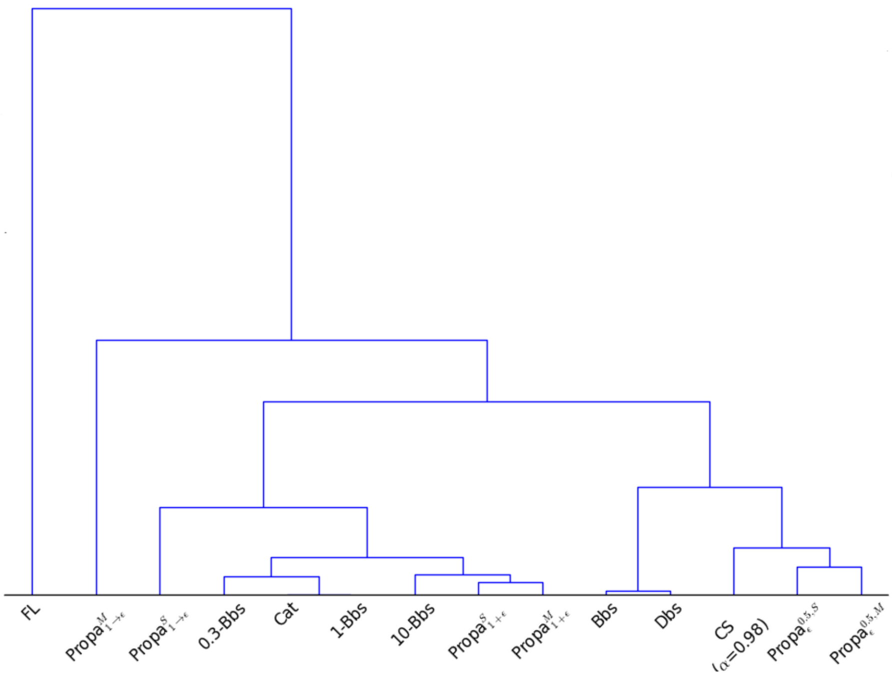


Figure 4.3 – Dendrogram representing the relationships between the ranking-based semantics studied in this thesis

On this dendrogram, the height of the branch between two clusters indicates how different they are from each other: the greater the height, the greater the difference. Two groups emerge from this study: one containing the semantics Dbs, Bbs, CS and $Propa_\epsilon$ (which have a dissimilarity degree always smaller than 4.5% in Table 4.2) and another one containing the semantics

Cat, α-Bbs (for three values of $\alpha \in \{0.3, 1, 10\}$), $Propa_{1+\epsilon}$ (which have a dissimilarity degree always smaller than 2.5% in Table 4.2). The propagation semantics $Propa^S_{1\to\epsilon}$ seems closer that the second group of semantics with a dissimilarity degree between 8% and 9% with all these ranking-based semantics. Among these groups, one can observe that some semantics are very close like Bbs and Dbs.
But, an important observation is that the categoriser-based semantics and the α-Burden-based semantics when $\alpha = 1$ always returned the same ranking (their dissimilarity degree is 0% in Table 4.2).

Proposition 12. ([AMGOUD *et al.* 2016]) Let AF be an argumentation framework,

$$Cat(AF) = 1\text{-}Bbs(AF)$$

4.2 Properties for ranking-based semantics

Our goal is now to explain the similarity and the dissimilarity between the ranking-based semantics, highlighted in the previous section. To this purpose, some properties were introduced in the literature (see Chapter 2, Section 2.5.1 for a recall). But, before checking which ones are satisfied by the different ranking-based semantics, we want to propose some additional properties in this section. These properties are separated into two parts. The first one contains the properties which are a generalization of properties only introduced in the context of a particular semantics. The second part includes some additional properties which capture characteristics we think important to satisfy in a particular context.

4.2.1 Generalized properties

Cayrol and Lagasquie-Schiex [CAYROL & LAGASQUIE-SCHIEX 2005b] introduced properties checking if some change related to the branches in an argumentation framework can improve or degrade the ranking of one argument. Indeed, what is the effect on the acceptability of a given argument with an additional attack branch? Is the effect the same if it is a defense branch? Does the length of the branch matter? Such questions seem interesting to answer in order to better understand the behavior of semantics. However, these properties have been proposed informally, in the context of the tuples-based semantics. It is why we propose a formal definition of these properties, that generalizes them for any argumentation frameworks.

First of all, let us introduce how we formally define the addition of an attack branch and the addition of a defense branch to an argument.

Definition 4.2.1 (Attack and defense branch added to an argument)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x \in \mathcal{A}$ be an argument. A defense branch added to x is $P^+(x) = \langle \mathcal{A}', \mathcal{R}' \rangle$, with $\mathcal{A}' = \{x_0, \ldots, x_n\}$ such that $n \in 2\mathbb{N}$, $x_0 = x$, $\mathcal{A} \cap \mathcal{A}' = \{x\}$, and $\mathcal{R}' = \{(x_i, x_{i-1}) \mid i \leq n\}$. The attack branch added to x, denoted $P^-(x)$ is defined similarly except that the sequence is of odd length (i.e. $n \in 2\mathbb{N} + 1$).

In order to evaluate the impact of an additional attack (or defense) branch on a given argument x of an argumentation framework AF, we "clone" this argumentation framework with an isomorphism γ (see Definition 2.5.1 page 63). Then, we can modify the argumentation framework $\gamma(AF)$ and analyse the impact on $\gamma(x)$ compared to x.

Addition of a branch

The first property concerns the attack branches and states that adding an attack branch to any argument decreases its level of acceptability.

Property 11 (Addition of an Attack Branch (+AB))**.**
A ranking-based semantics σ satisfies Addition of an Attack Branch if and only if for any $AF, AF' \in \mathbb{AF}$ and $x \in Arg(AF)$, for every isomorphism γ such that $AF' = \gamma(AF)$, if $AF^\star = AF \cup AF' \cup P^-(\gamma(x))$, then $x \succ^\sigma_{\text{AF}^\star} \gamma(x)$.

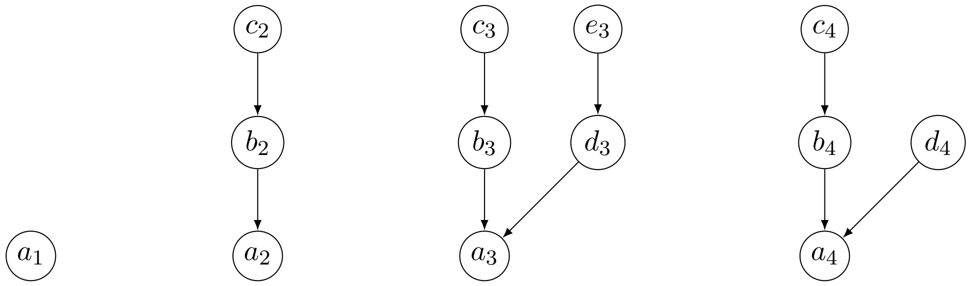


Figure 4.4 – Argumentation framework with different configurations of branches

Example 4.2.1. Let us consider the argumentation framework illustrated in Figure 4.4. If σ satisfies +AB then a_1, which has no attack branch, should be more acceptable than b_2, b_3, d_3 and b_4 which have one attack branch. In addition, a_2 should be more acceptable than a_4 because both have one defense branch with the same length but a_4 has also an attack branch while a_2 has not.

The two following properties concerns the defense branches. The first one states that adding a defense branch to any argument increases its level of acceptability.

Property 12 (Strict Addition of a Defense Branch ($\oplus$DB))**.**
A ranking-based semantics σ satisfies Strict Addition of a Defense Branch if and only if for any $AF, AF' \in \mathbb{AF}$ and $x \in Arg(AF)$, for every isomorphism γ such that $AF' = \gamma(AF)$, if $AF^\star = AF \cup AF' \cup P^+(\gamma(x))$, then $\gamma(x) \succ^\sigma_{\text{AF}^\star} x$.

Example 4.2.2 (cont.)**.** If σ satisfies $\oplus$DB then a_3 should be more acceptable than a_2 which should be more acceptable than a_1. Indeed, a_3 has one more defense branch than a_2 which has one more defense branch than a_1. In addition, a_4 with one defense branch and one attack branch should be more acceptable than b_2, b_3, d_3 and b_4 which have no defense branch.

However, it could make sense to treat differently non-attacked arguments. It is why, in [CAYROL & LAGASQUIE-SCHIEX 2005b], this property is defined in a more specific way: adding a defense branch to any attacked argument increases its level of acceptability.

Property 13 (Addition of a Defense Branch (+DB))**.**
A ranking-based semantics σ satisfies Addition of a Defense Branch if and only if for any $AF, AF' \in \mathbb{AF}$ and $x \in Arg(AF)$, for every isomorphism γ such that $AF' = \gamma(AF)$, if $AF^\star = AF \cup AF' \cup P^+(\gamma(x))$ and $\mathcal{R}_1(x) \neq \emptyset$, then $\gamma(x) \succ^\sigma_{\text{AF}^\star} x$.

Example 4.2.3 (cont.)**.** If σ satisfies +DB, the same conclusion as $\oplus$DB can be done except for a_2 and a_1. Indeed, a_1 is not attacked so nothing can be said about its ranking in comparison with the other arguments.

Increasing the length of a branch

Let us now define the properties based on the increase of the length of a branch. Formally, increasing the length of a branch consists in adding a defense branch [14] to the argument at the beginning of the branch.

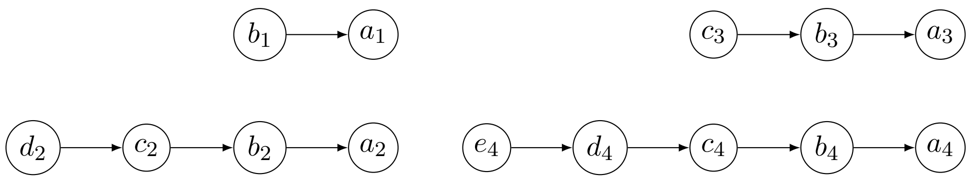


Figure 4.5 – Argumentation framework with different lengths of branch

The first property states that increasing the length of an attack branch of an argument increases its level of acceptability.

Property 14 (Increase of an Attack Branch ($\uparrow$AB))**.**
A ranking-based semantics σ satisfies Increase of an Attack Branch if and only if for any $AF, AF' \in \mathbb{AF}$ and $x \in Arg(AF)$, for every isomorphism γ such that $AF' = \gamma(AF)$, if $\exists y \in \mathcal{B}_-(x), y \notin \mathcal{B}_+(x)$ and $AF^\star = AF \cup AF' \cup P^+(\gamma(y))$, then $\gamma(x) \succ^\sigma_{\text{AF}^\star} x$.

Example 4.2.4. Let us consider the argumentation framework illustrated in Figure 4.5. If σ satisfies $\uparrow$AB then a_2 should be more acceptable than a_1 because a_2 has an attack branch of length 3 while a_1 has an attack branch of length 1.

The second property concerns the defense branch and states that increasing the length of a defense branch of an argument decreases its level of acceptability.

Property 15 (Increase of a Defense Branch ($\uparrow$DB))**.**
A ranking-based semantics σ satisfies Increase of a Defense Branch if and only if for any $AF, AF' \in \mathbb{AF}$ and $x \in Arg(AF)$, for every isomorphism γ such that $AF' = \gamma(AF)$, if $\exists y \in \mathcal{B}_+(x), y \notin \mathcal{B}_-(x)$ and $AF^\star = AF \cup AF' \cup P^+(\gamma(y))$, then $x \succ^\sigma_{\text{AF}^\star} \gamma(x)$.

Example 4.2.5 (cont.)**.** If σ satisfies $\uparrow$DB then a_3 should be more acceptable than a_4 because a_3 has a defense branch of length 2 while a_4 has a defense branch of length 4.

14. We add here a defense branch in order to leave the "role" of the branch unchanged: a defense (respectively attack) branch stays a defense (respectively attack) branch.

Please note that in [CAYROL & LAGASQUIE-SCHIEX 2005b], the dual properties also exist: removing a branch of an argument or decreasing the length of a branch of an argument. However, we intentionally defined the properties in order to catch these two aspects. Indeed, for example, a semantics which satisfies the property +AB means that adding an attack branch to an argument decreases its acceptability. But if we remove this branch then intuitively the acceptability should revert to its original level (see Abstraction), which means that removing an attack branch to an argument increases its acceptability.

4.2.2 Additional properties

In addition to the properties proposed until here, we want to add some other interesting properties.

The first one, called *Total*, allows to make a distinction between the semantics which return a total preorder or a partial preorder between arguments. Indeed, too many incompatibilities can be problematic, especially if we want to use argumentation for decision-making or for the online debate platforms (see the discussion in [LEITE & MARTINS 2011]), the users could be requested to give arguments for or against two opposite topics in order to compare them and know which one is the most popular. Thus, it could be frustrating for the users to obtain an incomparability between both arguments after spending some time to deliberate. In this case, one will prefer to select a semantics that satisfies Total.

Property 16 (Total (Tot))**.**
A ranking-based semantics σ satisfies Total if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, $x \succeq^{\sigma}_{\text{AF}} y$ or $y \succeq^{\sigma}_{\text{AF}} x$.

Argument Equivalence ensures that the acceptability of an argument depends only on (the structure of) its attackers and defenders. This property is related to a well-known property satisfied by the classical semantics, called *Directionality* [BARONI *et al.* 2011], which states that an argument can only be affected by arguments following the direction of the attacks (*i.e.* an argument a cannot be affected by another argument b if there exists no path from b to a). Formally, if there exists an isomorphism between the ancestors' graph of two arguments, then they are equally acceptable.

Property 17 (Argument Equivalence (AE))**.**
A ranking-based semantics σ satisfies Argument Equivalence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, for every isomorphism γ such that $Anc_{\text{AF}}(x) = \gamma(Anc_{\text{AF}}(y))$ then $x \simeq^{\sigma}_{\text{AF}} y$.

Please note that the reverse is not true because two arguments can be equally acceptable but with different ancestors' graphs.

The property *Non-attacked Equivalence* is a particular case of Argument Equivalence because it focuses on the comparison between the non-attacked arguments. Indeed, if the arguments are affected only by the arguments in their ancestors' graph, then the non-attacked arguments should be unaffected by the remaining part of the argumentation framework (because they have no attacker and defender). Thus, they should have the same ranking. If one agrees with this idea then Non-attacked Equivalence must be satisfied.

Property 18 (Non-attacked Equivalence (NaE))**.**
A ranking-based semantics σ satisfies Non-Attacked Equivalence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, $\mathcal{R}_1(x) = \emptyset$ and $\mathcal{R}_1(y) = \emptyset$ then $x \simeq^{\sigma}_{\text{AF}} y$.

Another possibility to detect when two arguments are equally acceptable consists in just taking into account their direct attackers. Suppose that two arguments, x and y, have the same number of direct attackers. If, for each direct attacker of x, there exists a direct attacker of y such that the two attackers are equally acceptable, then x and y are equally acceptable too.

Property 19 (Ordinal Equivalence (OE))**.**
A ranking-based semantics σ satisfies Ordinal Equivalence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if there exists a bijective function f from $\mathcal{R}_1(x)$ to $\mathcal{R}_1(y)$ such that $\forall z \in \mathcal{R}_1(x), z \simeq^{\sigma}_{\text{AF}} f(z)$ then $x \simeq^{\sigma}_{\text{AF}} y$.

The last property describes the behavior adopted by a semantics concerning the notion of defense, which plays a key role in the obtained rankings. Indeed, back to AF_c, it is clear that there is a consensus between the ranking-based semantics (see Table 4.1) to say that c, which is defended once, is always more acceptable than b which is directly attacked by a non-attacked argument. But if we compare b and f, which have two distinct defense branches, we can remark, that for some semantics, b is either more acceptable (e.g. Discussion-based semantics), equally acceptable (e.g. Categoriser-based ranking semantics) or less acceptable (e.g. Tuples or $Propa_{1\to\epsilon}$) than f. However, there is no guarantee that the ranking will change again if we increase the number of defense branches. Moreover, existing properties which are related to the defense (e.g. Defense Precedence (DP) or Addition of Defense Branch (+DB)) are not able to catch this difference. It is why we introduce the property *Attack vs Full Defense* which allows to make this distinction between the semantics which consider a defense as a weak attack and the semantics which consider a defense as a reinforcement for the targeted argument.

Property 20 (Attack vs Full Defense (AvsFD))**.**
A ranking-based semantics σ satisfies Attack vs Full Defense if and only if for any acyclic $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $|\mathcal{B}_-(x)| = 0$, $|\mathcal{R}_1(y)| = 1$ and $|\mathcal{R}_2(y)| = 0$ then $x \succ^{\sigma}_{\text{AF}} y$.

For example, as illustrated in Figure 4.6, the property states that an argument which is (only) attacked once by a non-attacked argument (it is the case of b only attacked by b_1) is worse than an argument that have any number of attacks that all belong to defense branches (it is the case of a which have four defense branches and no attack branch).

4.2.3 Relationships between properties

Each property studied in this thesis aims to capture a particular principle. However, some of them can focus on the same aspect of the argumentation framework (*e.g.* direct attackers, the number of defenders). Thus, one can wonder whether some overlaps exist between them.

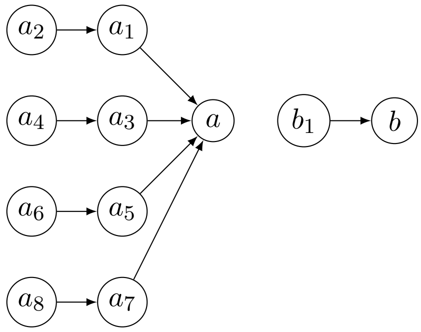


Figure 4.6 – Illustration of the property Attack vs Full Defense

Conversely, like it is the case with the properties Cardinality Precedence and Quality Precedence (see Figure 2.21 page 65), one can also wonder whether some additional incompatibilities exist. To this purpose, we continue the work initiated in [AMGOUD & BEN-NAIM 2013, BESNARD *et al.* 2017] about the incompatibilities and the dependencies between properties. All the results obtained in this section are summed up in Figure 4.7 (page 103).

Let us first recall when two properties are incompatible (*i.e.* they cannot be simultaneously satisfy).

Definition 4.2.2 (Incompatibility)**.**
Two properties are incompatible if there exists an argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $x, y \in \mathcal{A}$ such that when one property states that $x \succ_{\text{AF}} y$, the other property states that $y \succeq_{\text{AF}} x$.

The next proposition recalls some results, and proves new ones, about the incompatibility of some properties.

Proposition 13. For every ranking-based semantics, the following pairs of properties are incompatible :

(1) Cardinality Precedence (CP) and Quality Precedence (QP) [AMGOUD & BEN-NAIM 2013]

(2) Self-Contradiction (SC) and Cardinality Precedence (CP) [BESNARD *et al.* 2017]

(3) Self-Contradiction (SC) and Counter-Transitivity (CT) [BESNARD *et al.* 2017]

(4) Self-Contradiction (SC) and Strict Counter-Transitivity (SCT) [BESNARD *et al.* 2017]

(5) Cardinality Precedence (CP) and Attack vs Full Defense (AvsFD)

(6) Cardinality Precedence (CP) and Addition of a Defense Branch (+DB)

(7) Cardinality Precedence (CP) and Strict Addition of a Defense Branch ($\oplus$DB)

(8) Void Precedence (VP) and Strict Addition of a Defense Branch ($\oplus$DB)

(9) Strict Counter-Transitivity (SCT) and Strict Addition of a Defense Branch ($\oplus$DB)

(10) Argument Equivalence (AE) and Self-Contradiction (SC)

Proposition 14. No ranking-based semantics can simultaneously satisfy Addition of a Defense Branch (+DB), Strict Counter-Transitivity (SCT) and Argumentation Equivalence (AE).

Some of these results are not surprising. Indeed, some properties have different views on the notion of defense (see the discussion in the previous section when we introduced the property Attack vs Full Defense). It is the case, for example, with the properties CP and SCT which consider that any additional (defense or attack) branch should have a negative effect on a given argument while +DB or $\oplus$DB state that an additional defense branch should have a positive impact on this argument.

Then, let us first define when a property implies another property.

Definition 4.2.3 (Implication)**.**
A property P implies another property Q if and only if for any ranking-based semantics σ, if σ satisfies P then σ satisfies Q.

The next proposition recalls some results, and proves new ones, about the implication between properties.

Proposition 15.

(1) Strict Counter-Transitivity (SCT) implies Void Precedence (VP) [AMGOUD & BEN-NAIM 2013]

(2) Counter-Transitivity (CT) and Strict Counter-Transitivity (SCT) imply Defense Precedence (DP) [AMGOUD & BEN-NAIM 2013]

(3) Counter Transitivity (CT) implies Non-attacked Equivalence (NaE)

(4) Counter Transitivity (CT) implies Ordinal Equivalence (OE)

(5) Strict Counter-Transitivity (SCT) and Ordinal Equivalence (OE) imply Counter-Transitivity (CT)

(6) Strict Addition of Defense Branch ($\oplus$DB) implies Addition of a Defense Branch (+DB)

(7) Argument Equivalence (AE) implies Non-attacked Equivalence (NaE)

(8) Ordinal Equivalence (OE) implies Non-attacked Equivalence (NaE)

(9) Void Precedence (VP) and Quality Precedence (QP) imply Attack vs Full Defense (AvsFD)

(10) Cardinality Precedence (CP) implies Addition of an Attack Branch (+AB)

Interestingly, even if each property aims to catch a particular behavior, some of them remain connected. For example, if the properties SCT and OE are both satisfied, then one can directly considered VP, DP, CT and NaE satisfied too.

4.3 Properties $\times$ Ranking-based semantics

We are now able to check which properties are satisfied by the ranking-based semantics studied in this thesis. Recall that, among these ranking-based semantics, some of them (*e.g.* α-burden-based semantics, the propagation semantics) are configurable with one or several parameters. Thus, two values of a parameter could give different rankings. It is why we consider that a property is satisfied by these ranking-based semantics only if the property is satisfied for all the values of a parameter.

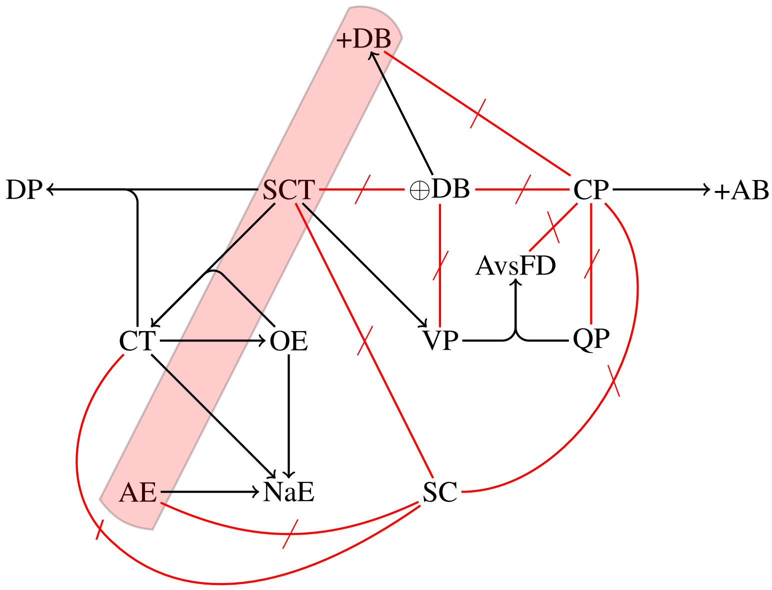


Figure 4.7 – Graph which represents the relation between properties ($X \rightarrow Y$ means that X implies Y, $X \not\!\!-\!\!\!- Y$ means that X and Y are not compatible and the properties into the red rectangle cannot be simultaneously satisfied.)

Proposition 16. The properties that are satisfied by each ranking-based semantics (the other properties [15] are not satisfied by the corresponding ranking-based semantics):

- The categoriser-based ranking semantics (Cat) satisfies Abs, In, VP, DP, CT, SCT, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.
- The discussion-based semantics (Dbs) satisfies Abs, In, VP, DP, CT, SCT, CP, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.
- The burden-based semantics (Bbs) satisfies Abs, In, VP, DP, CT, SCT, CP, DDP, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.
- Let $\alpha \in]0, +\infty[$. The α-burden-based semantics (α-Bbs) satisfies Abs, In, VP, DP, CT, SCT, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.
- The fuzzy labeling (FL) satisfies Abs, In, CT, QP, Tot, NaE, OE and AvsFD.
- Let $\alpha \in]0, 1[$. The counting semantics (CS) satisfies Abs, VP, DP, CT, SCT, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.
- Tuples-based semantics (Tuples) satisfies Abs, In, VP, +DB, ↑AB, ↑DB, +AB, NaE, AE, OE and AvsFD.
- The ranking-based semantics M&T satisfies Abs, In, VP, +AB, SC, Tot, NaE and AvsFD.
- The iterated graded defense semantics (IGD) satisfies Abs, In, VP, +AB, and NaE.

15. except for the property AE with the semantics FL, M&T and IGD and the property OE with the semantics IGD

For a better reading, we do not include the proofs directly in the main text but we strongly recommend to the reader to look at the Appendix B (page 141) where the proofs and the counter-examples are detailed because this is really illuminating on the behavior of the different semantics.

Let us now check which properties are satisfied by our propagation semantics.

Proposition 17. Let $\oplus \in \{M, S\}$ and $\epsilon \in]0, 1]$.

- The ranking-based semantics $Propa_{\epsilon}^{\epsilon,\oplus}$ satisfies Abs, In, VP, DP, ↑AB, ↑DB, +AB, NaE, Tot and AE. The other properties are not satisfied.
- The ranking-based semantics $Propa_{1+\epsilon}^{\epsilon,\oplus}$ satisfies Abs, In, VP, DP, DDP, ↑AB, ↑DB, +AB, Tot, NaE, AE and AvsFD. The other properties are not satisfied.
- The ranking-based semantics $Propa_{1\rightarrow\epsilon}^{\epsilon,\oplus}$ satisfies Abs, In, VP, DP, DDP, +DB, ↑AB, ↑DB, +AB, Tot, NaE, AE and AvsFD. The other properties are not satisfied.

Among the set of properties, some of them allow to make a distinction between the rankings returned by the propagation semantics when the multiset or the set is used to select the attackers or defenders.

Proposition 18. Let $\epsilon \in]0, 1]$.

- The ranking-based semantics $Propa_{\epsilon}^{\epsilon,M}$ satisfies CT, SCT and OE.
- The ranking-based semantics $Propa_{1+\epsilon}^{\epsilon,M}$ satisfies CT, SCT and OE.
- The ranking-based semantics $Propa_{1\rightarrow\epsilon}^{\epsilon,M}$ satisfies OE.

We also checked what are the properties satisfied by the usual Dung's grounded semantics which is the only semantics to return an unique extension for any argumentation framework. The idea is to give some hints on the compatibility of these properties with classical semantics. Note that, in this case, this is a degenerate ranking semantics with only two levels (accepted/rejected):

Proposition 19. The grounded semantics (Gr) satisfies Abs, In, Tot, NaE, AE and AvsFD. The other properties are not satisfied.

We summarize all the results obtained in this section in Table 4.3 where a cross $\times$ means that the property is not satisfied, symbol ✓ means that the property is satisfied, symbol $✓_M$ is specific to the propagation semantics and means that the property is only satisfied when $\oplus = M$, symbol ? means that we do not know if the property is satisfied or not, and the shaded cells highlight the results already proven in the literature.

4.4 Discussion

Several observations can be made regarding the properties and the results reported in Table 4.3:

Properties	Cat	Dbs	Bbs	α-Bbs	FL	CS	$Propa_{\epsilon}$	$Propa_{1+\epsilon}$	$Propa_{1\rightarrow\epsilon}$	Tuples	M&T	IGD	Gr
Abs	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
In	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
VP	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$
DP	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\times$	$\times$
CT	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark_M$	$\checkmark_M$	$\times$	$\times$	$\times$	$\times$	$\times$
SCT	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\checkmark_M$	$\checkmark_M$	$\times$	$\times$	$\times$	$\times$	$\times$
CP	$\times$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
QP	$\times$	$\times$	$\times$	$\times$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
DDP	$\times$	$\times$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\times$	$\times$
SC	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\checkmark$	$\times$	$\times$
$\oplus$DB	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
+DB	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\times$
$\uparrow$AB	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\times$
$\uparrow$DB	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\times$
+AB	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\times$
Tot	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\times$	$\checkmark$
NaE	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
AE	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	?	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	?	$\checkmark$
OE	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark_M$	$\checkmark_M$	$\checkmark_M$	$\checkmark$	$\times$	?	$\times$
AvsFD	$\times$	$\times$	$\times$	$\times$	$\checkmark$	$\times$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$

Table 4.3 – Properties satisfied by the ranking-based semantics studied in this thesis.

• *Some properties seem to be widely shared by almost all semantics.* It is the case of the properties Abs, In, VP, +AB, Tot, NaE and AE. We recall that the input is a Dung's abstract argumentation framework where there is no information about the nature of arguments (abstract arguments), so only the attacks have to be taken into account, hence the importance of the property Abstraction (Abs). Concerning the property Independence (In), it seems difficult to justify the fact that an argument can influence other arguments without being linked, even indirectly, to them. The only semantics which does not satisfy this property is the counting semantics which needs the maximal indegree of the argument graph to guarantee the convergence. One of the remarks, done from the rankings in Table 4.1 (pages 90), was that the non-attacked arguments are often more acceptable than the attacked arguments. This observations is confirmed because all the semantics (except FL) satisfy the property Void Precedence (VP).

Non-attacked Equivalence (NaE) and Argument Equivalence (AE) are satisfied by all semantics. This is a kind of compatibility principle with usual Dung's semantics (the grounded semantics satisfies them too), where only your attackers should impact your ranking, not the arguments you attack.

It is also interesting to note that almost all the semantics satisfy the property Total (Tot) which can be a possible request (but not necessary) in applications like online debate platforms where distinguishing all the arguments is necessary.

A last property satisfied by almost all semantics is +AB, which states that adding an attack branch towards an argument degrades its level of acceptability. This also seems to be a perfectly natural requirement for ranking-based semantics: the more you have attackers, the less acceptable you are. This property is one of the main reasons to explain the high degree of dissimilarity, observed in section 4.1, between the semantics FL, which does not satisfy +AB, and all the other semantics.

• *Some properties are compatible with the grounded semantics.* One can note that Abs, In, AE, NaE, Tot and AvsFD are satisfied by the grounded semantics. However, among the properties widely accepted by the ranking-based semantics, VP and +AB are not satisfied by the grounded semantics. An explanation is related to the fact that the extension-based semantics (and in particular the grounded semantics) consider that the impact of an attack from an argument to another one is drastic. In other words, the grounded semantics falsifies VP and +AB because an attack can "kill" another argument (see the Killing principle in Section 2.1 page 34) while the ranking-based semantics suppose that an attack does not "kill" but just weaken the attacked argument. Thus, VP and +AB implicitly suppose that the ranking-based semantics satisfied this principle (called Resilience in [AMGOUD & BEN-NAIM 2016]). However, these two principles are not totally incompatible with grounded semantics to some extent. Indeed, as suggested in [THIMM & KERN-ISBERNER 2014], a weak version of Void Precedence, which states that non-attacked arguments should at least as acceptable as (and not strictly more acceptable) attacked arguments, can also be defined.

Property 21 (Weak Void Precedence (WVP))**.** [THIMM & KERN-ISBERNER 2014]
A ranking-based semantics σ satisfies Weak Void Precedence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $\mathcal{R}_1(x) = \emptyset$ and $\mathcal{R}_1(y) \neq \emptyset$ then $x \succeq^{\sigma}_{\text{AF}} y$.

Clearly, Void Precedence implies Weak Void Precedence so all the semantics which satisfy VP also satisfy WVP. But it is interesting to note that the grounded semantics satisfies WVP because the non-attacked arguments are always accepted but can be equal to some other attacked arguments. Following the same reasoning, it might be interesting to define the weak version of some other existing properties (*e.g.* DP, +AB).

• *Some properties are very discriminatory and provide a classification of semantics.* If some incompatibilities between properties exist (see Proposition 13 page 101), some other properties allow to separate the ranking-based semantics into sub-classes. It is the case with the semantics which satisfy the properties (S)CT and the semantics which satisfy the properties AvsFD or +DB. Indeed, one can remark that a semantics always belongs to at least one of these two groups (except for $Propa_{1+\epsilon}$ where all these properties are accepted). Different visions (without being incompatible) concerning the defense are considered by these properties. The semantics which satisfy AvsFD take care of the whole defense branches. Whereas for the semantics which satisfy (S)CT, a defense branch (that still ends by an attack towards the argument) always penalizes it.

• *More specific properties.* As already mentioned, the properties operate at different levels. There are "local" properties (*e.g.* CP, QP, DP, DDP, (S)CT) focusing on the direct attackers or direct defenders, which can be justified in some situations, but seem hardly general (and sometimes impossible to reconcile with some global properties, as Proposition 13 page 101 shows). Properties related to "change" (*e.g.* $\oplus$DB, +DB, $\uparrow$AB, $\uparrow$DB, +AB) seem very appealing because they specify how the ranking should be affected on the basis of the comparison of attack and defense branches. They allow, for example, to categorize the semantics according to the behavior towards some basic requirements. These properties are also interesting because they allow to make a distinction between some semantics which satisfied the same "local" properties. It is the case with the tuples-based semantics (Tuples), which satisfies +DB, ↑AB, ↑DB, +AB, and the semantics M&T, which satisfies none of them.

• *Defining axiomatically the least acceptable arguments is not obvious.* Interestingly, while all semantics agree axiomatically on which arguments should be the most acceptable in an argumentation framework (see the Void Precedence property), there is no consensus regarding the least acceptable arguments. The Self Contradiction property (SC) is very interesting in that respect. It makes the observation that a self-contradicting argument is intrinsically flawed, without even requiring other arguments to defeat it. But, as can be observed, none of the semantics comply with it, except the one from [MATT & TONI 2008] (M&T) who introduced the property. The explanation is that all ranking-based semantics consider that an argument that attacks itself is just a single path, like the other ones. So an argument which is attacked by itself (and by no other argument) is more acceptable than an argument which is directly attacked several times.
On the other hand, another possibility is when the properties +AB and ↑AB are satisfied together. Indeed, one can consider the least acceptable argument as the one which is directly attacked by a maximum number (+AB) of non-attacked arguments (↑AB).

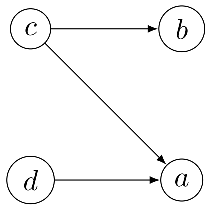
• *The interplay of properties is often instructive.* In section 4.2.3, we have identified some implications and incompatibilities between properties. Let us focus, for example, on the relation of incompatibility between VP and ⊕DB. One can easily remark that ⊕DB is more general than +DB, and in a sense more natural: the property is stated for *any* cases, it does not treat some arguments (the non-attacked arguments here) differently. But it contradicts VP in this case. +DB is a less "systematic" property (it was the original one proposed in [CAYROL & LAGASQUIE-SCHIEX 2005b]) but is compatible with VP : if one accepts that non-attacked arguments should be the best (VP), then adding a defense branch cannot *always* improve the situation of a given argument.

• *This set of properties is yet to be augmented ...* This can be observed with the semantics Categoriser, α-Burden-based semantics and $Propa_{\epsilon}^{\epsilon,M}$ which satisfy the same set of properties, whereas they have quite different definitions and behaviors, as it is revealed in Section 4.1 (except for Categoriser and α-Burden-based semantics when $\alpha = 1$, which return similar rankings). This means that some property is missing to discriminate these operators. In this direction, [AMGOUD & BEN-NAIM 2016] have recently introduced a set of properties for scoring semantics, aiming, in their words, to "set up the foundations of acceptability semantics". Some semantics like Abstraction (Abs) or Independence (In) are kept but other properties (*e.g.* Strict Counter-Transitivity (SCT), Void Precedence (VP)) are deconstructed in several other "primitive" properties. However, only Dung's classical semantics and the categoriser-based ranking semantics are studied.
... but some differences between semantics can be hardly caught with properties. Indeed, in some cases, the differences between two semantics concern just a very specific part of an argumentation framework or a subset of the argumentation frameworks. For example, the semantics M&T does not satisfy the property Addition of an Attack Branch (+AB) because an argument that attacks itself has the minimal score and adding an attack branch cannot decrease its score.

But, if the self-attacks are not allowed, then the property would be satisfied. Another example concerns the ranking-based semantics using a parameter to capture different notions. It is the case for example with α-Burden-based semantics, where α allows to choose the degree of importance between the quality and the quantity of the direct attackers, or with $Propa_\epsilon$, where ϵ controls the impact of the non-attacked arguments on the others arguments. We know that two values of ϵ (or α) can lead to different rankings (see Table 4.1 page 90). Recall that in order to consider a property satisfied by these particular semantics, the property must be satisfied for all the values of the parameter. However, it seems difficult to capture these differences specific to rankings with classical properties (either accepted or rejected). A possible solution could be to define parametrized properties in order to capture these specific aspects.

4.5 Conclusion

In this chapter, we provided a general comparison of all ranking-based semantics studied in this thesis with respect to all the properties of the literature. This study allows to understand the similarities and the differences of behavior between the existing ranking-based semantics revealed by our empirical comparison and which cannot be directly interpreted from the mathematical definition of each ranking-based semantics. However, following the previous discussion, there is still work needed on this topic.
Concerning the properties, an ambitious goal would be to fully characterize classes of semantics with respect to a subset of properties. Thus, a possible user would have the opportunity to select a set of consistent properties (*i.e.* without incompatibility between properties) that she thinks important to respect for a ranking-based semantics, and then checking which semantics is the more appropriate according to her choice. In that sense, a preliminary work [BESNARD *et al.* 2017, DAVID 2017] adapts and selects some existing properties in order to build a "custom-made" ranking-based semantics which respects the principles caught by the selected properties. For example, if one considers that the properties Void Precedence (VP), Non-attacked Equivalence (NaE) and Cardinality Precedence (CP) should be satisfied, then the ranking computed from the argumentation framework depicted in Figure 4.8, in using the ranking-based semantics associated, should be: $c \simeq d \succ b \succ a$.



NaE : $c \simeq d$

VP : $c \succ b,\ c \succ a,\ d \succ b,\ d \succ a$

CP : $c \succ b \succ a,\ d \succ b \succ a$

$\boxed{c \simeq d \succ b \succ a}$

Figure 4.8 – How properties selected by the user constrain the resulting ranking

We could also imagine another process where the user does not need to know the properties. To do this, it could be interesting to ask her to directly rank/order the arguments in a given argumentation framework from the most to the least acceptable ones. From the results, the goal would be to infer which properties she wants to satisfy and thus propose her an appropriate ranking-based semantics.

Concerning the ranking-based semantics, this comparative study also allows to know which kind of semantics (and especially their behavior) exists or not. Our analysis is applied to existing semantics which provide a good basis for comparison, and thus, with the rising number of ranking semantics, any new ranking-based semantics could be inspected through the same lens.

Finally, in Section 4.1, we explained that we developed a software which allows to compute the ranking of existing ranking-based semantics (Cat, Dbs, Bbs, α-Bbs, FL, CS and the propagation semantics). But, in order to avoid the accumulation of softwares, we plan to directly include our software in an existing software (*e.g.* CoQuiAAS [LAGNIEZ *et al.* 2015]).

Chapter 5

Ranking-based Semantics for Persuasion

In the previous chapter, many ranking-based semantics were compared on the basis of properties. However, the relevance of some properties may be very much dependent on the context of application. What is often missing to compare these approaches is thus a clear indication of the applications they target.

In this chapter, we question the ability of the existing ranking-based semantics for argumentation to capture persuasion settings, emphasizing in particular the phenomena of protocatalepsis (the fact that it is often efficient to anticipate the counter-arguments of the audience), and of fading (the fact that long lines of argumentation become ineffective). It turns out that some widely accepted principles of ranking-based semantics are incompatible with a faithful treatment of these phenomena. We thus propose a parametrized ranking-based semantics based on the propagation of values, which allows to control the scope of arguments to be considered for evaluation. We investigate its properties (identifying in particular threshold values guaranteeing that some properties hold), and report experimental results showing that the family of rankings that may be returned have a high coherence rate.

This chapter develops the results from [BONZON *et al.* 2017a, BONZON *et al.* 2017b]

Contents

5.1 Persuasion principles

In the previous chapter, we explained that checking which properties are satisfied or not by a given ranking-based semantics allows to better understand its behavior. But, this also makes it possible to decide what semantics is more suited for a given application. Indeed, in a specific context, it seems necessary to satisfy or not some subset of properties in order to obtain an appropriate ranking. For example, for online debate platforms, satisfying the property Total (Tot) may seem natural to ensure the comparison between all the arguments and thus guarantee a result to the users. Conversely, for the same platforms where votes are assigned to each argument and represent their social support, a possibility would be to reward a more aggressive non-attacked argument. Thus, such property like Non-attacked Equivalence (NaE), considering that all the non-attacked arguments should be equally acceptable, should not be satisfied.

In this chapter, we want to focus on a specific context in argumentation: persuasion. Persuasion is an activity that involves one party (the persuader) trying to induce another party (the persuadee) to believe or do something.

We shall concentrate on two well documented phenomena in persuasion and draw a parallel between each of them and existing properties for ranking-based semantics.

Procatalepsis

Procatalepsis, or prolepsis, is a figure of speech in which the speaker raises an objection to their own argument and then immediately answers it. The goal is to strengthen this argument by dealing with possible counter-arguments before their audience can raise them [WALTON 2007]. To illustrate this, we extend an example from [BESNARD & HUNTER 2008, p.85]: a (made-up) sales pitch intended to persuade to buy a specific car. The representation of this example as argumentation framework is depicted in Figure 5.1.

($a1$) The car *x* is a high performance family car with a diesel engine and a price of 32000

($a2$) In general, diesel engines have inferior performance compared with gazoline engines

($a3$) But, with these new engines, the difference in performance [...] is negligible

($a4$) In addition, even if the price of the car seems high

($a5$) It will be amortized because the Diesel engines run longer before breaking than any other kind of engines.

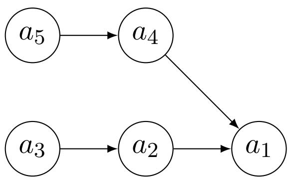


Figure 5.1 – Argumentation framework illustrating a (made-up) sales pitch using the procatalepsis principle

In this kind of persuasion contexts, it is clearly more convincing to state the more plausible counter-argument to (a_1) in order to provide some convincing defenses against them, than simply stating (a_1) alone. These anticipations allow to persuade the interlocutor that any attack against (a_1) is vain. In addition, it becomes difficult to the persuadee to find arguments against (a_1) if the persuader anticipate most of them. Thus, in term of ranking, (a_1) with several defense branches could be seen as strictly more acceptable than (a_1) without any branch.
What is striking is that procatalepsis blatantly contradicts the property Void Precedence (VP), considering that a non-attacked argument is strictly more acceptable than an attacked argument. Recall that, as remarked in the discussion part of the previous chapter, this property is satisfied by almost all the existing ranking-based semantics (or by all the ranking-based semantics if we include its weak version WVP defined in Property 21 (page 201) which is satisfied by FL and the grounded semantics). Thus, no ranking-based semantics has yet been proposed where (Weak) Void Precedence is not satisfied. So there exists no ranking-based semantics which can capture the procatalepsis principle.

Fading

The fading principle states that long lines of argumentation become ineffective in practice, because the audience easily looses track of the relation between the arguments.

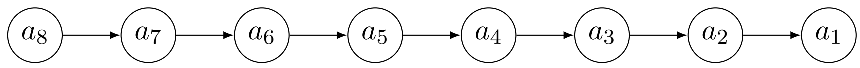


Figure 5.2 – Argumentation framework with a long line of arguments

In focusing on the argumentation framework depicted in Figure 5.2, the fading principle concerns the limit until which the length of path between an argument and another one is too long to have an impact on the targeted argument. For example, if one considers that the arguments situated at the beginning of the paths with a length greater or equal to 5 have no impact on a given argument, then arguments a_6, a_7 and a_8 have no impact on a_1. This limit is however not "radical" in the sense that before the limit, the arguments have the same impact and, after the limit, the arguments have no impact. Indeed, we think that a closer attacker (respectively, defender) of an argument has more effect than a further one on the argument. For example, argument a_2 should have more impact on a_1 than any other argument because a_2 is the direct attacker of a_1. So, the impact is gradually reduced when the length of the path between two arguments increases.
In practice, this principle is supported by the work of [Tan *et al.* 2016] which shows (in the context of their study, an extensive analysis of persuasive debates which took place on the subreddit "ChangeMyView" [16]), that arguments located at a distance greater than 10 from another argument, have no impact in the debate.
While some ranking semantics incorporate features which can be used to *discount* the strength of arguments relatively to their distance, this is not the case of all semantics.

16. https://www.reddit.com/r/changemyview/

While our method is general, we shall also pay special attention to tree shaped argumentation frameworks where an argument a, called *targeted argument*, has only defense branches (*i.e.* $\mathcal{B}_-(x) = \emptyset$ and $\mathcal{B}_+(x) \neq \emptyset$). Such frameworks will be called **persuasion pitches**. The argumentation framework depicted in Figure 5.1 is an example of persuasion pitch with a_1 as targeted argument.

5.2 Ranking-based semantics taking into account the persuasion principles

In this section, our goal is to build a ranking-based semantics which allows to catch the procatalepsis principle and the fading effect. For the fading effect, a solution could be to use an attenuation factor to gradually decrease the impact of arguments. It is the method used by the counting semantics [Pu *et al.* 2015c] with the damping factor α.
For the procatalepsis principle, we want that an attacked argument with many defense roots (like a_1 in Figure 5.1) can be more acceptable than a non-attacked argument. To achieve this, a solution could be to only take into account the defense roots and the attack roots of an argument. Indeed, if we consider that a defense (respectively, attack) root has a positive (respectively, negative) effect on an argument, then, after a number of defense root (which can be catch by a parameter), this argument could be more acceptable than a non-attacked argument.

5.2.1 Propagation with attenuation

We propose to adapt the propagation principle introduced in Chapter 3, Section 3.2 with the elements previously put forward. Recall that the idea of propagation is to assign a positive initial value to each argument in the argumentation framework (arguments may start with the same initial value or start with distinct values where non-attacked arguments have greater value than attacked ones). Then each argument propagates its value into the argumentation framework, alternating the polarity according to the considered path (negatively if it is an attack path, positively if it is a defense one).
But, in order to catch the persuasion principle, we formally redefine the propagation principle by including a damping factor δ which allows to decrease the impact of attackers situated further away along a path (the longer the path length i, the smaller the δ^i).

Definition 5.2.1 (Attenuated propagation)**.**
Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. The valuation P of $x \in \mathcal{A}$, at step i, is given by:

$$P_i^{\epsilon,\delta}(x) = \begin{cases} v_\epsilon(x) & \text{if } i = 0 \\ P_{i-1}^{\epsilon,\delta}(x) + (-1)^i \delta^i \sum\limits_{y \in \mathcal{R}_i^S(x)} v_\epsilon(y) & \text{otherwise} \end{cases}$$

with $\delta \in]0, 1[$ be an attenuation factor and $v_\epsilon : \mathcal{A} \to \mathbb{R}^+$ is a valuation function giving an initial weight to each argument, with $\epsilon \in [0, 1]$ such that $\forall y \in \mathcal{A}$,

$$v_\epsilon(y) = \begin{cases} 1 & \text{if } \mathcal{R}_1^S(y) = \emptyset \\ \epsilon & \text{otherwise} \end{cases}$$

One can remark that the parameter $\oplus$, allowing to make a distinction between the use of the set ($\oplus = S$) or the multiset ($\oplus = M$) to select the attackers or defenders of an argument, is missing in the previous definition compared to the original definition of the propagation principle (see Definition 3.2.2 page 76). Indeed, in this new definition, we choose to only use the set ($\oplus = S$) to guarantee a result to the method. We will provide more details about this point further in the section.

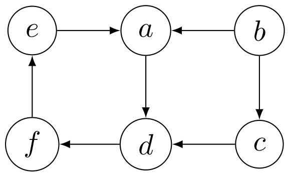


Figure 5.3 – The argumentation framework AF_1

Example 5.2.1. Let us compute the valuation P of each argument in AF_1, depicted in Figure 5.3, when $\epsilon = 0.5$ and $\delta = 0.4$. The results, at each step, are given in Table 5.1.
Let us focus on the argument f. One can see that f begins with an initial weight of 0.5 because it is attacked,

$$P_0^{0.5,0.4}(f) = 0.5$$

Then, during the step $i = 1$, it negatively receives the value attenuated by δ and sent by its direct attacker d which is also attacked,

$$P_1^{0.5,0.4}(f) = P_0^{0.5,0.4}(f) - 0.4 \times v_{0.5}(d) = 0.3$$

During the second step ($i = 2$), it positively receives the weights from a and c attenuated by δ^2,

$$P_2^{0.5,0.4}(f) = P_1^{0.5,0.4}(f) + 0.4^2 \times (v_{0.5}(a) + v_{0.5}(c)) = 0.46$$

When $i = 3$, it negatively receives the weight of 1 from b and the weight of 0.5 from e attenuated by δ^3,

$$P_3^{0.5,0.4}(f) = P_2^{0.5,0.4}(f) - 0.4^3 \times (v_{0.5}(b) + v_{0.5}(e)) = 0.364$$

And so on and so forth.

The following proposition answers the question of convergence of the valuation P. The convergence is guaranteed by the use of the damping factor, but also because the set of arguments which attack or defend an argument for a given length of path is finite and limited by the number of arguments in an argumentation framework. It is why the use of the multiset is impossible here, because when a high number of cycles exists, the multiset of arguments can increase very fast with respect to the length of the considered path and the damping factor (even small) is not enough to guarantee the convergence of the formula.

Proposition 20. Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $\delta \in]0, 1[$ and $\epsilon \in]0, 1]$. For all $x \in \mathcal{A}$, the sequence $\{P_i^{\epsilon,\delta}(x)\}_{i=0}^{+\infty}$ converges.

$P_i^{0.5,0.4}$	a	b	c	d	e	f
0	0.5	1	0.5	0.5	0.5	0.5
1	-0.1	1	0.1	0.1	0.3	0.3
2	-0.02	1	0.1	0.34	0.38	0.46
3	-0.052	1	0.1	0.308	0.316	0.364
⋮	⋮	⋮	⋮	⋮	⋮	⋮
14	-0.0402	1	0.1	0.3161	0.3506	0.3736

Table 5.1 – Computation of the valuation P of each argument from AF_1 when $\epsilon = 0.5$ and $\delta = 0.4$

Let us now compute the propagation number of an argument in using a fixed-point iteration (the outcome is guaranteed with the previous proposition).

Definition 5.2.2 (Propagation number)**.**
Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $\delta \in]0,1[$ and $\epsilon \in]0,1]$. The **propagation number** of an argument $x \in \mathcal{A}$ is:

$$P^{\epsilon,\delta}(x) = \lim_{i \to +\infty} P_i^{\epsilon,\delta}(x)$$

Example 5.2.1 (cont.)**.** The propagation number of each argument in AF_1 is represented in the shaded cell in Table 5.1. Thus, $P^{0.5,0.4}(a) = -0.0402$, $P^{0.5,0.4}(b) = 1$, $P^{0.5,0.4}(c) = 0.1$, $P^{0.5,0.4}(d) = 0.3161$, $P^{0.5,0.4}(e) = 0.3506$ and $P^{0.5,0.4}(f) = 0.3736$.

5.2.2 Variable-depth propagation

Let us now define a ranking-based semantics using the propagation number and taking into consideration the persuasion principles. As said in the introduction of this section, a solution to catch the procatalepsis principle is to only take into account the roots of the arguments. Formally, it is possible when $\epsilon = 0$. Indeed, in this case, non-attacked arguments propagate their weights (=1) in the argumentation graph, while attacked arguments have a weight of 0. Thus, the propagation number of each argument is only based on the value received by their attack or defense roots. Any pairwise strict comparison (based on propagation number) resulting from this process is fixed.
A second phase can be necessary to break ties among arguments equally valued in the first phase. For example, with the argumentation framework represented in Figure 5.4, a_1 and b_1 have the same propagation number when $\epsilon = 0$: $P^{0,\delta}(a) = P^{0,\delta}(b) = 2\delta^2$. However, b_1 is directly attacked only once while a_1 is directly attacked twice, so one can consider that b_1 should be more acceptable than a_1. So, finally, we re-run the propagation phase, this time setting an initial weight $\epsilon \neq 0$ in order to take into account the attacked arguments.

Definition 5.2.3 (Variable-Depth Propagation)**.**
Let $\epsilon \in]0,1]$ and $\delta \in]0,1[$. The ranking-based semantics Variable-Depth Propagation $\mathsf{vdp}^{\epsilon,\delta}$ associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq_{\mathrm{AF}}^{\mathrm{vdp}}$ on $\mathcal{A}$ such that $\forall x, y \in \mathcal{A}$,

$$x \succeq_{\mathrm{AF}}^{\mathrm{vdp}} y \text{ if and only if } P^{0,\delta}(x) > P^{0,\delta}(y) \text{ or } (P^{0,\delta}(x) = P^{0,\delta}(y) \text{ and } P^{\epsilon,\delta}(x) \geq P^{\epsilon,\delta}(y))$$

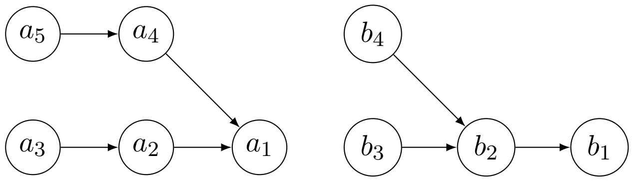


Figure 5.4 – Two arguments a_1 and b_1 with the same propagation number when $\epsilon = 0$

Example 5.2.1 (cont.)**.** According to the previous definition, we need first to compute the propagation number of each argument with $\epsilon = 0$. Argument b is the only non-attacked argument, so the propagation number of each argument is only based on the value that it propagates (the valuations of each argument at each step is given in Table 5.2). It is why, until step $i = 3$, e has a valuation of 0, but during step $i = 4$, it receives a positive value from b, so $P_4^{0,0.4}(e) = 0.4^4 \times v_0(b) = 0.0256$.
We obtain the following propagation numbers: $P^{0,0.4}(a) = -0.4105$, $P^{0,0.4}(b) = 1$, $P^{0,0.4}(c) = -0.4$, $P^{0,0.4}(d) = 0.1642$, $P^{0,0.4}(e) = 0.0263$ and $P^{0,0.4}(f) = -0.0657$.
Thus, in comparing them, we obtain the following ranking:

$$b \succ d \succ e \succ f \succ c \succ a$$

Note that no arguments are equally acceptable here, so it is not necessary to perform the second phase. Thus, $\forall \epsilon \in]0, 1]$, $\mathsf{vdp}^{\epsilon,0.4}$ returns:

$$b \succ^{\text{vdp}} d \succ^{\text{vdp}} e \succ^{\text{vdp}} f \succ^{\text{vdp}} c \succ^{\text{vdp}} a$$

$P_i^{0,0.4}$	a	b	c	d	e	f
0	0	1	0	0	0	0
1	-0.4	1	-0.4	0	0	0
2	-0.4	1	-0.4	0.16	0	0
3	-0.4	1	-0.4	0.308	0	-0.064
4	-0.4	1	-0.4	0.308	0.0256	-0.064
⋮	⋮	⋮	⋮	⋮	⋮	⋮
14	-0.4105	1	-0.4	0.1642	0.0263	-0.0657

Table 5.2 – Computation of the valuation P of each argument from AF_1 when $\epsilon = 0$ and $\delta = 0.4$

Let us give another example where the second phase is needed to distinguish two arguments.

Example 5.2.2. Let us compute the ranking returned by $\mathsf{vdp}^{0.5,0.4}$ for the argumentation framework depicted in Figure 5.4, in beginning by the case $\epsilon = 0$ and then the case $\epsilon = 0.5$ (see Figure 5.5).

According to the definition of vdp, we first compare the propagation number of each argument when $\epsilon = 0$, and we obtain the following ranking:

$$a_3 \simeq a_5 \simeq b_3 \simeq b_4 \succ a_1 \simeq b_1 \succ a_2 \simeq a_4 \succ b_2$$

$P_i^{0,0.4}$	a_3, a_5, b_3, b_4	a_2, a_4	b_2	a_1	b_1
0	1	0	0	0	0
1	1	-0.4	-0.8	0	0
2	1	-0.4	-0.8	0.32	0.32

$P_i^{0.5,0.4}$	a_3, a_5, b_3, b_4	a_2, a_4	b_2	a_1	b_1
0	1	0.5	0.5	0.5	0.5
1	1	0.1	-0.3	0.1	0.3
2	1	0.1	-0.3	0.42	0.62

Figure 5.5 – Valuation P for each argument in the argumentation framework depicted in Figure 5.4 when $\epsilon = 0$ (left) and when $\epsilon = 0.5$ (right) with $\delta = 0.4$

We can see that some arguments still equally acceptable, in particular a_1 and b_1. So, according to the definition of vdp, we restart the process with a non-zero ϵ (here $\epsilon = 0.5$):

$$a_3 \simeq^{\text{vdp}} a_5 \simeq^{\text{vdp}} b_3 \simeq^{\text{vdp}} b_4 \succ^{\text{vdp}} b_1 \succ^{\text{vdp}} a_1 \succ^{\text{vdp}} a_2 \simeq^{\text{vdp}} a_4 \succ^{\text{vdp}} b_2$$

With this second process, a_1 and b_1 can be distinguished. Indeed, they have two defense branches of length 2, so during the first step, they receive the same values from their defense roots. But, one can remark that a_1 is directly attacked twice while b_1 is directly attacked once. So, during the second process, which taking into account the attacked arguments, a_1 receives one more negative value than b_1 ($P_1^{0.5,0.4}(a_1) = 0.1 < 0.3 = P_1^{0.5,0.4}(b_1)$).

5.3 Influence of the parameters

The definition of the propagation number (see Definition 5.2.2 page 116) is based on two parameters independent of the argumentation framework: ϵ and δ. Let us, in this section, characterise their roles and their impacts on the ranking computed with the variable-depth propagation.

Recall that the parameter ϵ has a key role to distinguish the two phases aiming to compute the ranking between arguments. However, a concern might be that the value of ϵ might change the ranking obtained. We show that this is not the case:

Proposition 21. Let $\delta \in]0, 1[$ and $\epsilon, \epsilon' \in]0, 1]$. For any argumentation framework AF,

$$\mathsf{vdp}^{\epsilon,\delta}(AF) = \mathsf{vdp}^{\epsilon',\delta}(AF)$$

Please note that even if different values of ϵ do not change the ranking, it is necessary to keep it in the process in order to make a distinction between non-attacked and attacked arguments (see Definition 5.2.1 about the valuation function v_ϵ). However, this is a purely internal artifact without any effect on the outcome of the method. To make this clear, we note vdp^{δ} instead of $\mathsf{vdp}^{\epsilon,\delta}$ to describe our parametrized ranking semantics in general.

5.3.1 Controlling the scope of influence of the arguments

The parameter δ is defined as the damping factor allowing to decrease the impact of the argument when the length of the path increase. Following this, there intuitively exists a length such that the impact of arguments situating at the beginning of this path is negligible compared to the nearest arguments. Thus, the role of this parameter is to choose the scope of influence of the arguments in the argumentation framework, in addition to allow the convergence of the valuation P. For instance, with a value of δ close to 0, only the nearest arguments (so a little part of the argumentation framework) are taken into consideration to compute the different propagation numbers, whereas with a value of δ close to 1, (almost) all the argumentation framework will be inspected. Consequently, two different values of δ can produce different rankings for a same argumentation framework. Following the principle of the fading effect, it is natural to assume that arguments located at a long distance from another argument become ineffective. In terms of design, it seems very interesting to have the ability to control this parameter so as to specify a maximal depth after which arguments see their influence on the value of others vanish.

To better understand how to take the fading principle into account in using δ, let us detail the algorithm used to compute the propagation numbers.

1) A positive number is assigned to each argument: $\forall a \in \mathcal{A}$, $P_0^{\epsilon,\delta}(a) = 1$ if a is non-attacked or $P_0^{\epsilon,\delta}(a) = \epsilon$ otherwise,

2) We increase the step i by 1 and we add (or subtract) the score computed during the previous step ($P_{i-1}^{\epsilon,\delta}(a)$) and the attenuated weights (v_ϵ and δ^i) received from defenders (or attackers) at the beginning of a path with a length of i ($\mathcal{R}_i^S(a)$):

$$P_i^{\epsilon,\delta}(a) = P_{i-1}^{\epsilon,\delta}(a) + (-1)^i \delta^i \sum_{b \in \mathcal{R}_i^S(a)} v_\epsilon(b)$$

3) If, between two steps, the difference, for all the valuations P, is smaller than a fixed precision threshold μ (*i.e.* $\forall a \in \mathcal{A}, |P_i^{\epsilon,\delta}(a) - P_{i-1}^{\epsilon,\delta}(a)| < \mu$) then the process is stopped [17] and the last values correspond to the propagation number of each argument. If it is not the case, we go back to 2).

Thus, given a precision threshold, one can choose δ according to the maximal expected depth.

Proposition 22. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $i \in \mathbb{N} \backslash \{0\}$ be the maximal depth and μ be the precision threshold. If $\delta < \sqrt[i]{\frac{\mu}{\max\limits_{a \in \mathcal{A}} \left(|\mathcal{R}_i^S(a)| \right)}}$ then, for all $a \in \mathcal{A}$, the sequence $\{P_i^{\epsilon,\delta}(a)\}_{i=0}^{+\infty}$ converges before step $i+1$.

Example 5.3.1 (cont.)**.** Consider the argumentation framework AF_1 depicted in Figure 5.3 (page 115). Suppose that one considers that the maximal depth should be 5. In using the previous formula with a precision $\mu = 0.0001$, then δ should be smaller than $\sqrt[5]{\frac{0.0001}{2}} \simeq 0.1379$. Thus, a value close to this limit, for instance $\delta = 0.137$, ensures that only the arguments until a depth of 5 (included) are considered.

Using the formula in Proposition 22, we can determine, for each maximal depth, which value of δ used. For example, Figure 5.6 represents the arguments which propagate their initial value to f (from AF_1 illustrated in Figure 5.3) before the convergence and the associated interval of values of δ. One can remark that AF_1 contains a cycle $\langle a, d, f, e, a \rangle$, it is why f can receive (according to the value of δ) a value from itself.

17. In practice, we consider that a process is finished when, for each valuation, the difference between two steps is smaller than a precision threshold.

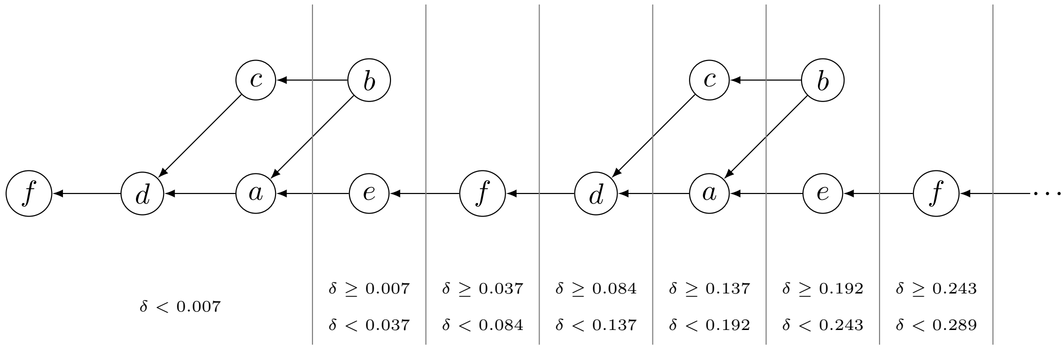


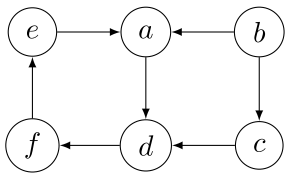
Figure 5.6 – Arguments which propagates their value to f according to the value of δ.

If Proposition 22 allows to select an appropriate δ in order to capture a given maximal expected depth (representing the fading effect), a legitimate question could concern the interval of values of δ to ensure the convergence at a specific depth i. However, it is not possible to answer this question in the general case, because of the diversity of argumentation frameworks. For example, suppose that one wants to consider a length up to 5 (no more no less). With an argumentation frameworks without cycle and with a maximal path smaller than 5, the process will be obviously stopped before. Thus the condition cannot be respected. It is why, we only focus on the maximal depth.

Finally, we can also find a computational advantage to represent the fading effect. Indeed, as the number of steps needed to find the propagation number of each argument is smaller as if we need to browse all the argumentation framework, the ranking is computed faster.

5.3.2 On the diversity of rankings

As shown in Figure 5.7, for a same argumentation framework, different values of δ can produce different rankings. Indeed, when $\delta \in \{0.0001, 0.2, 0.4, 0.6\}$, vdp^δ provides the same ranking. However, when $\delta \geq 0.8$, c becomes more acceptable than f and d becomes more acceptable than b.



δ	vdp^δ
0.0001	$b \succ d \succ e \succ f \succ c \succ a$
0.2	
0.4	
0.6	
0.8	$d \succ b \succ e \succ c \succ f \succ a$
0.9	$d \succ e \succ b \succ c \succ f \succ a$

Figure 5.7 – An argumentation framework and the rankings returned by vdp^δ for different values of δ

In light of these differences, one may be worried that the diversity of rankings could be so high that the semantics becomes too sensitive to small modifications of the parameter δ. To check this, we applied the same method used in Chapter 4, Section 4.1 to compare the rankings returned by existing ranking-based semantics. Thus, we applied our variable-depth propagation on the same 1000 randomly generated argumentation frameworks for different values of $\delta \in \{0.001, 0.2, 0.4, 0.6, 0.8, 0.9\}$. Then, we measure the dissimilarity degree between two rankings from two different values of δ in using the Kendall's tau coefficient (see Definition 4.1.1 page 93) which returns a value between 0 (both rankings are similar) and 1 (both rankings are opposite).

The Table 5.3 contains, for each pair of δ, the average Kendall's tau coefficient, from the results previously computed, that we multiply by 100 to obtain a percentage of dissimilarity.

δ	**0.001**	**0.2**	**0.4**	**0.6**	**0.8**	**0.9**
0.001	0	0.06	0.55	4.09	10.62	13.74
0.2	0.06	0	0.52	4.13	10.63	13.64
0.4	0.55	0.52	0	3.71	10.13	13.3
0.6	4.09	4.13	3.71	0	6.82	9.86
0.8	10.62	10.63	10.13	6.82	0	3.16
0.9	13.74	13.64	13.3	9.86	3.16	0

Table 5.3 – Percentage of dissimilarity between the rankings from vdp^{δ} with different values of δ

The results show that the obtained rankings stay pretty close since the biggest dissimilarity between the smallest and largest value of δ is 13.74%. This dissimilarity remains overall very small, showing that the semantics remain quite stable as the parameter varies.
The goal is now to understand if these differences are only caused by the fading effect or if δ has an impact on other domains too.

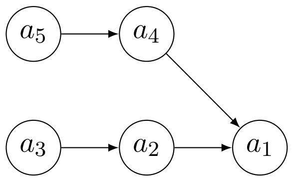
5.4 Properties satisfied by vdp

We now investigate the properties satisfied by our variable-depth propagation semantics vdp. We start by inspecting the case of Void Precedence, before checking other properties discussed in the literature.

5.4.1 Void Precedence

One of the very distinctive feature of vdp is that an attacked argument can have a better score (and so a better rank) than a non-attacked argument. Indeed, when a given argument has many defense branches, it receives many positive weights. However, as depicted with the following example, this feature is not guaranteed for all the value of δ.

Example 5.4.1. Let us compute the rankings of the argumentation framework, which represents the sales pitch aiming to persuade someone to buy a car used to explain the procatalepsis principle (see Section 5.1), in using variable-depth propagation with several values of δ. The result are given in Figure 5.8.



<table>
<tr><th>δ</th><th>vdp^{δ}</th></tr>
<tr><td>0.0001</td><td rowspan="4">$a_5 \simeq a_3 \succ a_1 \succ a_2 \simeq a_4$</td></tr>
<tr><td>0.2</td></tr>
<tr><td>0.4</td></tr>
<tr><td>0.6</td></tr>
<tr><td>0.8</td><td rowspan="2">$a_1 \succ a_5 \simeq a_3 \succ a_2 \simeq a_4$</td></tr>
<tr><td>0.9</td></tr>
</table>

Figure 5.8 – The different rankings computed with vdp for several values of δ applying to an argumentation framework

Indeed, one can remark that the non-attacked argument a_3 (respectively a_5) is strictly more acceptable than all the attacked arguments when $\delta \in \{0.0001, 0.2, 0.4, 0.6\}$ but a_1 becomes strictly more acceptable than a_3 (respectively a_5) for the value of $\delta \in \{0.8, 0.9\}$. Thus, according to the choice of δ, this argument, which is attacked, can obtain a greater score than the score of non-attacked arguments.

Let us formally determine which are, for a given argumentation framework, the values of δ which ensure that the non-attacked arguments are more acceptable than the attacked arguments:

Proposition 23. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x, y \in \mathcal{A}$ such that $\mathcal{R}_1^S(x) = \emptyset$ and $\mathcal{R}_1^S(y) \neq \emptyset$.

$$\text{If } \delta < \delta^M \text{ such that } \delta^M = \sqrt{\frac{1}{\max_{z \in \mathcal{A}}(|\mathcal{R}_2^S(z)|)}} \text{ then } P^{0,\delta}(x) > P^{0,\delta}(y)$$

De facto, there exists a threshold for the parameter δ which VP is satisfied. Let us recall its formal definition.

Void Precedence (VP) A non-attacked argument is ranked strictly higher than any attacked argument: $\mathcal{R}_1(a) = \emptyset$ and $\mathcal{R}_1(b) \neq \emptyset \Rightarrow a \succ b$

Corollary 1. For any argumentation framework, if $\delta < \delta^M$ then vdp^{δ} satisfies VP.

Example 5.4.1 (cont.)**.** The argument a_1 has the highest number of direct defenders with $|\mathcal{R}_2^S(a_1)| = 2$. The value of δ should be $\delta < \delta^M = \sqrt{1/2} \simeq 0.7071$ if one wants to satisfy VP.
So if $\delta = 0.7$, we obtain $P^{0,0.7}(a_1) = 0.98$, $P^{0,0.7}(a_2) = P^{0,0.7}(a_4) = -0.7$ and $P^{0,0.7}(a_3) = P^{0,0.7}(a_5) = 1$ when $\epsilon = 0$ and $P^{0.5,0.7}(a_1) = 0.78$, $P^{0.5,0.7}(a_2) = P^{0.5,0.7}(a_4) = -0.2$ and $P^{0.5,0.7}(a_3) = P^{0.5,0.7}(a_5) = 1$ when $\epsilon = 0.5$. These results allow to obtain the following ranking, which shows that the non-attacked arguments are strictly more acceptable than the attacked arguments:

$$a_3 \simeq a_5 \succ a_1 \succ a_2 \simeq a_4$$

Thus, our method departs from other approaches in its treatment of the Void Precedence property, but to a certain extent only. For instance, in a persuasion pitch, a single line of defense is not enough to be more convincing than a non-attacked argument. On the other hand, when this condition is met, a simple condition for the violation of VP in persuasion pitches can be stated:

Proposition 24. Let $PP = \langle \mathcal{A}, \mathcal{R} \rangle$ be a persuasion pitch with $x \in \mathcal{A}$ as the targeted argument and $y \in \mathcal{A}$ be a non-attacked argument. Then,
(i) if $|\mathcal{B}_+(x)| < 2$ then $y \succ^{\text{vdp}}_{PP} x$;
(ii) if $|\mathcal{B}_+(x)| \geq 2$ and $\delta > \sqrt[m]{\frac{1}{|\mathcal{B}_+(x)|}}$ with m the length of the longest defense branch of x then $x \succ^{\text{vdp}}_{PP} y$.

Interestingly, it turns out that in the context of our method, the Void Precedence property is related to the property: Defense Precedence. Let us recall its formal definition:
Defense Precedence (DP) For two arguments with the same number of direct attackers, a defended argument is ranked higher than a non-defended argument:

$$|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)|, \mathcal{R}_2(a) \neq \emptyset \text{ and } \mathcal{R}_2(b) = \emptyset \Rightarrow \; a \succ b$$

Proposition 25. If vdp^{δ} satisfies VP then it satisfies DP.

Note that this is not the case in general (some ranking-based semantics satisfy VP but not DP).

5.4.2 Other properties

Let us now check which properties, among those defined in this thesis [18], are satisfied by the variable-depth propagation semantics vdp:

Proposition 26. Let $\delta \in]0, 1[$. vdp^{δ} satisfies Abs, In, Tot, NaE, +AB, AE and AvsFD. The other properties are not satisfied.

Some global properties like +DB, ↑DB and ↑AB are not satisfied because of the fading effect. Indeed, when the branch, which is added or extended, is too long, the arguments at the end of this branch have no impact on the targeted argument. It is why we propose to define the corresponding properties (+DB$_i$, ↑DB$_i$ and ↑AB$_i$) which capture the same idea but with the additional condition that the property holds when the maximal length of the branch is i.
Formally, we need to redefine how an attack or a defense branch is added:

Definition 5.4.1 (Attack and defense branch added to an argument with a limited length)**.**
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $x \in \mathcal{A}$ be an argument and $i \in \mathbb{N}^*$ be a length. The defense branch added to x is $P^+_i(x) = \langle \mathcal{A}', \mathcal{R}' \rangle$, with $\mathcal{A}' = \{x_0, \ldots, x_n\}$ such that $n \in 2\mathbb{N}$ and $n \leq i$, $x_0 = x$, $\mathcal{A} \cap \mathcal{A}' = \{x\}$, and $\mathcal{R}' = \{(x_i, x_{i-1}) \mid i \leq n\}$. The attack branch added to x, denoted $P^-_i(x)$ is defined similarly except that the sequence is of odd length (i.e. $n \in 2\mathbb{N} + 1$).

18. The complete list of properties studied in this thesis is reproduced in Appendix D

We are now able to define the "i-version" of +DB, ↑DB and ↑AB.

Property 22 (Addition of a Defense Branch with a maximal length i (+DB$_i$))**.**
Let $i \in \mathbb{N}^*$. A ranking-based semantics σ satisfies i-addition of a defense branch if and only if for any $AF, AF' \in \mathbb{AF}$ and $x \in Arg(AF)$, for every isomorphism γ such that $AF' = \gamma(AF)$, if $AF^\star = AF \cup AF' \cup P_i^+(\gamma(x))$ and $\mathcal{R}_1(x) \neq \emptyset$, then $\gamma(x) \succ^\sigma_{\text{AF}^\star} x$.

Property 23 (Increase of an Attack Branch with a maximal length i (↑AB$_i$))**.**
Let $i \in \mathbb{N}^*$. A ranking-based semantics σ satisfies i-increase of an attack branch if and only if for any $AF, AF' \in \mathbb{AF}$ and $x \in Arg(AF)$, for every isomorphism γ such that $AF' = \gamma(AF)$, if $\exists y \in \mathcal{B}_-(x)$, $y \notin \mathcal{B}_+(x)$ and $AF^\star = AF \cup AF' \cup P_i^+(\gamma(y))$, then $\gamma(x) \succ^\sigma_{\text{AF}^\star} x$.

Property 24 (Increase of a Defense Branch with a maximal length i (↑DB$_i$))**.**
Let $i \in \mathbb{N}^*$. A ranking-based semantics σ satisfies i-increase of a defense branch if and only if for any $AF, AF' \in \mathbb{AF}$ and $x \in Arg(AF)$, for every isomorphism γ such that $AF' = \gamma(AF)$, if $\exists y \in \mathcal{B}_+(x)$, $y \notin \mathcal{B}_-(x)$ and $AF^\star = AF \cup AF' \cup P_i^+(\gamma(y))$, then $x \succ^\sigma_{\text{AF}^\star} \gamma(x)$.

Consequently, for a given maximal depth i, it is enough to choose a value of δ large enough to guarantee that the sequence converges (see Proposition 22 page 119) after taking into account the added or extended branch. As we want that vdp satisfies these properties for all argumentation frameworks, we need to take the largest value of δ. Thus, according to the proposition, if μ and i are fixed, this happens when $\max\limits_{a \in \mathcal{A}} \left(|\mathcal{R}_i^S(a)|\right)$ is minimal so when $\delta < \sqrt[i]{\mu}$.

Proposition 27. Let μ be a precision threshold and i the expected maximal length.

If $\delta \in]\delta^m, 1[$ such that $\delta^m = \sqrt[i]{\mu}$ then vdp^δ satisfies also +DB$_i$, ↑DB$_i$ and ↑AB$_i$

All these results are reported in Table 5.4 (page 125) which is the continuation of Table 4.3 (page 105), which sums up the properties satisfied by the others existing ranking-based semantics. Recall that a cross $\times$means that the property is not satisfied, symbol ✓means that the property is satisfied, symbol ? means that we do not know if the property is satisfied or not, symbol $\checkmark_M$ is specific to the propagation semantics and means that the property is only satisfied when $\oplus = M$, and $\checkmark_i$ means that the i-version of the property (cf Property 27 page 124) is satisfied. Shaded cells are results proved in this chapter.

We first remark that for any value of δ, vdp satisfies the properties accepted by almost all the existing ranking-based semantics (Abs, In, +AB, NaE, AE and Tot). The only exception concerns VP, but it is intended by design and discussed earlier. We can also note that vdp always satisfies property AvsFD, and for a specific δ ($\delta^m < \delta$) the property +DB. These three conditions are necessary to catch the procatalepsis principle. Indeed, AvsFD and +DB states that increasing the number of defense branches improve the acceptability of an argument, and the failure to satisfy VP is necessary to allow the attacked arguments to become more acceptable than non-attacked arguments.
It is clear that, like the Tuples-based semantics, the "local" properties like CT, SCT, CP, QP, DDP or SC cannot be satisfied by our semantics which mainly focus on the branch and not only on the direct attackers and direct defenders.

Properties	Cat	Dbs	Bbs	α-Bbs	FL	CS	Propa$_\epsilon$	Propa$_{1+\epsilon}$	Propa$_{1\to\epsilon}$	Tuples	M&T	IGD	vdp$^\delta$	vdp$^{\delta'}$	vdp$^{\delta''}$
Abs	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
In	✓	✓	✓	✓	✓	×	✓	✓	✓	✓	✓	✓	✓	✓	✓
VP	✓	✓	✓	✓	×	✓	✓	✓	✓	✓	✓	✓	×	×	✓
DP	✓	✓	✓	✓	×	✓	✓	✓	✓	×	×	×	×	×	✓
CT	✓	✓	✓	✓	✓	✓	✓$_M$	✓$_M$	×	×	×	×	×	×	×
SCT	✓	✓	✓	✓	×	✓	✓$_M$	✓$_M$	×	×	×	×	×	×	×
CP	×	✓	✓	×	×	×	×	×	×	×	×	×	×	×	×
QP	×	×	×	×	✓	×	×	×	×	×	×	×	×	×	×
DDP	×	×	✓	×	×	×	×	✓	✓	×	×	×	×	×	×
SC	×	×	×	×	×	×	×	×	×	×	✓	×	×	×	×
$\oplus$DB	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
+DB	×	×	×	×	×	×	×	×	✓	✓	×	×	×	✓$_i$	✓$_i$
↑AB	✓	✓	✓	✓	×	✓	✓	✓	✓	✓	×	×	×	✓$_i$	✓$_i$
↑DB	✓	✓	✓	✓	×	✓	✓	✓	✓	✓	×	×	×	✓$_i$	✓$_i$
+AB	✓	✓	✓	✓	×	✓	✓	✓	✓	✓	×	✓	✓	✓	✓
Tot	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	✓	×	✓	✓	✓
NaE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
AE	✓	✓	✓	✓	?	✓	✓	✓	✓	✓	×	?	✓	✓	✓
OE	✓	✓	✓	✓	✓	✓	✓$_M$	✓$_M$	✓$_M$	✓	×	?	×	×	×
AvsFD	×	×	×	×	✓	×	×	✓	✓	✓	✓	×	✓	✓	✓

Table 5.4 – Summary of the properties satisfied by vdp ($\forall\delta$, for $max(\delta^m, \delta^M) < \delta'$ and for $\delta^m < \delta'' < \delta^M$) and all the existing ranking semantics studied in this thesis.

5.5 Conclusion

In this chapter, we have highlighted the fact that none of the existing ranking-based semantics is really appropriate for the context of persuasion, emphasizing in particular two well-documented phenomena occurring in practice: protocatalepsis and fading. Indeed, all ranking-based semantics commit for instance to the Void Precedence property (or its weak version), which is incompatible with the procatalepsis principle. However, while this property is considered as mandatory in [Amgoud & Ben-Naim 2013], Void Precedence is also called into question in [Besnard & Hunter 2008, Thimm & Kern-Isberner 2014], arguing that arguments which are not attacked can be seen as arguments which have not yet proven their strength against counter-arguments. We think that this question relates to the status of the missing information in argumentation frameworks. If all the information are available, then "really unattacked" arguments should be better that any attacked argument. But there are cases where the argumentation frameworks encode the information currently available, and that is susceptible to be completed. This is this case that we consider here with the procatalepsis principle.

This motivated us to introduce a new parametrized ranking-based semantics based on the notion of propagation. The role of this parameter is manifold because it allows:

1. the convergence of the method which guarantees the existence of a result,
2. the decrease of the impact of further arguments and then to capture the fading effect, by selecting a maximal influence depth,
3. to give the possibility to choose if one wants to satisfy the Void Precedence property or

not (and then represent protocatalepsis in persuasion pitches).

We believe that this method offers a useful tool for persuasion, for instance to evaluate the relative impact that may have different persuasion pitches.

This work opens several perspectives for further research. Indeed, to go further on the example used in the introduction about the salesman, it could be interesting, if more information are available, to use some model of trust. This would allow to evaluate the trust of the customer after the salesman's persuasion pitch and see the possible impact when the procatalepsis principle is used.

Then, our methodology clearly focus on persuasion. However, it may also prove inspiring in other settings: by questioning the relevance of the existing semantics in other application contexts like negotiation (trying to resolve a conflict of interest by reaching a deal), deliberation (trying to reach a decision on a course of action), etc. We may find out that some specific phenomena are not properly captured, and that other adjustments are required.

Conclusion and Future Work

Conclusion

The overall aim of this work was to propose and study ranking-based semantics in the context of abstract argumentation. These semantics began to be actively studied recently and have been introduced as an intuitive alternative way to the classical semantics (extension-based semantics and labelling-based semantics) that are not appropriate for some applications.

In Chapter 2, we put forward the limits of Dung's semantics for some applications (*e.g.* decision-making or online debate platforms) and explained why the ranking-based semantics are a better choice for those applications. Then, we provided the first overview of existing ranking-based semantics which return a unique ranking between arguments from the most to the least acceptable one. We also presented the different properties that have been introduced in the literature, aiming to underline the difference of behavior between these semantics.

In Dung's semantics, non-attacked arguments have a great impact on the acceptability of the arguments of the argumentation framework while, in existing ranking-based semantics, they have no special impact. This motivated us to introduce, in Chapter 3, three ranking-based semantics which allow us to control the **influence of non-attacked arguments** on the acceptability of the other argument. In order to control the impact of non-attacked arguments while preserving as much as possible those concerning the quality and the quantity of attackers or defenders, we defined our ranking-based semantics on the basis of the propagation principle. We showed that ours semantics satisfy interesting properties and that there are some relationships between the ranking retuned by our semantics and the one returned by existing ranking-based semantics in some specific cases (*e.g.* when all the arguments have the same impact). We also checked if the propagation semantics giving the more impact to the non-attacked arguments, refine the Dung's semantics. We showed that it is not the case because, contrary to the ranking-based semantics, extension-based semantics do not consider the number of attacker or defender (one non-attacked argument is enough to rejected an argument), nor the length of an attack or defense path.

In Chapter 4, we proposed a **comparative study** of ranking-based semantics. An experimental comparison is first done by computing a dissimilarity degree between each pair of semantics on the basis of the ranking returned by these semantics from benchmarks of randomly generated argumentation frameworks. The results allowed to conclude that ranking-based semantics globally share a solid common basis. In particular, we observed and proved that the categoriser-based ranking semantics and the 1-burden-based semantics always return the same ranking. We also provided a general comparison of all these semantics with respect to the proposed properties. This study allows to understand the similarities and differences obtained during the experimental comparison.

Finally, in Chapter 5, we questioned the ability of existing ranking-based semantics to capture **persuasion** settings, emphasizing in particular the phenomena of protocatalepsis and of fading. We explained that some widely accepted principles of ranking-based semantics (like Void Precedence) are incompatible with a faithful treatment of these phenomena, which means that no existing ranking-based semantics can caught these two principles together. This motivated us to introduce a new parametrized ranking semantics based on the notion of propagation. This parameter gives the possibility to choose if one wants to satisfy the property Void Precedence or not (and then capture protocatalepsis). We also showed that this parameter allows to control the scope of impact of the arguments (and then to capture fading principle). In general, this work shows that, despite detailed studies of their properties, it is important to also evaluate existing semantics with respect to each targeted application.

Future work

In addition to the future works which have been discussed at the end of each chapter, this thesis opens the way for several important developments.

Debate systems

A problem often raised in abstract argumentation concerns the lack of "real data". Indeed, most of the time, we need to create homemade examples to check the good behavior of our methods. A domain of argumentation, called argument mining (see [Lippi & Torroni 2016] for a recent state of the art), aims at automatically recognizing argumentation structures in unstructured textual documents. Despite the progress made in this domain in recent years, it remains difficult to extract argumentation frameworks from these data. But, recently, more structured data has emerged on the web with the debate systems. We can consider, for example, the recent development of online debate platforms that allows people to participate in debates using argumentation graphs (*e.g.* Debategraph [19] or Argüman [20]), and its more and more usual use on political public consultation: recently the project of a "digital republic bill" has been publicly discussed through an online platform [21], where people could give arguments for or against the different parts of the bill. As shown in Figure 9 (page 129) which represents a debate from Argüman about computer science, we can clearly distinguish the arguments but also the relations (attack or support) between them.

19. http://debategraph.org
20. http://arguman.org
21. https://www.republiquenumerique.fr/pages/in-english

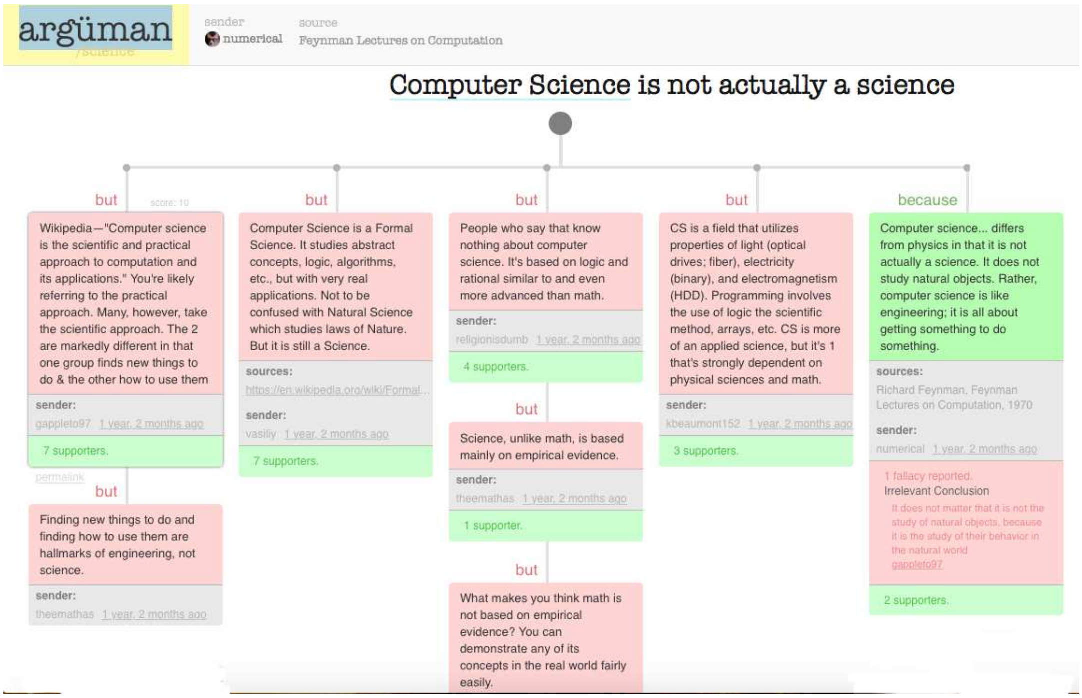


Figure 9 – Example of debate from the website argüman.com about computer science

However, for the moment, they are mainly interfaces where people can give arguments for or against a given issue without any particular processing and evaluation of those arguments. This is due to the fact that most of the existing platforms mainly focus on the debate's representation and use a naive method (just counting the number of argument pro and con) or let the users find the conclusion(s) of the debate themselves. The problems, in this case, are that the conclusion is not always objective or it could be difficult to find the conclusions when the debate is complicated (when many conflicting arguments exist). Using ranking-based semantics and scoring semantics could be a good solution to provide automatic reasoning to these platforms.

In this thesis, we studied and proposed ranking-based semantics for Dung's argumentation framework but, in such platforms, additional information (*e.g.* support, weights, votes, etc.) are available and must be taken into consideration. It is why, recent works propose to adapt some existing ranking-based semantics, initially introduced for Dung's argumentation framework, to framework with more information. Among them, one can find:

- Social argumentation framework (SAF) which contains an attack relation between arguments and a couple of integers, representing the positive votes and the negative votes, is assigned to each arguments/attacks. [LEITE & MARTINS 2011, EGILMEZ *et al.* 2013]
- "Weighted" argumentation framework which contains an attack relation between arguments and a weight is assigned to each arguments. [AMGOUD *et al.* 2017a]
- Bipolar argumentation framework (BAF) which contains an attack relation between arguments but also a support relation between them.
[CAYROL & LAGASQUIE-SCHIEX 2005a, BARONI *et al.* 2015, RAGO *et al.* 2016]

- Bipolar weighted argumentation framework (BWAF) which is a bipolar argumentation framework where a weight is assigned to each argument.
  [EVRIPIDOU & TONI 2012, AMGOUD & BEN-NAIM 2017]

For example, the debate from Argüman depicted in Figure 9 (page 129) can be easily represented by the bipolar weighted argumentation framework depicted in Figure 10 (the votes can be aggregated to obtain a value between 0 and 1). From this debate, the goal would be to know if the conclusion of the debate is that "Computer Science is not actually a science" or the converse. Thus, it could be interesting to check the result of existing semantics introduced for the bipolar weighted argumentation frameworks in such debates.

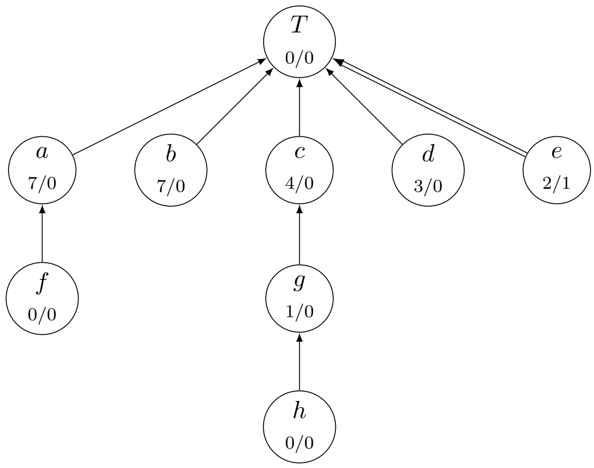


Figure 10 – Representation of the debate from Figure 9 with a bipolar weighted argumentation framework

In this thesis, we saw that our propagation semantics can be distinguished from existing ranking-based semantics, by proposing alternative behaviors. So, we could adapt the propagation principle introduced in this thesis for such framework. The following definition, which extends Definition 5.2.1 (page 114), could be used in this purpose. The initial value of each argument corresponds to the weight associated to each argument in the bipolar weighted argumentation framework. Then, each argument positively (respectively negatively) propagates its initial value to the arguments it defends and supports (respectively attacks) with less and less impact when the length of the paths increases.

Definition 5.5.1 (Attenuated propagation for bipolar weighted argumentation framework)**.**
Let $\langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w \rangle$ be a BWAF where $\mathcal{A}$ is a finite and non-empty set of arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is the attack relation, $\mathcal{S} \subseteq \mathcal{A} \times \mathcal{A}$ is the support relation and w is a function from $\mathcal{A}$ to $[0, 1]$. The valuation P of $x \in \mathcal{A}$, at step i, is given by:

- $P^{\delta}_0(x) = w(x)$ and
- $P^{\delta}_i(x) = P^{\delta}_{i\text{-}1}(x) + \delta^i(\sum\limits_{y \in P^+_i(x)} w(y) \quad - \sum\limits_{z \in P^-_i(x)} w(z))$

with $\delta \in]0,1[$ be an attenuation factor and $P_i^+(x)$ (respectively $P_i^-(x)$) be the set of defenders and supporters (respectively attackers) of x.

We could use a fixed-point iteration to compute the propagation value of each argument in order to compare them.

Several studies can be considered to improve the behavior of semantics for debate platforms:

- Recall that the existing ranking-based semantics and the scoring semantics are currently defined in the abstract case. So, they do not consider the content of the arguments. In many cases, this can be problematic. For example, the current semantics cannot differentiate an argument repeated several times. With the same idea, two "different" arguments, but with the same meaning, are considered as two distinct arguments by the semantics (for example, "No, I do not want to run because it is hot today" and "No, the temperature is too high to run" have the same meaning but have different syntaxes). In a work conducted in the context of an internship I co-supervised, [SAIDI 2017] suggests to first compute a degree of similarity between each arguments, by using methods introduced in natural language processing, in order to know how similar are two arguments. Then, these degrees of similarity are taken into account by the semantics to compute the score of each argument.
- We could also study the possibility to do strategic choices in such debate. Indeed, during a debate, the users could employed some strategies in order to increase the possibility to achieve their goal. For example, such questions can be considered: Does it better to add an attack, a support, a vote on an argument, a vote on an attack ... to improve the acceptability of an argument? Is it the good time to introduce an argument into the debate?

Abstract dialectical framework

Recall that the abstract dialectical frameworks [BREWKA & WOLTRAN 2010] is a generalization of Dung's argumentation frameworks aiming to express a wide range of relations (*e.g.* attack relation, support relation). The meaning of the links between arguments are expressed by the acceptance condition assigned to each argument, which define when an argument can be accepted or not. Thus, an argument is accepted if its acceptance condition is satisfied and is rejected if it is not ($C_x : 2^{par(x)} \to \{\top, \bot\}$). But in the case of scoring semantics, we assign a continuous value to each argument. So, it could be interesting to see if it makes sense to redefine the "acceptance condition" with a formula used by the scoring semantics which assign a value to an argument depending on the value of its direct attacker. Thus, the "acceptance condition" could be defined like that: $C_x : 2^{par(x)} \to \mathbb{R}$ (or any other ordered scale like $[0,1]$). This could allow us to keep the benefit of abstract dialectical frameworks (no need to define new framework) in including several levels of acceptability thanks to the scoring semantics.

Aggregation of argumentation frameworks

Simultaneously to the work done in this thesis, we worked on the problem of aggregation of Dung's abstract argumentation frameworks. This problem is an important one for multi-agent

systems: each agent can be associated with a different abstract argumentation framework (*i.e.* each agent may have different views on what constitutes a valid attack) that represents his beliefs. The problem is then to define a suitable representation at the beliefs of the group. In a first work [DELOBELLE *et al.* 2015], we studied existing aggregation methods from the literature [COSTE-MARQUIS *et al.* 2007, TOHMÉ *et al.* 2008] in the light of the proposed properties [DUNNE *et al.* 2012]. We also proposed three additional methods based on weighted argumentation frameworks where we endorsed one of the possible interpretations of the weights on the attacks (*i.e.* the weight represents the number of agents in a group that agree with this attack) and shown how to use them to define aggregation methods. The results clearly show that two families of aggregation operators of argumentation frameworks exist: one focuses on the attack relations, like it is the case with all the existing operators, and the other one focuses on the extensions. However, there exists no extension-based approach to aggregate argumentation frameworks. It is what we proposed in [DELOBELLE *et al.* 2016]. The idea is to aggregate the extensions from each argumentation framework in input, in choosing the more appropriate extension(s) according to a given distance operator (*e.g.* Hamming distance). Then, as result of the aggregation, we used extension-based generation operators to generate argumentation framework(s) corresponding to the selected extensions.
Recall that the ranking-based semantics can be seen as an alternative to extension-based semantics. So it could be interesting to check if the same conclusions can be done when ranking-based semantics are used: if all the agents agree with the fact that an argument a is strictly more acceptable than another argument b, is it always the case for the result of the aggregation? An interesting application can be made with the online debate platforms. Indeed, the number of platforms is constantly increasing so it could happen that the same topic is addressed in different locations (especially for famous topics like global warming or religion). So, thanks to the aggregation, we could imagine aggregating all these debates and see if one can reach to the same conclusion. For example, if each debate separately concludes that the global warming is dangerous but for different reasons, should the conclusion of the aggregating framework be the same? If yes, is it also the case if only a strict majority of debates concludes that the global warming is dangerous? Such questions could be interesting to answer.

Appendix

Appendix A

Proofs of the Results from Chapter 3

Proposition 5. Let $\oplus \in \{M, S\}$. For any argumentation framework AF, for any $\epsilon, \epsilon' \in]0, 1]$, it holds that

$$Propa^{\epsilon,\oplus}_{1\to\epsilon}(AF) = Propa^{\epsilon',\oplus}_{1\to\epsilon}(AF)$$

Proof. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $\oplus \in \{S, M\}$.
Recall that $Propa_{1\to\epsilon}$ is divided in two steps. During the first one, a pre-order between arguments is established from the scores obtained when $\epsilon = 0$. If at least two arguments could not be distinguished (i.e. $x, y \in \mathcal{A}, \forall i \in \{0, 1, \ldots\} P^{0,\oplus}_i(x) = P^{0,\oplus}_i(y)$) then, during the second step, we restart with $\epsilon \neq 0$.
The goal of this proof consists in showing that for all values of $\epsilon \in]0, 1]$, the pre-order obtained stays the same. In other words, we want to show that the value of ϵ (except the fact to be strictly positive) has no influence on the result.
During the first step, it is obvious that the pre-order obtained is identical whatever the value of ϵ because $\epsilon = 0$. Let us show, that during the second step, where $\epsilon \neq 0$, it is not necessary to know the value of ϵ. Indeed, this step aims to distinguish arguments which cannot be distinguished by non-attacked arguments during the first step. Let $x, y \in \mathcal{A}$ both of these arguments where $\mathcal{R}^{\oplus}_1(x) \neq \emptyset$, $\mathcal{R}^{\oplus}_1(y) \neq \emptyset$ and $\forall i \in \{0, 1, \ldots\} P^{0,\oplus}_i(x) = P^{0,\oplus}_i(y)$.
We first rewrite on a different (but equivalent) way the formula that allows to calculate the score of both arguments. $\forall i \in \{1, 2, \ldots\}$:

$$P^{\epsilon,\oplus}_i(x) = \begin{cases} \epsilon & \text{if } i = 0 \\ P^{\epsilon,\oplus}_{i-1}(x) - (k + n\epsilon) & \text{if } i \text{ is odd} \\ P^{\epsilon,\oplus}_{i-1}(x) + (k + n\epsilon) & \text{if } i \text{ is even} \end{cases} \quad P^{\epsilon,\oplus}_i(y) = \begin{cases} \epsilon & \text{if } i = 0 \\ P^{\epsilon,\oplus}_{i-1}(y) - (k + m\epsilon) & \text{if } i \text{ is odd} \\ P^{\epsilon,\oplus}_{i-1}(y) + (k + m\epsilon) & \text{if } i \text{ is even} \end{cases}$$

where $k = |\mathcal{B}^{\oplus}_i(x)| = |\mathcal{B}^{\oplus}_i(y)|$ and $n = |\mathcal{R}^{\oplus}_i(x)| - |\mathcal{B}^{\oplus}_i(x)|$ (resp. $m = |\mathcal{R}^{\oplus}_i(y)| - |\mathcal{B}^{\oplus}_i(y)|$).

Let us show now that for any value of i, only the values of n and m allow to distinguish both arguments:

i = 0: x and y have the same score because they are both attacked so $P^{\epsilon,\oplus}_0(x) = P^{\epsilon,\oplus}_0(y) = \epsilon$.

<u>i is odd</u>: Until here, both arguments could not be distinguished (otherwise this step is useless because of the lexicographical order) so $P_{i-1}^{\epsilon,\oplus}(x) = P_{i-1}^{\epsilon,\oplus}(y)$. Idem for the value of k (otherwise there was a difference during the first step where $\epsilon = 0$). So there exists three ways to compare x and y :

- Both arguments keep the same score:

$$\begin{aligned} P_i^{\epsilon,\oplus}(x) &= P_i^{\epsilon,\oplus}(y) \\ P_{i-1}^{\epsilon,\oplus}(x) - (k + n\epsilon) &= P_{i-1}^{\epsilon,\oplus}(y) - (k + m\epsilon) \\ -(k + n\epsilon) &= -(k + m\epsilon) \\ k + n\epsilon &= k + m\epsilon \\ n\epsilon &= m\epsilon \\ n &= m \end{aligned}$$

- x becomes better than y ($x \succ_{\text{AF}}^{\overline{P}} y$) : $P_i^{\epsilon,\oplus}(x) > P_i^{\epsilon,\oplus}(y) \Rightarrow n < m$
- y becomes better than x ($y \succ_{\text{AF}}^{\overline{P}} x$) : $P_i^{\epsilon,\oplus}(x) < P_i^{\epsilon,\oplus}(y) \Rightarrow n > m$

Consequently, the value of ϵ is not significant when i is odd.

<u>i is even</u>: This case is quite similar to the previous one :

- Both arguments keep the same score: $P_i^{\epsilon,\oplus}(x) = P_i^{\epsilon,\oplus}(y) \Rightarrow n = m$
- x becomes better than y ($x \succ_{\text{AF}}^{\overline{P}} y$) : $P_i^{\epsilon,\oplus}(x) > P_i^{\epsilon,\oplus}(y) \Rightarrow n > m$
- y becomes better than x ($y \succ_{\text{AF}}^{\overline{P}} x$) : $P_i^{\epsilon,\oplus}(x) < P_i^{\epsilon,\oplus}(y) \Rightarrow n < m$

Once again, we can see that only the values of n and m play a key role in the order between x and y.

So, whatever the value of i, the value of ϵ is not significative to establish an order between x and y in agreement with the property. □

Proposition 6. Let $\oplus \in \{M, S\}$. For any argumentation framework AF, for any $\epsilon, \epsilon' \in]0, 1]$, it holds that

$$Propa_{1+\epsilon}^{\epsilon,\oplus}(AF) = Propa_{1+\epsilon}^{\epsilon',\oplus}(AF)$$

Proof. The proof is exactly the same as for $Propa_{1\to\epsilon}$ because, according to the formula allowing to give a score to each argument, the only thing to check is that the value of k remains the same during the steps where $\epsilon \neq 0$. And this is exactly the case with $Propa_{1+\epsilon}$ because, before to look at the case where $\epsilon \neq 0$, the score must be identical during the first step where $\epsilon = 0$ (thanks to the lexicographical order). Consequently, k keeps the same value so $k = |P_i^{0,\oplus}(x) - P_{i-1}^{0,\oplus}(x)| = |P_i^{0,\oplus}(y) - P_{i-1}^{0,\oplus}(y)|$ and this bring us to same conclusions as for $Propa_{1\to\epsilon}$. □

Proposition 7. Let $\oplus \in \{M, S\}$. For any argumentation framework AF,

$$Propa_{\epsilon}^{0,\oplus}(AF) = Propa_{1+\epsilon}^{0,\oplus}(AF) = Propa_{1\to\epsilon}^{0,\oplus}(AF)$$

Proof. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $x, y \in \mathcal{A}$ and $\oplus \in \{S, M\}$.
In order to show that the three semantics return the same pre-order when $\epsilon = 0$, we will proceed in two steps. The first consists in applying the case $\epsilon = 0$ to the definition of $Propa_\epsilon$:

$$x \succeq^{P}_{\text{AF}} y \Rightarrow P^{0,\oplus}(x) \succeq_{lex} P^{0,\oplus}(y)$$

The second step aims to prove that when the case $\epsilon = 0$ is applied to $Propa_{1\to\epsilon}$ and $Propa_{1+\epsilon}$, their definitions are exactly the same as $Propa_\epsilon$.
Let us begin with $Propa_{1\to\epsilon}$ where we replace ϵ by 0:

$$\begin{aligned} x \succeq^{\overline{P}}_{\text{AF}} y &\Rightarrow P^{0,\oplus}(x) \succeq_{lex} P^{0,\oplus}(y) \textit{ or } (P^{0,\oplus}(x) \simeq_{lex} P^{0,\oplus}(y) \textit{ and } P^{\epsilon,\oplus}(x) \succeq_{lex} P^{\epsilon,\oplus}(y)) \\ &\Rightarrow P^{0,\oplus}(x) \succeq_{lex} P^{0,\oplus}(y) \textit{ or } (P^{0,\oplus}(x) \simeq_{lex} P^{0,\oplus}(y) \textit{ and } P^{0,\oplus}(x) \succeq_{lex} P^{0,\oplus}(y)) \\ &\Rightarrow P^{0,\oplus}(x) \succeq_{lex} P^{0,\oplus}(y) \textit{ or } P^{0,\oplus}(x) \simeq_{lex} P^{0,\oplus}(y) \\ &\Rightarrow P^{0,\oplus}(x) \succeq_{lex} P^{0,\oplus}(y) \end{aligned}$$

Finally, we apply the same reasoning to $Propa_{1+\epsilon}$:

$$\begin{aligned} x \succeq^{\widehat{P}}_{\text{AF}} y &\Rightarrow (P^{0,\oplus}(x) \cup_s P^{\epsilon,\oplus}(x)) \succeq_{lex} (P^{0,\oplus}(y) \cup_s P^{\epsilon,\oplus}(y)) \\ &\Rightarrow (P^{0,\oplus}(x) \cup_s P^{0,\oplus}(x)) \succeq_{lex} (P^{0,\oplus}(y) \cup_s P^{0,\oplus}(y)) \\ &\Rightarrow P^{0,\oplus}(x) \succeq_{lex} P^{0,\oplus}(y) \end{aligned}$$

Because it is obvious that if $P^{0,\oplus}(x) \cup_s P^{0,\oplus}(x) \succeq_{lex} P^{0,\oplus}(y) \cup_s P^{0,\oplus}(y)$ then $P^{0,\oplus}(x) \succeq_{lex} P^{0,\oplus}(y)$ (even if $P^{0,\oplus}(x) \cup_s P^{0,\oplus}(x) \neq P^{0,\oplus}(x)$).

So whatever the propagation semantics chosen, when $\epsilon = 0$, $x \succeq y$ if $P^{0,\oplus}(x) \succeq_{lex} P^{0,\oplus}(y)$. □

Proposition 8. Let $\oplus \in \{M, S\}$ and $\epsilon \in]0, 1]$. For any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ such that $\nexists x \in \mathcal{A}, \mathcal{R}^{\oplus}_1(x) = \emptyset$, $Propa^{\epsilon,\oplus}_{\epsilon}(AF) = Propa^{\epsilon,\oplus}_{1+\epsilon}(AF) = Propa^{\epsilon,\oplus}_{1\to\epsilon}(AF)$.

Proof. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework without non-attacked argument, $x, y \in A$ and $\oplus \in \{S, M\}$.
The fact to have no non-attacked argument in AF makes useless the use of the case $\epsilon = 0$ for $Propa_{1+\epsilon}$ and $Propa_{1\to\epsilon}$. Indeed, in this case, all the arguments begin with a score of 0 so no one can propagate its value when $\epsilon = 0$: $\forall x \in \mathcal{A}, \forall i \in \{0, 1, \ldots\}$, $P^{0,\oplus}_i(x) = 0$. So the only case where a distinction between arguments can be done is when $\epsilon \neq 0$. All the arguments begin with a score of ϵ : $\forall x \in \mathcal{A}, v_\epsilon(x) = \epsilon$. So whatever the propagation semantics, when there is no non-attacked argument $x \succeq y$ if $P^{\epsilon,\oplus}(x) \succeq_{lex} P^{\epsilon,\oplus}(y)$. □

Proposition 9. Let $\oplus \in \{M, S\}$. For any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$,

$$\text{If } \epsilon < \frac{1}{maxdeg(AF)}, \text{ then } Propa^{\epsilon,\oplus}_{\epsilon}(AF) = Propa^{\epsilon,\oplus}_{1+\epsilon}(AF)$$

Proof. Let us show that for any couple of arguments $x, y \in \mathcal{A}$, if $\epsilon < \frac{1}{maxdeg(AF)}$ then the ranking between x and y is the same for $Propa_\epsilon$ and $Propa_{1+\epsilon}$.
If x and y are not attacked then they are equally acceptable for both semantics whatever the value

of ϵ because $Propa_\epsilon$ and $Propa_{1+\epsilon}$ satisfy the property Non-attacked Equivalence (Chapter 4 Proposition 17 page 104).
If x is non-attacked and y is attacked then x is strictly more acceptable than y for both semantics whatever the value of ϵ because $Propa_\epsilon$ and $Propa_{1+\epsilon}$ satisfy the property Void Precedence (Chapter 4 Proposition 17 page 104).
So the last case is when x and y are both attacked. Let us rewrite the formula which allows us to compute the score of both arguments at step $i \in \{0, 1, 2, \ldots\}$,

$$P_i^{\epsilon,\oplus}(x) = \begin{cases} \epsilon & \text{if } i = 0 \\ P_{i-1}^{\epsilon,\oplus}(x) - (k + n\epsilon) & \text{if } i \text{ is odd} \\ P_{i-1}^{\epsilon,\oplus}(x) + (k + n\epsilon) & \text{if } i \text{ is even} \end{cases} \quad P_i^{\epsilon,\oplus}(y) = \begin{cases} \epsilon & \text{if } i = 0 \\ P_{i-1}^{\epsilon,\oplus}(y) - (k' + m\epsilon) & \text{if } i \text{ is odd} \\ P_{i-1}^{\epsilon,\oplus}(y) + (k' + m\epsilon) & \text{if } i \text{ is even} \end{cases}$$

where $k = |\mathcal{B}_i^\oplus(x)|$ (resp. $k' = |\mathcal{B}_i^\oplus(y)|$) and $n = |\mathcal{R}_i^\oplus(x)| - |\mathcal{B}_i^\oplus(x)|$ (resp. $m = |\mathcal{R}_i^\oplus(y)| - |\mathcal{B}_i^\oplus(y)|$).
We know that n (resp. m) is lower or equal to $maxdeg(AF)$ and ϵ is a positive value:

$$n \leq maxdeg(AF) \Rightarrow n\epsilon \leq maxdeg(AF)\epsilon \tag{A.1}$$

And we know that:

$$\epsilon < \frac{1}{maxdeg(AF)} \Rightarrow maxdeg(AF)\epsilon < 1 \tag{A.2}$$

Combining the equations A.1 and A.2, we obtain:

$$n\epsilon < 1 \text{ (resp. } m\epsilon < 1) \tag{A.3}$$

Let us show now that for any value of i, the ranking between x and y is the same for $Propa_\epsilon$ and $Propa_{1+\epsilon}$:

<u>i = 0</u>: x and y have the same score for both semantics ($P_0^{\epsilon,\oplus}(x) = P_0^{\epsilon,\oplus}(y) = \epsilon$ and $P_0^{0,\oplus}(x) = P_0^{0,\oplus}(y) = 0$).

<u>i is odd</u>: Until here, both arguments could not be distinguished for both semantics (otherwise this step is useless because of the lexicographical order) so $P_{i-1}^{\epsilon,\oplus}(x) = P_{i-1}^{\epsilon,\oplus}(y)$ including for $\epsilon = 0$.

- if $k = k'$ then $P_i^{0,\oplus}(x) = P_i^{0,\oplus}(y)$ so the difference is done with the value of n and m:
  - if $n = m$ then $P_i^{\epsilon,\oplus}(x) = P_i^{\epsilon,\oplus}(y)$ so both semantics go to the next step because x and y have the same score.
  - if $n > m$ then $P_i^{\epsilon,\oplus}(x) < P_i^{\epsilon,\oplus}(y)$ so $y \succ_{AF}^{P} x$ and $y \succ_{AF}^{\widehat{P}} x$.
  - if $n < m$ then $P_i^{\epsilon,\oplus}(x) > P_i^{\epsilon,\oplus}(y)$ so $x \succ_{AF}^{P} y$ and $x \succ_{AF}^{\widehat{P}} y$.

- if $k < k'$ then $P_i^{0,\oplus}(x) > P_i^{0,\oplus}(y)$ so $x \succ_{AF}^{\widehat{P}} y$. Let us show that we obtain the same ranking with $Propa_\epsilon$. In using the result A.3, we have:

$$\begin{gathered} k - n\epsilon < k' - m\epsilon \\ -(k - n\epsilon) > -(k' - m\epsilon) \\ P_{i-1}^{\epsilon,\oplus}(x) - (k - n\epsilon) > P_{i-1}^{\epsilon,\oplus}(y) - (k' - m\epsilon) \\ P_i^{\epsilon,\oplus}(x) > P_i^{\epsilon,\oplus}(y) \\ x \succ_{AF}^{P} y \end{gathered}$$

- if $k > k'$ then $P_i^{0,\oplus}(x) < P_i^{0,\oplus}(y)$ so $y \succ_{AF}^{\widehat{P}} x$ and following the same idea that the previous case $y \succ_{AF}^{P} x$.

i is even: Following the same reasoning, we obtain the same result with the opposite ranking.

So $Propa_\epsilon$ and $Propa_{1+\epsilon}$ return the same ranking between arguments. □

Proposition 10. Let $\epsilon \in]0,1]$. For any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ such that $\nexists x \in \mathcal{A}, \mathcal{R}_1^M(x) = \emptyset$, $Propa_\epsilon^{\epsilon,M}(AF) = Propa_{1+\epsilon}^{\epsilon,M}(AF) = Propa_{1\to\epsilon}^{\epsilon,M}(AF) = Dbs(AF)$.

Proof. The previous proposition shows that all semantics based on propagation return the same ranking between arguments when there is no non-attacked arguments. And we want to show that this ranking is also the same for the discussion-based semantics (Dbs). Thus, it is sufficient to show that the ranking computed by one propagation semantics is always the same that the one computed by the discussion-based semantics. It is why, in this proof, we will focus on $Propa_\epsilon$. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x, y \in \mathcal{A}$.
We first rewrite on different way (because there is no attacked argument) the formula that allows to calculate the score of both arguments. $\forall i \in \{0, 1, 2, \ldots\}$:

$$P_i^{\epsilon,M}(x) = \begin{cases} \epsilon & \text{if } i = 0 \\ P_{i-1}^{\epsilon,M}(x) + (\text{-}1)^i n\epsilon & \text{otherwise} \end{cases} \quad P_i^{\epsilon,M}(y) = \begin{cases} \epsilon & \text{if } i = 0 \\ P_{i-1}^{\epsilon,M}(y) + (\text{-}1)^i m\epsilon & \text{otherwise} \end{cases}$$

where $n = |\mathcal{R}_i^M(x)|$ and $m = |\mathcal{R}_i^M(y)|$.
And with the discussion-based semantics, the score of each argument is computed with the following formula. $\forall i \in \{1, 2, \ldots\}$:

$$Dis_i(a) = (-1)^{i+1} \times n \qquad\qquad Dis_i(b) = (-1)^{i+1} \times m$$

It is obvious that only the values of n and m allow to differentiate the arguments x and y for $Propa_\epsilon$ and Dbs. So, let us show that for any value of i, the ranking between x and y stay the same for both semantics.

i = 0: The function Dis is not defined for $i = 0$ and $P_0^{\epsilon,M}(a) = P_0^{\epsilon,M}(b) = \epsilon$ so the process continue for both semantics.

i is odd: Until here, both arguments could not be distinguished because the lexicographical order is used in both semantics so $P_{i-1}^{\epsilon,M}(x) = P_{i-1}^{\epsilon,M}(y)$ and $Dis_{i-1}(x) = Dis_{i-1}(y)$.

- If $n = m$ then $P_i^{\epsilon,M}(x) = P_i^{\epsilon,M}(y)$ and $Dis_i(x) = Dis_i(y)$ so x and y still equally acceptable for both semantics.
- If $n > m$ then $P_i^{\epsilon,M}(x) < P_i^{\epsilon,M}(y)$ and $Dis_i(x) > Dis_i(y)$ so y becomes strictly better than x for both semantics: $y \succ_{\text{AF}}^{P} x$ and $y \succ_{\text{AF}}^{\text{Dbs}} x$.
- If $n < m$ then $P_i^{\epsilon,M}(x) > P_i^{\epsilon,M}(y)$ and $Dis_i(x) < Dis_i(y)$ so x becomes strictly better than y for both semantics: $x \succ_{\text{AF}}^{P} y$ and $x \succ_{\text{AF}}^{\text{Dbs}} y$.

i is even: This case is quite similar to the previous one but here only defense path are taken into consideration.

- If $n = m$ then $P_i^{\epsilon,M}(x) = P_i^{\epsilon,M}(y)$ and $Dis_i(x) = Dis_i(y)$ so x and y still equally acceptable for both semantics.
- If $n > m$ then $P_i^{\epsilon,M}(x) > P_i^{\epsilon,M}(y)$ and $Dis_i(x) < Dis_i(y)$ so x becomes strictly better than y for both semantics: $y \succ_{\text{AF}}^{P} x$ and $y \succ_{\text{AF}}^{\text{Dbs}} x$.
- If $n < m$ then $P_i^{\epsilon,M}(x) < P_i^{\epsilon,M}(y)$ and $Dis_i(x) > Dis_i(y)$ so y becomes strictly better than x for both semantics: $y \succ_{\text{AF}}^{P} x$ and $y \succ_{\text{AF}}^{\text{Dbs}} x$.

For each step i, if x becomes more acceptable than y under the semantics $Propa_\epsilon$ then it is also the case with Dbs (idem for the other possibilities). Consequently, $Propa_\epsilon$ (and consequently $Propa_{1+\epsilon}$ and $Propa_{1\rightarrow\epsilon}$) and Dbs return the same ranking between arguments. □

Proposition 11. For any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$, $Propa_\epsilon^{1,M}(AF) = Dbs(AF)$.

Proof. The proof is similar to the previous one except that $\epsilon = 1$ and take into account all the argumentation frameworks. □

Appendix B

Proofs of the Results from Chapter 4

Proposition 12. Let AF be an argumentation framework, if $\alpha = 1$ then $Cat(AF)$ = α-$Bbs(AF)$.

Proof. Let $x \in Arg(AF)$ be an argument.
If x is not attacked then $Cat(x) = s_\alpha(x) = 1/s_\alpha(x) = 1$.
If x is attacked then:

$$\left.\begin{array}{r} Cat(x) = \frac{1}{1+\sum\limits_{y\in\mathcal{R}_1(x)} Cat(y)} \\ s_\alpha(x) = 1 + \sum\limits_{y\in\mathcal{R}_1(x)} \frac{1}{s_\alpha(y)} \Rightarrow 1/s_\alpha(x) = \frac{1}{1+\sum\limits_{y\in\mathcal{R}_1(x)} \frac{1}{s_\alpha(y)}} \end{array}\right\} Cat(x) = 1/s_\alpha(x)$$

So for all arguments x and y in AF,

$$\begin{aligned} x \succeq^{\text{Cat}}_{\text{AF}} y \iff Cat(x) \geq Cat(y) &\iff 1/s_\alpha(x) \geq 1/s_\alpha(y) \\ &\iff s_\alpha(x) \leq s_\alpha(y) \iff x \succeq^{\alpha\text{-Bbs}}_{\text{AF}} y \end{aligned}$$

□

Proposition 13. For every ranking-based semantics, the following pairs of properties are not compatible :

(1) Cardinality Precedence (CP) and Quality Precedence (QP) [AMGOUD & BEN-NAIM 2013]
(2) Self-Contradiction (SC) and Cardinality Precedence (CP) [BESNARD *et al.* 2017]
(3) Self-Contradiction (SC) and Counter-Transitivity (CT) [BESNARD *et al.* 2017]
(4) Self-Contradiction (SC) and Strict Counter-Transitivity (SCT) [BESNARD *et al.* 2017]
(5) Cardinality Precedence (CP) and Attack vs Full Defense (AvsFD)
(6) Cardinality Precedence (CP) and Addition of Defense Branch (+DB)
(7) Cardinality Precedence (CP) and Strict Addition of Defense Branch (⊕DB)
(8) Void Precedence (VP) and Strict Addition of Defense Branch (⊕DB)
(9) Strict Counter-Transitivity (SCT) and Strict Addition of Defense Branch (⊕DB)
(10) Argument Equivalence (AE) and Self-Contradiction (SC)

Proof. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $a, b \in \mathcal{A}$ and σ be a ranking semantics.

(1) See [AMGOUD & BEN-NAIM 2013]

(2) See [BESNARD *et al.* 2017]

(3) See [BESNARD *et al.* 2017]

(4) See [BESNARD *et al.* 2017]

(5) Let us suppose that $|\mathcal{R}_1(b)| = 1$, $|\mathcal{R}_2(b)| = 0$ and $|\mathcal{B}_-(a)| = 0$. If σ satisfied AvsFD then $a \succ^{\sigma}_{\text{AF}} b$. However, there is no restriction about the number of defense branches of a, so there exists cases where $|\mathcal{R}_1(a)| > |\mathcal{R}_1(b)|$. In these cases, the property CP says that $a \prec^{\sigma}_{\text{AF}} b$ which contradicts AvsFD.

(6) Let $AF^{\star} = AF \cup AF^{\gamma} \cup P_+(\gamma(a))$ be an argumentation framework such that $AF^{\gamma} = \gamma(AF)$ and a is attacked ($\mathcal{R}_1(a) \neq \emptyset$). In $AF^{\star}$, $\gamma(a)$ has one more defense branch (and so one more direct attacker) than a ($\mathcal{R}_1(a) < \mathcal{R}_1(\gamma(a))$). If σ satisfies CP then $a \succ^{\sigma}_{\text{AF}^{\star}} \gamma(a)$ whereas if +DB is satisfied then $\gamma(a) \succ^{\sigma}_{\text{AF}^{\star}} a$.

(7) Same proof that +DB except that a can be non-attacked too.

(8) Let $AF^{\star} = AF \cup AF^{\gamma} \cup P_+(\gamma(a))$ be an argumentation framework such that $AF^{\gamma} = \gamma(AF)$ and a is a non-attacked argument ($\mathcal{R}_1(a) = \emptyset$). If σ satisfies $\oplus$DB then $\gamma(a) \succ^{\sigma}_{\text{AF}^{\star}} a$ whereas if σ satisfies VP then $a \succ^{\sigma}_{\text{AF}^{\star}} \gamma(a)$ because $\gamma(a)$ becomes attacked ($\mathcal{R}_1(a) = \emptyset$ and $\mathcal{R}_1(\gamma(a)) \neq \emptyset$).

(9) Same proof that +DB except that a can be non-attacked too.

(10) Let us show with the argumentation framework illustrated in Figure B.1 that the ranking between two arguments suggests by the properties AE and SC are different. It is clear that

Figure B.1 – Incompatibility between Argument Equivalence (AE) and Self-Contradiction (SC)

it exists an isomorphism between the ancestor's graph of a and b (which is a infinite line of arguments) so, according to the property AE, a and b are equally acceptable ($a \simeq b$). In addition, the argument a attacks itself contrary to b so, according to the property SC, b is strictly more acceptable than a ($b \succ a$). Thus, according to the definition of the incompatibility between two properties, AE and SC are incompatible.

□

Proposition 14. No ranking-based semantics can simultaneously satisfy Addition of a Defense Branch (+DB), Strict Counter-Transitivity (SCT) and Argumentation Equivalence (AE).

Proof. Let AF, AF^γ be two argumentation framework such that there exists an isomorphism γ between AF and AF^γ ($AF^\gamma = \gamma(AF)$) and σ be a ranking semantics.
According to the property AE, it is clear that each argument and its image are equally acceptable ($\forall x \in Arg(AF), x \simeq^\sigma_{\text{AF}\cup\text{AF}^\gamma} \gamma(x)$).
Let $AF^\star = AF \cup AF^\gamma \cup P_+(\gamma(a))$ and $a \in Arg(AF)$ is attacked ($\mathcal{R}_1(a) \neq \emptyset$). If σ satisfies +DB then $\gamma(a) \succ^\sigma_{\text{AF}^\star} a$ whereas if σ satisfies SCT then $a \succ^\sigma_{\text{AF}^\star} \gamma(a)$ because it exists an injective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(\gamma(a))$ such that $\forall c \in \mathcal{R}_1(a), f(c) \succeq^\sigma_{\text{AF}^\star} c$ (thanks to the property AE) and $|\mathcal{R}_1(\gamma(a))| > |\mathcal{R}_1(a)|$. □

Proposition 15. The following properties are not independent:

(1) Strict Counter-Transitivity (SCT) implies Void Precedence (VP) [AMGOUD & BEN-NAIM 2013]

(2) Counter-Transitivity (CT) and Strict Counter-Transitivity (SCT) imply Defense Precedence (DP) [AMGOUD & BEN-NAIM 2013]

(3) Counter Transitivity (CT) implies Non-attacked Equivalence (NaE)

(4) Counter Transitivity (CT) implies Ordinal Equivalence (OE)

(5) Strict Counter-Transitivity (SCT) and Ordinal Equivalence (OE) imply Counter-Transitivity (CT)

(6) Strict addition of Defense Branch (⊕DB) implies Addition of Defense Branch (+DB)

(7) Argument Equivalence (AE) implies Non-attacked Equivalence (NaE)

(8) Ordinal Equivalence (OE) implies Non-attacked Equivalence (NaE)

(9) Void Precedence (VP) and Quality Precedence (QP) imply Attack vs Full Defense (AvsFD)

(10) Cardinality Precedence (CP) implies Addition of Attack Branch (+AB)

Proof. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $a, b \in \mathcal{A}$ and σ be a ranking semantics.

(1) See [AMGOUD & BEN-NAIM 2013]

(2) See [AMGOUD & BEN-NAIM 2013]

(3) Let us suppose that a and b are non-attacked ($\mathcal{R}_1(a) = \mathcal{R}_1(b) = \emptyset$). As the sets of direct attackers are empty, it is clear that there exists an injective function from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(b)$ (resp. from $\mathcal{R}_1(b)$ to $\mathcal{R}_1(a)$) such that $\forall c \in \mathcal{R}_1(a), f(c) \succeq^\sigma_{\text{AF}} c$ (resp. $\forall c \in \mathcal{R}_1(b), f(c) \succeq^\sigma_{\text{AF}} c$). As σ satisfies CT, so $a \simeq^\sigma_{\text{AF}} b$ (because $a \succeq^\sigma_{\text{AF}} b$ and $b \succeq^\sigma_{\text{AF}} a$) in agreement with the property NaE.

(4) Let us suppose that there exists a bijective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(b)$ such that $\forall c \in \mathcal{R}_1(a), c \simeq^\sigma_{\text{AF}} f(c)$ (*i.e.* $c \succeq^\sigma_{\text{AF}} f(c)$ and $f(c) \succeq^\sigma_{\text{AF}} c$). By definition, a bijective function is also an injective function, so f is injective and $\mathcal{R}_1(b) \geq_S \mathcal{R}_1(a)$ (because $\forall c \in \mathcal{R}_1(a), f(c) \succeq^\sigma_{\text{AF}} c$). As σ satisfies CT, one can conclude that $a \succeq^\sigma_{\text{AF}} b$. But the existence of the bijective function f implies that there also exists a bijective function g ($g = f^{-1}$) from $\mathcal{R}_1(b)$ to $\mathcal{R}_1(a)$. So following the same reasoning, one can conclude that $b \succeq^\sigma_{\text{AF}} a$. So, by definition, $a \succeq^\sigma_{\text{AF}} b$ and $b \succeq^\sigma_{\text{AF}} a$ implies that $a \simeq^\sigma_{\text{AF}} b$ in agreement with OE.

(5) Let us suppose that there exists an injective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(b)$ such that $\forall c \in \mathcal{R}_1(a), f(c) \succeq^{\sigma}_{\text{AF}} c$ and that σ satisfies SCT and OE. Let us show that for all a, b which satisfy this condition then $a \succeq^{\sigma}_{\text{AF}} b$.

   (a) If $\mathcal{R}_1(b) > \mathcal{R}_1(a)$ or $\exists c \in \mathcal{R}_1(a), f(a) \succ^{\sigma}_{\text{AF}} a$ then according to SCT $a \succ^{\sigma}_{\text{AF}} b$. By definition, $a \succ^{\sigma}_{\text{AF}} b$ is equivalent to $a \succeq^{\sigma}_{\text{AF}} b$ and $b \not\succeq^{\sigma}_{\text{AF}} a$ so CT is satisfied.

   (b) If $\mathcal{R}_1(b) = \mathcal{R}_1(a)$ and $\nexists c \in \mathcal{R}_1(a), f(a) \succ^{\sigma}_{\text{AF}} a$ then $\forall c \in \mathcal{R}_1(a), f(a) \simeq^{\sigma}_{\text{AF}} a$. But, as $\mathcal{R}_1(b) = \mathcal{R}_1(a)$ then f is also surjective so f is bijective and $\forall c \in \mathcal{R}_1(a), f(a) \simeq^{\sigma}_{\text{AF}} a$, so according to OE, we have $a \simeq^{\sigma}_{\text{AF}} b$. By definition, $a \simeq^{\sigma}_{\text{AF}} b$ is equivalent to $a \succeq^{\sigma}_{\text{AF}} b$ and $b \succeq^{\sigma}_{\text{AF}} a$ so CT is satisfied.

(6) Obvious because +DB is a particular case of $\oplus$DB (if it is true for all the arguments then it is also true for the attacked arguments).

(7) Obvious because if a and b are non-attacked, then they have the same ancestors' graph which is empty. Thus according to AE, they are equally acceptable ($a \simeq^{\sigma} b$) in agreement with NaE.

(8) Let us suppose that a and b are non-attacked ($\mathcal{R}_1(a) = \mathcal{R}_1(b) = \emptyset$). As the sets of direct attackers are empty, it is clear that there exists an bijective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(b)$ such that $\forall c \in \mathcal{R}_1(a), c \simeq^{\sigma}_{\text{AF}} f(c)$. As σ satisfies OE, so $a \simeq^{\sigma}_{\text{AF}} b$ in agreement with NaE.

(9) Let us suppose that $|\mathcal{B}_-(a)| = 0$ which means that a is either not attacked or attacked but defended. Let us also assume that $|\mathcal{R}_1(b)| = 1$ and $|\mathcal{R}_2(b)| = 0$. According to the property AvsFD, in this case, a is strictly more acceptable than b ($a \succ^{\sigma}_{\text{AF}} b$). We will show that when the properties VP and QP are satisfied we obtain the same result.
Let $\mathcal{R}_1(a) = \{a_1, \ldots, a_n\}$ and $\mathcal{R}_1(b) = \{b_1\}$.
$\underline{\mathcal{R}_1(a) = \emptyset}$: From VP, we have $a \succ^{\sigma}_{\text{AF}} b$ because $\mathcal{R}_1(a) = \emptyset$ and $\mathcal{R}_1(b) \neq \emptyset$.
$\underline{\mathcal{R}_1(a) \neq \emptyset}$: By VP, $\forall a_i \in \mathcal{R}_1(a)$, $b_1 \succ^{\sigma}_{\text{AF}} a_i$ because $\mathcal{R}_1(b_1) = \emptyset$. So, by QP, we have $a \succ^{\sigma}_{\text{AF}} b$.

(10) Let $AF^{\star} = AF \cup AF^{\gamma} \cup P_-(\gamma(a))$ be an argumentation framework such that $AF^{\gamma} = \gamma(AF)$. In $AF^{\star}$, $\gamma(a)$ has one more attack branch (and so one more direct attacker) than a ($\mathcal{R}_1(a) < \mathcal{R}_1(\gamma(a))$). As σ satisfies CP, so $a \succ^{\sigma}_{AF^{\star}} \gamma(a)$ in agreement with +AB.

□

Proposition 16. The properties that are satisfied by each ranking-based semantics (the other properties are not satisfied by the corresponding ranking-based semantics):

- The categoriser-based ranking semantics (Cat) satisfies Abs, In, VP, DP, CT, SCT, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.
- The discussion-based semantics (Dbs) satisfies Abs, In, VP, DP, CT, SCT, CP, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.
- The burden-based semantics (Bbs) satisfies Abs, In, VP, DP, CT, SCT, CP, DDP, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.

- Let $\alpha \in]0, +\infty[$. The α-burden-based semantics (α-Bbs) satisfies Abs, In, VP, DP, CT, SCT, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.
- The fuzzy labeling (FL) satisfies Abs, In, CT, QP, Tot, NaE, OE and AvsFD.
- Let $\alpha \in]0, 1[$. The counting semantics (CS) satisfies Abs, VP, DP, CT, SCT, ↑AB, ↑DB, +AB, Tot, NaE, AE and OE.
- Tuples-based semantics (Tuples) satisfies Abs, In, VP, +DB, ↑AB, ↑DB, +AB, NaE, AE, OE and AvsFD.
- The ranking-based semantics M&T satisfies Abs, In, VP, +AB, SC, Tot, NaE and AvsFD.
- The iterated graded defense semantics (IGD) satisfies Abs, In, VP, +AB, and NaE.

Proof.

Categoriser-based ranking semantics

The results concerning the properties Abstraction (Abs), Independence (In), Void Precedence (VP), Defense Precedence (DP), (Strict) Counter-Transitivity ((S)CT), Cardinality Precedence (CP), Quality Precedence (QP) and Distributed-Defense Precedence (DDP) can be found in [Pu *et al.* 2014].

Properties satisfied

(OE) OE is implied by CT which is satisfied.

(NaE) NaE is implied by OE which is satisfied.

(AE) According to the definition of the categoriser function, the categoriser value of an argument is computed from the categoriser values of its direct attackers which depend themselves of the categoriser values of their direct attackers and so on. So the only arguments which directly or indirectly impact a given argument x are the attacker and the defender of x ($x \cup \mathcal{R}_+(x) \cup \mathcal{R}_-(x)$), *i.e.* the arguments in its ancestors' graph.
Pu et al. [Pu *et al.* 2014, Theorem 1] show that for every argumentation framework there always exists a unique categoriser valuation, which means that two AFs with the same topology assign the same value to their arguments (and so have the same ranking). So if two arguments x and y have the same ancestors' graph (and it is the case because there exists an isomorphism between $Anc(x)$ and $Anc(y)$) then $Cat(x) = Cat(y)$ and so $x \simeq^{\text{Cat}} y$, in agreement with the property.

(Tot) The categoriser-based ranking semantics guarantees a comparison between all the arguments because all arguments have a score between 0 and 1 which is a totally ordered set of real number and the Theorem 1 [Pu *et al.* 2014] ensure the existence of a result. So all pairs of arguments can be compared.

(+AB,↑AB,↑DB) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that an isomorphism γ exists such that $AF = \gamma(AF')$. Let $a \in \mathcal{A}$ and its image $\gamma(a) \in \mathcal{A}'$

be two arguments such that $\mathcal{R}_1(a) = \{a_1, \ldots, a_n\}$ and $\mathcal{R}_1(\gamma(a)) = \{\gamma(a_1), \ldots, \gamma(a_n)\}$. As the semantics satisfies Argument Equivalence, each argument and its image are equally acceptable and so have the same score: $\forall x \in \mathcal{A}, Cat(x) = Cat(\gamma(x))$.

$\boxed{\text{+AB}}$ Let us add an attack branch to $\gamma(a)$ where the argument b is the direct attacker of $\gamma(a)$ belonging to the new attack branch. If a is not attacked, then $\gamma(a)$ is now attacked by b so according to the property VP which is satisfied, we have $a \succ^{\text{Cat}} \gamma(a)$. If a is attacked then the result is the same. Indeed, the score of b is strictly positive ($Cat(b) > 0$) because the function $f(x) = \frac{1}{1+x}$ cannot be equal to 0, so we have:

$$Cat(a_1) + \cdots + Cat(a_n) + 0 < Cat(\gamma(a_1)) + \cdots + Cat(\gamma(a_n)) + Cat(b)$$
$$\frac{1}{1 + Cat(a_1) + \cdots + Cat(a_n)} > \frac{1}{1 + Cat(\gamma(a_1)) + \cdots + Cat(\gamma(a_n)) + Cat(b)}$$
$$Cat(a) > Cat(\gamma(a))$$

Consequently, we have $a \succ^{\text{Cat}} \gamma(a)$ in agreement with the property.

$\boxed{\uparrow\text{AB}}$ Let us suppose $\exists b \in \mathcal{B}_-(a)$, $b \notin \mathcal{B}_+(a)$ and consider a branch from b to a with a length of $n \in 2\mathbb{N} + 1$: $p = \langle b, b_{n-1}, \ldots, b_2, a_1, a \rangle$. Let us now add a defense branch to the non-attacked argument $\gamma(b)$. As the property VP is satisfied, the score of $\gamma(b)$ which is now attacked becomes lower than the score of b (so $b \succ^{\text{Cat}} \gamma(b)$). Combining with the fact that SCT is satisfied, then $\gamma(b_{n-1}) \succ^{\text{Cat}} b_{n-1}$. With the same reasoning, we obtain $b_{n-2} \succ^{\text{Cat}} \gamma(b_{n-2})$ and so on until that $\gamma(a) \succ^{\text{Cat}} a$, in agreement with the property.

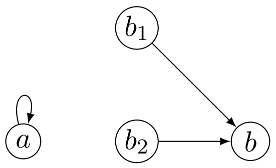
$\boxed{\uparrow\text{DB}}$ The reasoning is similar to the proof of ↑AB.

$\boxed{\text{Counter-examples}}$

(+DB) Incompatible with SCT which is satisfied.

($\oplus$DB) Incompatible with VP which is satisfied.

(SC) To show that the categoriser-based ranking semantics does not satisfy the property Self-Contradiction (SC), consider the argumentation framework AF from Figure B.2.

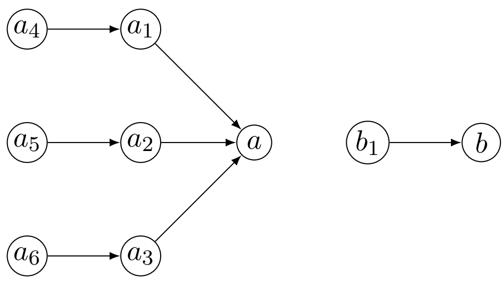


$Cat(b_1) = Cat(b_2) = 1$
$Cat(b) \approx 0.33$
$Cat(a) \approx 0.62$

$b_1 \simeq^{\text{Cat}} b_2 \succ^{\text{Cat}} a \succ^{\text{Cat}} b$

Figure B.2 – The categoriser-based ranking semantics falsifies the property SC

The property says that b should be ranked higher than a because a attacks itself while b does not attack itself. But, using the categoriser-based ranking semantics, a is ranked higher than b, contradicting the property.



$Cat(a_4) = Cat(a_5) = Cat(a_6) = Cat(b_1) = 1$
$Cat(a_1) = Cat(a_2) = Cat(a_3) = Cat(b) = 0.5$
$Cat(a) = 0.4$

$$a_4 \simeq^{\text{Cat}} a_5 \simeq^{\text{Cat}} a_6 \simeq^{\text{Cat}} b_1 \succ^{\text{Cat}} a_1 \simeq^{\text{Cat}} a_2 \simeq^{\text{Cat}} a_3 \simeq^{\text{Cat}} b \succ^{\text{Cat}} a$$

Figure B.3 – The categoriser-based ranking semantics falsifies the property AvsFD

(AvsFD) To show that the categoriser-based ranking semantics does not satisfy the property Attack vs Full Defense (AvsFD), consider the argumentation framework AF from Figure B.3.

The property says that a should be strictly more acceptable than b because a has only defense branches while b has exactly one direct attacker and no defense branch. But in using the categoriser-based ranking semantics, b is strictly more acceptable than a, contradicting the property.

Discussion-based semantics

The results concerning the properties Abstraction (Abs), Independence (In), Void Precedence (VP), Defense Precedence (DP), (Strict) Counter-Transitivity ((S)CT), Cardinality Precedence (CP), Quality Precedence (QP) and Distributed-Defense Precedence (DDP) can be found in [AMGOUD & BEN-NAIM 2013].

Properties satisfied

(OE) OE is implied by CT which is satisfied.

(NaE) NaE is implied by OE which is satisfied.

(AE) It is clear that, following the definition, the discussion count of an argument only depends on the attackers and the defenders of this argument, and so only on the arguments in its ancestors' graph. So if two arguments x and y have the same ancestors' graph (and it is the case because there exists an isomorphism between $Anc(x)$ and $Anc(y)$) then it is obvious to say that $\forall i \in \mathbb{N} \setminus \{0\}$, $|\mathcal{R}_i(x)| = |\mathcal{R}_i(y)|$. Consequently, $Dis(x) = Dis(y)$ which implies that $x \simeq^{\text{Dbs}} y$, in agreement with the property.

(Tot) The discussion-based ranking semantics guarantees a comparison between all the arguments because $\succeq^{\text{Dbs}}$ is total [AMGOUD & BEN-NAIM 2013, Definition 2].

(+AB) +AB is implied by CP which is satisfied.

(↑AB,↑DB) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that an isomorphism γ exists such that $AF = \gamma(AF')$. As the semantics satisfies Argument Equivalence, each argument and its image are equally acceptable and so have the same score: $\forall x \in \mathcal{A}, Dis(x) = Dis(\gamma(x))$ (*i.e.* $\forall i > 0$, $Dis_i(x) = Dis_i(\gamma(x))$).

↑AB Let us suppose $\exists b \in \mathcal{B}_-(a)$, $b \notin \mathcal{B}_+(a)$ and consider a branch from b to a with a length of $n \in 2\mathbb{N}+1$. Let us now add a defense branch to the non-attacked argument $\gamma(b)$. So, $\forall i \leq n$, $Dis_i(a) = Dis_i(\gamma(a))$ but during the step $n+1$, $\gamma(a)$ has now one additional defender ($|\mathcal{R}_{n+1}(\gamma(a))| > |\mathcal{R}_{n+1}(a)|$) so $Dis_{n+1}(\gamma(a)) = -|\mathcal{R}_{n+1}(\gamma(a))| < -|\mathcal{R}_{n+1}(a)| = Dis_{n+1}(a)$. Consequently, $Dis(a) \succ_{lex} Dis_{n+1}(\gamma(a))$ implies that $\gamma(a) \succ^{\text{Dbs}} a$, in agreement with the property.

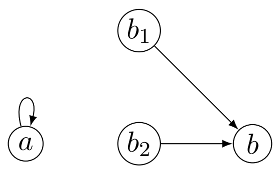
↑DB The reasoning is similar to the proof of ↑AB except that the length of the branch from b to a is $n \in \mathbb{N}$. So $Dis_{n+1}(\gamma(a)) = |\mathcal{R}_{n+1}(\gamma(a))| > |\mathcal{R}_{n+1}(a)| = Dis_{n+1}(a)$ which implies that $Dis(\gamma(a)) \succ_{lex} Dis_{n+1}(a)$ and $a \succ^{\text{Dbs}} \gamma(a)$, in agreement with the property.

Counter-examples

(+DB) Incompatible with SCT which is satisfied.

(⊕DB) Incompatible with VP which is satisfied.

(SC) To show that the discussion-based semantics does not satisfy the property Self-Contradiction (SC), consider the argumentation framework AF from Figure B.4.



$Dis(b_1) = Dis(b_2) = \langle 0, 0, 0, 0, \ldots \rangle$
$Dis(a) = \langle 1, -1, 1, -1, \ldots \rangle$
$Dis(b) = \langle 2, 0, 0, 0, \ldots \rangle$

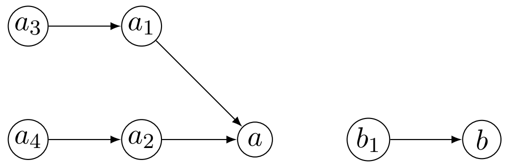
$b_1 \simeq^{\text{Dbs}} b_2 \succ^{\text{Dbs}} a \succ^{\text{Dbs}} b$

Figure B.4 – The discussion-based semantics falsifies the property SC

The property says that b should be strictly more acceptable than a because a attacks itself while b does not attack itself. But, using the discussion-based semantics, a is strictly more acceptable than b, contradicting the property.

(AvsFD) To show that the discussion-based semantics does not satisfy the property Attack vs Full Defense (AvsFD), consider the argumentation framework AF from Figure B.5.

The property says that a should be strictly more acceptable than b because a has only defense branches while b has exactly one direct attacker and no defense branch. But in using the discussion-based semantics, b is strictly more acceptable than a, contradicting the property.



$Dis(a_3) = Dis(a_4) = Dis(b_1) = \langle 0, 0, 0 \rangle$
$Dis(a_1) = Dis(a_2) = Dis(b) = \langle 1, 0, 0 \rangle$
$Dis(a) = \langle 2, -2, 0 \rangle$

$$a_3 \simeq^{\text{Dbs}} a_4 \simeq^{\text{Dbs}} b_1 \succ^{\text{Dbs}} a_1 \simeq^{\text{Dbs}} a_2 \simeq^{\text{Dbs}} b \succ^{\text{Dbs}} a$$

Figure B.5 – The discussion-based semantics falsifies the property AvsFD

Burden-based semantics

The results concerning the properties Abstraction (Abs), Independence (In), Void Precedence (VP), Defense Precedence (DP), (Strict) Counter-Transitivity ((S)CT), Cardinality Precedence (CP), Quality Precedence (QP) and Distributed-Defense Precedence (DDP) can be found in [AMGOUD & BEN-NAIM 2013].

Properties satisfied

(OE) OE is implied by CT which is satisfied.

(NaE) NaE is implied by OE which is satisfied.

(AE) According to its definition, the burden number of an argument is computed from the burden number of its direct attackers which depend themselves of the burden number of their direct attackers and so on. So the only arguments which directly or indirectly impact a given argument x are the attacker and the defender of x ($x \cup \mathcal{R}_+(x) \cup \mathcal{R}_-(x)$), *i.e.* the arguments in its ancestors' graph. So if two arguments x and y have the same ancestors' graph (and it is the case because there exists an isomorphism between $Anc(x)$ and $Anc(y)$) then $\forall i \in \mathbb{N}, Bur_i(x) = Bur_i(y)$. Indeed, it is obviously true when $i = 0$ (see the definition), when $i = 1$ because they have the same number of direct attackers, when $i = 2$ because their direct attackers are attacked by the same number of arguments and so on. Consequently, $x \simeq^{\text{Bbs}} y$, in agreement with the property.

(Tot) The burden-based ranking semantics guarantees a comparison between all the arguments because $\succeq^{\text{Bbs}}$ is total [AMGOUD & BEN-NAIM 2013, Definition 2].

(+AB) +AB is implied by CP which is satisfied.

($\uparrow$AB,$\uparrow$DB) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that an isomorphism γ exists such that $AF = \gamma(AF')$. As the semantics satisfies Argument Equivalence, each argument and its image are equally acceptable and so have the same burden vector: $\forall x \in \mathcal{A}, Bur(x) = Bur(\gamma(x))$ (*i.e.* $\forall i \geq 0$, $Bur_i(x) = Bur_i(\gamma(x))$).

$\uparrow$AB Let us suppose $\exists b \in \mathcal{B}_-(a)$, $b \notin \mathcal{B}_+(a)$ and consider a branch from b to a with a length of $n \in 2\mathbb{N} + 1$: $p = \langle b, b_{n-1}, \ldots, b_2, a_1, a \rangle$. Let us now add a defense branch to the non-attacked argument $\gamma(b)$. As the property VP is satisfied, $\gamma(b)$ which is now attacked becomes less acceptable than b which is non-attacked ($b \succ^{\text{Bbs}} \gamma(b)$). Combining with the fact that SCT is satisfied, then $\gamma(b_{n-1}) \succ^{\text{Bbs}} b_{n-1}$. With the same reasoning, $b_{n-2} \succ^{\text{Bbs}} \gamma(b_{n-2})$ and so on until that $\gamma(a) \succ^{\text{Bbs}} a$, in agreement with the property.

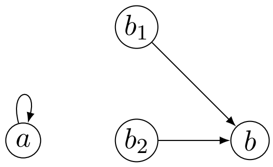
↑DB The same reasoning as the proof of ↑AB can be used.

Counter-examples

(+DB) Incompatible with SCT which is satisfied.

(⊕DB) Incompatible with VP which is satisfied.

(SC) To show that the burden-based semantics does not satisfy the property Self-Contradiction (SC), consider the argumentation framework AF from Figure B.6.



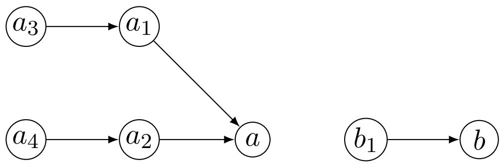
$Bur(b_1) = Bur(b_2) = \langle 1, 1, 1, 1, \ldots \rangle$
$Bur(a) = \langle 2, 1.5, 1.666, 1.6, \ldots \rangle$
$Bur(b) = \langle 3, 3, 3, 3, \ldots \rangle$

$b_1 \simeq^{\text{Bbs}} b_2 \succ^{\text{Bbs}} a \succ^{\text{Bbs}} b$

Figure B.6 – The burden-based semantics falsifies the property SC

The property says that b should be strictly more acceptable than a because a attacks itself while b does not attack itself. But, using the burden-based semantics, a is strictly more acceptable than b, contradicting the property.

(AvsFD) To show that the burden-based semantics does not satisfy the property Attack vs Full Defense (AvsFD), consider the argumentation framework AF from Figure B.7.



$Bur(a_3) = Bur(a_4) = Bur(b_1) = \langle 1, 1, 1 \rangle$
$Bur(a_1) = Bur(a_2) = Bur(b) = \langle 2, 2, 2 \rangle$
$Bur(a) = \langle 3, 2, 2 \rangle$

$a_3 \simeq^{\text{Bbs}} a_4 \simeq^{\text{Bbs}} b_1 \succ^{\text{Bbs}} a_1 \simeq^{\text{Bbs}} a_2 \simeq^{\text{Bbs}} b \succ^{\text{Bbs}} a$

Figure B.7 – The burden-based semantics falsifies the property AvsFD

The property says that a should be strictly more acceptable than b because a has only defense branches while b has exactly one direct attacker and no defense branch. But in using the burden-based semantics, b is strictly more acceptable than a, contradicting the property.

α-burden-based semantics

The results concerning the properties Abstraction (Abs), Independence (In), Void Precedence (VP), Defense Precedence (DP), (Strict) Counter-Transitivity ((S)CT), Cardinality Precedence (CP), Quality Precedence (QP) and Distributed-Defense Precedence (DDP) can be found in [AMGOUD *et al.* 2016].

Properties satisfied

(OE) OE is implied by CT which is satisfied.

(NaE) NaE is implied by OE which is satisfied.

(AE) According to its definition, the burden number of an argument is computed from the burden number of its direct attackers which depend themselves of the burden number of their direct attackers and so on. So the only arguments which directly or indirectly impact a given argument x are the attacker and the defender of x ($x \cup \mathcal{R}_+(x) \cup \mathcal{R}_-(x)$), *i.e.* the arguments in its ancestors' graph. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $a, b \in \mathcal{A}$ such that there exists an isomorphism γ between $Anc_{AF}(a)$ and $Anc_{AF}(b)$. In [AMGOUD *et al.* 2016, Theorem 1], the authors ensure that the solution of a system of equations exists and is unique. So it is clear that the systems of equations from $Anc_{AF}(a)$ and from $Anc_{AF}(b)$ are similar (because there exists an isomorphism) and have the same solution. Consequently, $\forall a' \in Anc_{AF}(a)$ then $s_\alpha(a') = s_\alpha(\gamma(a'))$. It is particularly true for a and b, so $s_\alpha(a) = s_\alpha(b)$ which means that $a \simeq^{\alpha\text{-Bbs}}_{\text{AF}} b$.

(Tot) The α-burden-based ranking semantics guarantees a comparison between all the arguments because $\succeq^{\alpha\text{-Bbs}}$ is total [AMGOUD *et al.* 2016, Definition 2].

(+AB,↑AB,↑DB) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that an isomorphism γ exists such that $AF = \gamma(AF')$. Let $a \in \mathcal{A}$ and its image $\gamma(a) \in \mathcal{A}'$ be two arguments such that $\mathcal{R}_1(a) = \{a_1, \ldots, a_n\}$ and $\mathcal{R}_1(\gamma(a)) = \{\gamma(a_1), \ldots, \gamma(a_n)\}$. As the semantics satisfies Argument Equivalence, each argument and its image are equally acceptable and so have the same score: $\forall x \in \mathcal{A}, s_\alpha(x) = s_\alpha(\gamma(x))$.

+AB Let us add an attack branch to $\gamma(a)$ where the argument b is the direct attacker of $\gamma(a)$ belonging to the new attack branch. If a is not attacked, then $\gamma(a)$ is now attacked by b so according to the property VP which is satisfied, we have $a \succ^{\alpha\text{-Bbs}} \gamma(a)$. If a is attacked then the result is the same. Indeed, the score of b is strictly positive ($s_\alpha(b) > 0$) because the domain of s_α is $[1, \infty[$, so we have $\forall \alpha \in]0, \infty[$:

$$\frac{1}{(s_\alpha(a_1))^\alpha} + \cdots + \frac{1}{(s_\alpha(a_n))^\alpha} + 0 < \frac{1}{(s_\alpha(\gamma(a_1)))^\alpha} + \cdots + \frac{1}{(s_\alpha(\gamma(a_n)))^\alpha} + \frac{1}{(s_\alpha(b))^\alpha}$$

$$\left(\frac{1}{(s_\alpha(a_1))^\alpha} + \cdots + \frac{1}{(s_\alpha(a_n))^\alpha}\right)^{1/\alpha} < \left(\frac{1}{(s_\alpha(\gamma(a_1)))^\alpha} + \cdots + \frac{1}{(s_\alpha(\gamma(a_n)))^\alpha} + \frac{1}{(s_\alpha(b))^\alpha}\right)^{1/\alpha}$$

$$1 + \left(\frac{1}{(s_\alpha(a_1))^\alpha} + \cdots + \frac{1}{(s_\alpha(a_n))^\alpha}\right)^{1/\alpha} < 1 + \left(\frac{1}{(s_\alpha(\gamma(a_1)))^\alpha} + \cdots + \frac{1}{(s_\alpha(\gamma(a_n)))^\alpha} + \frac{1}{(s_\alpha(b))^\alpha}\right)^{1/\alpha}$$

$$s_\alpha(a) < s_\alpha(\gamma(a))$$

Consequently, according to the definition of the semantics, we have $a \succ^{\alpha\text{-Bbs}} \gamma(a)$ in agreement with the property.

↑AB Let us suppose $\exists b \in \mathcal{B}_-(a)$, $b \notin \mathcal{B}_+(a)$ and consider a branch from b to a with a length of $n \in 2\mathbb{N}+1$: $p = \langle b, b_{n-1}, \dots, b_2, a_1, a \rangle$. Let us now add a defense branch to the non-attacked argument $\gamma(b)$. As the property VP is satisfied, the score of $\gamma(b)$, which is now attacked, becomes greater than the score of b (so $b \succ^{\alpha\text{-Bbs}} \gamma(b)$). Combining with the fact that SCT is satisfied, then $\gamma(b_{n-1}) \succ^{\alpha\text{-Bbs}} b_{n-1}$. With the same reasoning, we obtain $b_{n-2} \succ^{\alpha\text{-Bbs}} \gamma(b_{n-2})$ and so on until that $\gamma(a) \succ^{\alpha\text{-Bbs}} a$, in agreement with the property.

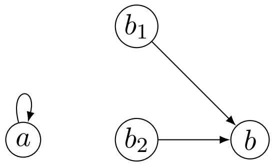
↑DB The reasoning is similar to the proof of ↑AB.

Counter-examples

(+DB) Incompatible with SCT which is satisfied.

(⊕DB) Incompatible with VP which is satisfied.

(SC) To show that the α-burden-based semantics does not satisfy the property Self-Contradiction (SC), consider the argumentation framework AF from Figure B.8.



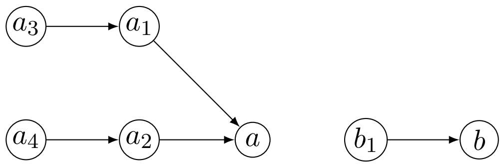
$s_\alpha(b_1) = s_\alpha(b_2) = 1$
$s_\alpha(a) \simeq 1.618$
$s_\alpha(b) = 3$

$$b_1 \simeq^{\alpha\text{-Bbs}} b_2 \succ^{\alpha\text{-Bbs}} a \succ^{\alpha\text{-Bbs}} b$$

Figure B.8 – The α-burden-based semantics falsifies the property SC

The property says that b should be strictly more acceptable than a because a attacks itself while b does not attack itself. But, using the α-burden-based semantics, a is strictly more acceptable than b, contradicting the property.

(AvsFD) To show that the α-burden-based semantics does not satisfy the property Attack vs Full Defense (AvsFD), consider the argumentation framework AF from Figure B.9.



If $\alpha = 1$:
$s_\alpha(a_3) = s_\alpha(a_4) = s_\alpha(b_1) = 1$
$s_\alpha(a_1) = s_\alpha(a_2) = s_\alpha(b) = 2$
$s_\alpha(a) = 2$

$$a_3 \simeq^{\alpha\text{-Bbs}} a_4 \simeq^{\alpha\text{-Bbs}} b_1 \succ^{\alpha\text{-Bbs}} a_1 \simeq^{\alpha\text{-Bbs}} a_2 \simeq^{\alpha\text{-Bbs}} b \simeq^{\alpha\text{-Bbs}} a$$

Figure B.9 – The α-burden-based semantics falsifies the property AvsFD

The property says that a should be strictly more acceptable than b because a has only defense branches while b has exactly one direct attacker and no defense branch. But in using the α-burden-based semantics with $\alpha = 1$, a and b are equally acceptable, contradicting the property.

Fuzzy labeling

The results concerning the property (Tot) can be found in [DA COSTA PEREIRA *et al.* 2011][Definition 9].

Properties satisfied

(Abs) The nature of an argument is not used in the computation of its score. Only the attack relation is needed (see definition 2.3.14).

(In) Obvious because, according to the definition 2.3.14, an argument only depends on the score of its direct attacker, which depends on the score of its direct attackers and so on.

(QP) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$. Suppose that $\exists y' \in \mathcal{R}_1(y)$ such that $\forall x' \in \mathcal{R}_1(x)$, $y' \succ^{\text{FL}}_{\text{AF}} x'$ which implies that $f(y') > f(x')$. So $\max_{y' \in \mathcal{R}_1(y)} f(y') > \max_{x' \in \mathcal{R}_1(x)} f(x')$. According to the definition 2.3.14, we obtain $f(y) < f(x)$ and thus $x \succ^{\text{FL}}_{\text{AF}} y$.

(CT) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$. Suppose that it exists an injective function f from $\mathcal{R}_1(y)$ to $\mathcal{R}_1(x)$ such that $\forall z \in \mathcal{R}_1(y)$, $f(z) \succeq^{\text{FL}}_{z}$ which implies that $f(f(z)) \geq f(z)$. So $\max_{f(z) \in \mathcal{R}_1(x)} f(f(z)) \geq \max_{z \in \mathcal{R}_1(y)} f(z)$. According to the definition 2.3.14, we obtain $f(y) \geq f(x)$ and thus $y \succeq^{\text{FL}}_{\text{AF}} x$.

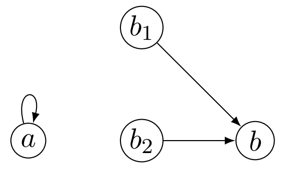
(OE) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$. Suppose that there exists a bijective function f from $\mathcal{R}_1(x)$ to $\mathcal{R}_1(y)$ such that $\forall z \in \mathcal{R}_1(x)$, $z \simeq^{\sigma}_{\text{AF}} f(z)$ which implies that $f(z) = f(f(z))$. So $\max_{f(z) \in \mathcal{R}_1(y)} f(f(z)) = \max_{z \in \mathcal{R}_1(x)} f(z)$. According to the definition 2.3.14, we obtain $f(y) = f(x)$ and thus $x \simeq^{\text{FL}}_{\text{AF}} y$.

(NaE) NaE is implied by OE which is satisfied.

(AvsFD) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework where $x \in \mathcal{A}$ is attacked by only one non-attacked argument and $y \in \mathcal{A}$ is only defended (i.e. without attack branch). The non-attacked arguments have a score of 1 which implies that all the arguments directly attacked by them have a score of 0, especially x: $f(x) = 0$. If y is non-attacked then $f(y) = 1$. So, $f(y) = 1 > 0 = f(x)$ which implies that $y \succ^{\text{FL}} x$. If y is attacked then it is clear that $f(y) > 0$ because it cannot have a direct attacker with a score of 1 (otherwise one of its branch will be an attack branch but it is not the case because it has only defense branches). So $f(y) > f(x)$ which implies that $y \succ^{\text{FL}} x$, in agreement with the property.

Counter-examples

(SC) To show that the fuzzy labeling does not satisfy the property Self-Contradiction (SC), consider the argumentation framework AF from Figure B.10.



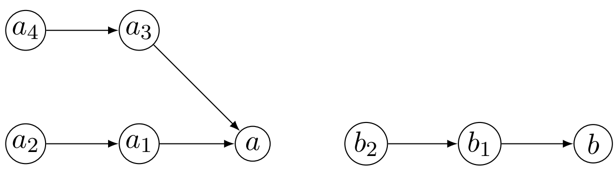
$f(b_1) = f(b_2) = 1$
$f(b) = 0$
$f(a) = 0.5$

$b_1 \simeq^{\text{FL}} b_2 \succ^{\text{FL}} a \succ^{\text{FL}} b$

Figure B.10 – The fuzzy labeling falsifies the property SC

The property says that b should be strictly more acceptable than a because a attacks itself while b does not attack itself. But, one can remark that a is strictly more acceptable than b, contradicting the property.

(SCT) To show that the fuzzy labeling does not satisfy the property Strict Counter-Transitivity (SCT), consider the argumentation framework AF from Figure B.11.



$f(a_2) = f(a_4) = f(b_2) = f(a) = f(b) = 1$
$f(a_1) = f(a_3) = f(b_1) = 0$

$$a_2 \simeq^{\text{FL}} a_4 \simeq^{\text{FL}} b_2 \simeq^{\text{FL}} a \simeq^{\text{FL}} b \succ^{\text{FL}} a_1 \simeq^{\text{FL}} a_3 \simeq^{\text{FL}} b_1$$

Figure B.11 – The fuzzy labeling falsifies the properties SCT, VP, +DB and $\oplus$DB.

The property says that b should be strictly more acceptable than a because it exists an injective function f from $\mathcal{R}_1(b)$ to $\mathcal{R}_1(a)$ such that $\forall b' \in \mathcal{R}_1(b)$, $f(b') \succeq b'$ ($a_1 \succeq^{\text{FL}} b_1$ because $a_1 \simeq^{\text{FL}} b_1$) so $\mathcal{R}_1(a) \geq_S^{\text{FL}} \mathcal{R}_1(b)$ and $\mathcal{R}_1(a) > \mathcal{R}_1(b)$. But the semantics considers that a and b are equally acceptable, contradicting the property.

(VP) To show that the fuzzy labeling does not satisfy the property Void Precedence (VP), consider the argumentation framework AF from Figure B.11.
Void Precedence says that a_2 should be strictly more acceptable than a because a_2 is a not attacked ($\mathcal{R}_1(a_2) = \emptyset$) while a is attacked ($\mathcal{R}_1(a) \neq \emptyset$). But the semantics considers that a_2 and a are equally acceptable, contradicting the property.

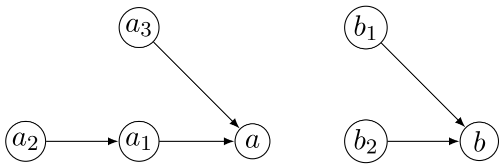
(+DB, $\oplus$DB) To show that FL does not satisfy the property Addition of Defense Branch (+DB) and the property Strict addition of Defense Branch ($\oplus$DB), consider the argumentation framework AF from Figure B.11.
Both properties say that a should be strictly more acceptable than b because a has one defense branch while b has no defense branch. But the semantics considers that a and b are equally acceptable, contradicting both properties.

(DP) To show that the fuzzy labeling does not satisfy the property Defense Precedence (DP), consider the argumentation framework AF from Figure B.12.

Defense Precedence says that a should be strictly more acceptable than b because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$ and $|\mathcal{R}_2(a)| = 1 > 0 = |\mathcal{R}_2(b)|$. But the semantics considers that a and b are equally acceptable, contradicting the property.

(CP) To show that the fuzzy labeling does not satisfy the property Cardinality Precedence (CP), consider the argumentation framework AF from Figure B.13.

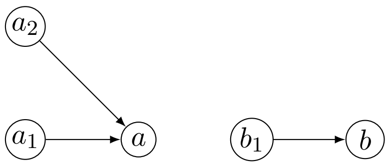
The property says that b should be strictly more acceptable than a because $|\mathcal{R}_1(b)| = 1 < 2 = |\mathcal{R}_1(a)|$. But the semantics considers that a and b are equally acceptable, contradicting the property.



$$f(a_2) = f(a_3) = f(b_1) = f(b_2) = 1$$
$$f(a_1) = f(a) = f(b) = 0$$

$$a_2 \simeq^{\text{FL}} a_3 \simeq^{\text{FL}} b_1 \simeq^{\text{FL}} b_2 \succ^{\text{FL}} a_1 \simeq^{\text{FL}} a \simeq^{\text{FL}} b$$

Figure B.12 – The fuzzy labeling falsifies the property DP



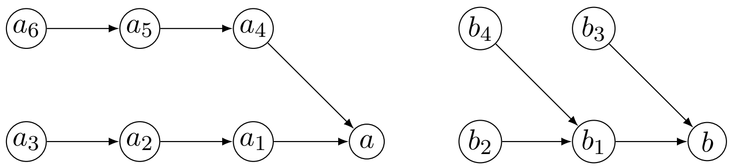
$$f(a_1) = f(a_2) = f(b_1) = 1$$
$$f(a) = f(b) = 0$$

$$a_1 \simeq^{\text{FL}} a_2 \simeq^{\text{FL}} b_1 \succ^{\text{FL}} a \simeq^{\text{FL}} b$$

Figure B.13 – The fuzzy labeling falsifies the properties CP and +AB

(+AB) To show that FL does not satisfy the property Addition of Attack Branch (+AB), consider the argumentation framework AF from Figure B.13.
The property says that b should be strictly more acceptable than a because a has two attack branches while b has one attack branch. But the semantics considers that a and b are equally acceptable, contradicting the property.

(DDP) To show that FL does not satisfy the property Distributed-Defense Precedence (DDP), consider the argumentation framework AF from Figure B.14.



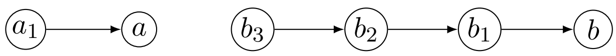
$$f(a_1) = f(a_3) = f(a_4) = f(a_6) = f(b_2) = f(b_3) = f(b_4) = 1$$
$$f(a_2) = f(a_5) = f(b_1) = f(a) = f(b) = 0$$

$$a_3 \simeq^{\text{FL}} a_6 \simeq^{\text{FL}} b_2 \simeq^{\text{FL}} b_3 \simeq^{\text{FL}} b_4 \simeq^{\text{FL}} a_1 \simeq^{\text{FL}} a_4 \succ^{\text{FL}} a_2 \simeq^{\text{FL}} a_5 \simeq^{\text{FL}} b \simeq^{\text{FL}} a \simeq^{\text{FL}} b_1$$

Figure B.14 – The fuzzy labeling falsifies the property DDP

The property says that a should be strictly more acceptable than b because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$, $|\mathcal{R}_2(a)| = |\mathcal{R}_2(b)| = 2$ and the defense of a is simple and distributed while the defense of b is simple but not distributed. But the semantics considers that a and b are equally acceptable, contradicting the property.

(↑AB) To show that FL does not satisfy the property Increase of Attack branch (↑AB), consider the argumentation framework AF from Figure B.15.



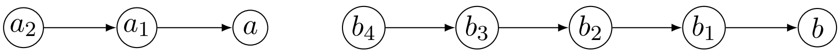
$$f(a_1) = f(b_3) = f(b_1) = 1$$
$$f(b_2) = f(a) = f(b) = 0$$

$$a_1 \simeq^{\text{FL}} b_3 \simeq^{\text{FL}} b_1 \succ^{\text{FL}} b_2 \simeq^{\text{FL}} a \simeq^{\text{FL}} b$$

Figure B.15 – The fuzzy labeling falsifies the property ↑AB

The property says that b should be strictly more acceptable than a because the length of the attack branch of b is greater than the length of the attack branch of a. But in using the semantics, we can see that a and b are equally acceptable, contradicting the property.

(↑DB) To show that FL does not satisfy the property Increase of Defense branch (↑DB), consider the argumentation framework AF from Figure B.16.



$$f(a_2) = f(b_4) = f(b_2) = f(a) = f(b) = 1$$
$$f(a_1) = f(b_3) = f(b_1) = 0$$

$$a_2 \simeq^{\text{FL}} b_4 \simeq^{\text{FL}} b_2 \simeq^{\text{FL}} a \simeq^{\text{FL}} b \succ^{\text{FL}} a_1 \simeq^{\text{FL}} b_3 \simeq^{\text{FL}} b_1$$

Figure B.16 – The fuzzy labeling falsifies the property ↑DB

The property says that a should be strictly more acceptable than b because the length of the defense branch of b is greater than the length of the defense branch of a. But in using the semantics, we can see that a and b are equally acceptable, contradicting the property.

Counting semantics

The results concerning the properties Abstraction (Abs), Independence (In), Void Precedence (VP), Defense Precedence (DP), (Strict) Counter-Transitivity ((S)CT), Cardinality Precedence (CP), Quality Precedence (QP) and Distributed-Defense Precedence (DDP) can be found in [Pu *et al.* 2015b].

Properties satisfied

(OE) OE is implied by CT which is satisfied.

(NaE) NaE is implied by OE which is satisfied.

(AE) Use the matrix approach ensures that the score of an argument only depends on its attackers and defenders. Thus, the score of an argument is the same in focusing on its ancestors' graph as in the full argumentation framework with the same normalization factor. If there exists an isomorphism between the ancestors' graph of x and y then the topology of the argumentation frameworks $Anc(x)$ and $Anc(y)$ are identical, which implies that the adjacency matrix of $Anc(x)$ and $Anc(y)$ are identical too. Pu et al. guarantee that the counting model always exists and is unique so the counting model is the same for

$Anc(x)$ and $Anc(y)$. It is particularly true for x and y so $w(x) = w(y)$ which implies that $x \simeq^{\text{CS}} y$.

(Tot) According [Pu *et al.* 2015c, Theorem 1], the counting model ranges the strength value of each argument into the interval $[0, 1]$ and converges to a unique solution. The interval $[0, 1]$ is a totally ordered set of real number so all its values can be compared using $\geq$. Consequently, all the arguments in an argumentation framework can be compared too.

(+AB,↑AB,↑DB) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that an isomorphism γ exists such that $AF = \gamma(AF')$. Let $a \in \mathcal{A}$ and its image $\gamma(a) \in \mathcal{A}'$ be two arguments. As the property AE is satisfied, we can say that $\forall x \in \mathcal{A}$ then $x \simeq^{\text{CS}}_{\text{AF}} \gamma(x)$ because if there exists an isomorphism between AF and AF', it is also true for the subgraphs of AF and more precisely for the ancestors' graph of each argument. Consequently, $a \simeq^{\text{CS}}_{\text{AF} \cup \text{AF}'} \gamma(a)$ and $\mathcal{R}_1(\gamma(a)) \geq^{\text{CS}}_S \mathcal{R}_1(a)$.

+AB If we add an attack branch $P^-(\gamma(a))$ to $\gamma(a)$ then we still have $\mathcal{R}_1(\gamma(a)) \geq^{\text{CS}}_S \mathcal{R}_1(a)$ but $|\mathcal{R}_1(\gamma(a))| > |\mathcal{R}_1(a)|$ which implies that $\mathcal{R}_1(\gamma(a)) >^{\text{CS}}_S \mathcal{R}_1(a)$. As the property SCT is satisfied then $a \succ^{\text{CS}}_{\text{AF}*} \gamma(a)$, in agreement with the property.

↑AB Let us suppose $\exists b \in \mathcal{B}_-(a)$, $b \notin \mathcal{B}_+(a)$ and consider a branch from b to a with a length of $n \in 2\mathbb{N} + 1$: $p = \langle b, b_{n-1}, \ldots, b_2, a_1, a \rangle$. Let us now add a defense branch to the non-attacked argument $\gamma(b)$. As the property VP is satisfied, the score of $\gamma(b)$ which is now attacked becomes lower than the score of b (so $b \succ^{\text{CS}}_{\text{AF}*} \gamma(b)$). Combining with the fact that SCT is satisfied, then $\gamma(b_{n-1}) \succ^{\text{CS}}_{\text{AF}*} b_{n-1}$. With the same reasoning, we obtain $b_{n-2} \succ^{\text{CS}}_{\text{AF}*} \gamma(b_{n-2})$ and so on until that $\gamma(a) \succ^{\text{CS}}_{\text{AF}*} a$, in agreement with the property.

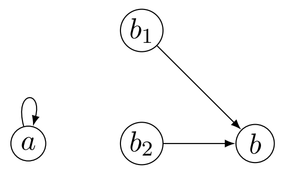
↑DB The reasoning is similar to the proof of ↑AB.

Counter-examples

(+DB) Incompatible with SCT which is satisfied.

(⊕DB) Incompatible with VP which is satisfied.

(SC) To show that the counting semantics does not satisfy the property Self-Contradiction (SC), consider the argumentation framework AF from Figure B.17.



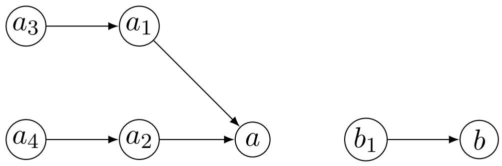
If $\alpha = 0.5$:
$w(b_1) = w(b_2) = 1$
$w(b) = 0.5$
$w(a) \approx 0.78$

$b_1 \simeq^{\text{CS}} b_2 \succ^{\text{CS}} a \succ^{\text{CS}} b$

Figure B.17 – The counting semantics falsifies the property SC

The property says that b should be strictly more acceptable than a because a attacks itself while b does not attack itself. But, the counting semantics considers that a is strictly more acceptable than b, contradicting the property.

(AvsFD) To show that the counting semantics does not satisfy the property Attack vs Full Defense (AvsFD), consider the argumentation framework AF from Figure B.18.



If $\alpha = 0.5$
$w(a_3) = w(a_4) = w(b_1) = 1$
$w(a_1) = w(a_2) = w(b) = 0.75$
$w(a) = 0.5625$

$$a_3 \simeq^{\text{CS}} a_4 \simeq^{\text{CS}} b_1 \succ^{\text{CS}} a_1 \simeq^{\text{CS}} a_2 \simeq^{\text{CS}} b \succ^{\text{CS}} a$$

Figure B.18 – The counting semantics falsifies the property AvsFD

The property says that a should be strictly more acceptable than b because a has only defense branches while b has exactly one direct attacker and no defense branch. But the counting semantics considers that b is strictly more acceptable than a, contradicting the property.

Tuples-based semantics

The results concerning the properties Void Precedence (VP), Addition of Defense Branch (+DB), Addition of Attack Branch (+AB), Increase of Attack branch (↑AB), Increase of Defense branch (↑DB) and Total (Tot) can be found in [CAYROL & LAGASQUIE-SCHIEX 2005b].

Properties satisfied

(Abs) If there exists an isomorphism γ between two argumentation frameworks AF and AF' then they have the same structure. So for each argument, its image has exactly the same number of branch with the same length (see the definition of isomorphism) which implies that an argument and it image have the same tupled value: $\forall x \in Arg(AF), v(x) = v(\gamma(x))$. Thus, for all arguments $a, b \in Arg(AF)$, following Algorithm 1, as $v(a) = v(\gamma(a))$ and $v(b) = v(\gamma(b))$, it is clear that if $a \succeq^{\text{T}}_{\text{AF}} b$ then we also have $\gamma(a) \succeq^{\text{T}}_{\text{AF}'} \gamma(b)$.

(In) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $AF' \in cc(AF)$ with $a, b \in Arg(AF')$ and $c \notin Arg(AF')$. The tupled value of an argument is only computed from its attack and defense roots which necessarily belongs to the same component as the arguments (because there exists a path between both). So as there exists no path between a (respectively b) and c, then c cannot be a root of a (respectively b). Consequently, it cannot influence the ranking between a and b.

(OE) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $a, b \in \mathcal{A}$ such that there exists a bijective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(b)$ such that $\forall z \in \mathcal{R}_1(a), z \simeq^{\sigma}_{\text{AF}} f(z)$. According to Algorithm 1, two arguments are equally acceptable if they have the same tupled value so $\forall z \in \mathcal{R}_1(a), v(z) = v(f(z))$. The original definition [CAYROL & LAGASQUIE-SCHIEX 2005b, Definition 10] compute the tupled value of each argument on the basis of the tupled value of its direct attackers. So, as the tupled values of the direct attackers of a and b are the same, then they obtain the same tupled value ($v(a) = v(b)$) which implies that $a \simeq^{\text{T}} b$.

(NaE) NaE is implied by OE which is satisfied.

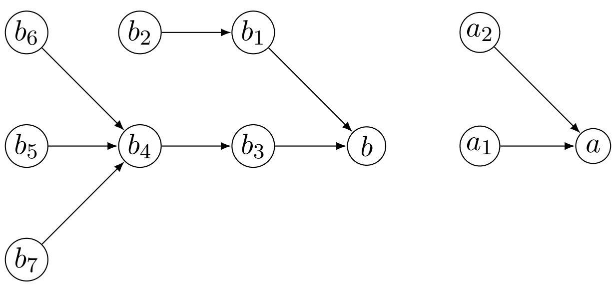
(AE) Obvious because if a and b have the same ancestors' graph, then they have the same number of branches with a length of 1, the same number of branches with a length of 2 and so on (see how the construction of the ancestors' graph is done). So $v_p(a) = v_p(b)$ and $v_i(a) = v_i(b) \Rightarrow v(a) = v(b) \Rightarrow a \simeq^{\mathrm{T}}_{AF} b$.

(AvsFD) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $a, b \in \mathcal{A}$ such that $|\mathcal{B}_{-}(b)| = 0$, $|\mathcal{R}_1(a)| = 1$ and $|\mathcal{R}_2(a)| = 0$. So $|v_i(a)| = 1$ and $|v_p(a)| = 0$ because $v(a) = [(), (1)]$. Concerning b, its tupled-value respects the following criteria: $|v_i(b)| = 0$ and $|v_p(b)| > 0$. Consequently, we have $|v_i(a)| > |v_i(b)|$ and $|v_p(a)| < |v_p(b)|$ and, according to Algorithm 1, b is strictly more acceptable than a ($b \succ^{\mathrm{T}} a$).

Counter-examples

($\oplus$DB) Incompatible with VP which is satisfied.

(DP) To show that the tuples-based semantics does not satisfy the property Defense Precedence (DP), consider the argumentation framework AF from Figure B.19.



$v(a_1) = v(a_2) = v(b_2) = v(b_5) = v(b_6) = v(b_7) = [0^{\infty}, ()]$
$v(b_1) = [(), (1)]$
$v(a) = [(), (1, 1)]$
$v(b_4) = [(), (1, 1, 1)]$
$v(b_3) = [(2, 2, 2), ()]$
$v(b) = [(2), (3, 3, 3)]$

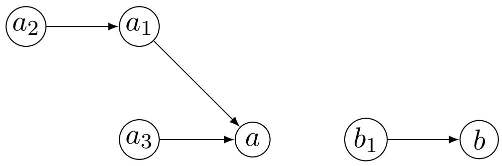
$$a_1 \simeq^{\mathrm{T}} a_2 \simeq^{\mathrm{T}} b_2 \simeq^{\mathrm{T}} b_5 \simeq^{\mathrm{T}} b_6 \simeq^{\mathrm{T}} b_7 \succ^{\mathrm{T}} b_3 \succ^{\mathrm{T}} b_1 \succ^{\mathrm{T}} a \succ^{\mathrm{T}} b_4$$
$$a_1 \simeq^{\mathrm{T}} a_2 \simeq^{\mathrm{T}} b_2 \simeq^{\mathrm{T}} b_5 \simeq^{\mathrm{T}} b_6 \simeq^{\mathrm{T}} b_7 \succ^{\mathrm{T}} b_3 \succ^{\mathrm{T}} b \succ^{\mathrm{T}} b_4$$
$$a \not\succeq^{\mathrm{T}} b \text{ and } b \not\succeq^{\mathrm{T}} a$$
$$b_1 \not\succeq^{\mathrm{T}} b \text{ and } b \not\succeq^{\mathrm{T}} b_1$$

Figure B.19 – The tuples-based semantics falsifies the properties DP and QP

The property says that b should be strictly more acceptable than a because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$ and $|\mathcal{R}_2(a)| = 0 < 2 = |\mathcal{R}_2(b)|$. But we obtain two incomparable tuples : $v(a) = [(), (1, 1)]$ and $v(b) = [(2), (3, 3, 3)]$ (see Algorithm 1 Case 7 [Cayrol & Lagasquie-Schiex 2005b] : $|v_i(a)| < |v_i(b)|$ and $|v_p(a)| < |v_p(b)|$), so a and b are incomparable, contradicting the property.

(QP) To show that the tuples-based semantics does not satisfy the property Quality Precedence (QP), consider the argumentation framework AF from Figure B.19.
The property says that b should be more acceptable that a because $a_2 \succeq^{\mathrm{T}} b_1$ and $a_2 \succeq^{\mathrm{T}} b_3$ (a_1 can also be used). But, using the tuples-based semantics, a and b are incomparable, contradicting the property.

(CT) To show that the tuples-based semantics does not satisfy the property Counter-Transitivity (CT), consider the argumentation framework AF from Figure B.20.



$v(a_2) = v(a_3) = v(b_1) = [0^\infty, ()]$
$v(a_1) = v(b) = [(), (1)]$
$v(a) = [(2), (1, 1)]$

$$a_2 \simeq^{\mathrm{T}} a_3 \simeq^{\mathrm{T}} b_1 \succ^{\mathrm{T}} a \succ^{\mathrm{T}} a_1 \simeq^{\mathrm{T}} b$$

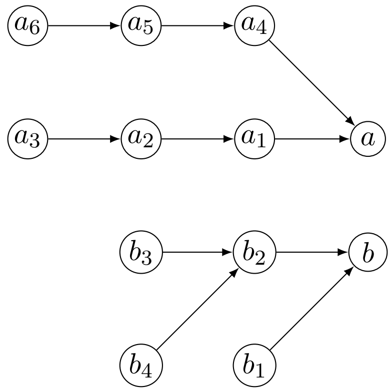
Figure B.20 – The tuples-based semantics falsifies the properties CT, SCT and CP

The definition says that b should be at least as acceptable as a because it exists an injective function f from $\mathcal{R}_1(b)$ to $\mathcal{R}_1(a)$ such that $\forall b' \in \mathcal{R}_1(b)$, $f(b') \succeq^{\mathrm{T}} b'$. Indeed, we have $\mathcal{R}_1(b) = \{b_1\}$ and $\mathcal{R}_1(a) = \{a_1, a_3\}$ and $a_3 \succeq^{\mathrm{T}} b_1$. But, using the tuples-based semantics, a is strictly more acceptable than b, contradicting the property.

(SCT) To show that the tuples-based semantics does not satisfy the property Strict Counter-Transitivity (SCT), consider the argumentation framework AF from Figure B.20.
The property says that b should be strictly more acceptable than a because it exists an injective function f from $\mathcal{R}_1(b)$ to $\mathcal{R}_1(a)$ such that $\forall b' \in \mathcal{R}_1(b)$, $f(b') \succeq^{\mathrm{T}} b'$ and $|\mathcal{R}_1(b)| < |\mathcal{R}_1(a)|$. Indeed, we have $\mathcal{R}_1(b) = \{b_1\}$ and $\mathcal{R}_1(a) = \{a_1, a_3\}$ (so $|\mathcal{R}_1(b)| = 1 < 2 = |\mathcal{R}_1(a)|$) where $a_3 \succeq^{\mathrm{T}} b_1$. But, using the tuples-based semantics, a is strictly more acceptable than b, contradicting the property.

(CP) To show that the tuples-based semantics does not satisfy the property Cardinality Precedence (CP), consider the argumentation framework AF from Figure B.20.
The property says that b should be strictly more acceptable than a because $|\mathcal{R}_1(a)| = 2 > 1 = |\mathcal{R}_1(b)|$. But, using the tuples-based semantics, a is strictly more acceptable than b, contradicting the property.

(DDP) To show that the tuples-based semantics does not satisfy the property Distributed-Defense Precedence (DDP), consider the argumentation framework AF from Figure B.21.



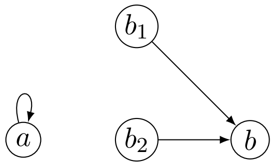
$v(a_3) = v(a_6) = v(b_1) = v(b_3) = v(b_4) = [0^\infty, ()]$
$v(a_2) = v(a_5) = [(), (1)]$
$v(a_1) = v(a_4) = [(2), ()]$
$v(b_2) = [(), (1, 1)]$
$v(a) = [(), (3, 3)]$
$v(b) = [(2, 2), (1)]$

$$b \succ^{\mathrm{T}} a$$

Figure B.21 – The tuples-based semantics falsifies the property DDP

The property says that a should be strictly more acceptable than b because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$, $|\mathcal{R}_2(a)| = |\mathcal{R}_2(b)| = 2$ and the defense of a is simple and distributed while the defense of b is simple but not distributed. But, using the tuples-based semantics, b is strictly more acceptable than a, contradicting the property.

(SC) To show that the tuples-based semantics does not satisfy the property Self-Contradiction (SC), consider the argumentation framework AF from Figure B.22.



$v(b_1) = v(b_2) = [0^\infty, ()]$
$v(b) = [(), (1,1)]$
$v(a) = [(2,4,6,\dots),(1,3,5,\dots)]$

$b_1 \simeq^{\text{Bbs}} b_2 \succ^{\text{Bbs}} b$
$b_1 \simeq^{\text{Bbs}} b_2 \succ^{\text{Bbs}} a$
$a \not\succeq^{\text{T}} b$ and $b \not\succeq^{\text{T}} a$

Figure B.22 – The tuples-based semantics falsifies the property SC

The property says that b should be strictly more acceptable than a because a attacks itself while b does not attack itself. But, with the tuples-based semantics, a and b are incomparable, which contradicts the property.

Ranking-based semantics M&T

The results concerning the properties Independence (In), Void Precedence (VP) and Self-Contradiction (SC) can be found in [MATT & TONI 2008].

Properties satisfied

(Abs) The nature of an argument is not used in the computation of its score. Only the attack relation is needed.

(Tot) This semantics guarantees a comparison between all the arguments because the score of an argument $a \in \mathcal{A}$ is such that $s(a) \in [0,1]$ which is a totally ordered set. In [MATT & TONI 2008], they ensure the existence of a value v thanks to the minimax theorem (von Neumann 1928).

(NaE) NaE is implies by AE which is satisfied.

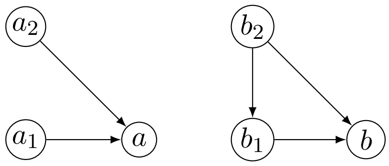
(AvsFD) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $a, b \in \mathcal{A}$ be two arguments where b is attacked by a non-attacked argument and a only have defense branches and no attack branch. The value of the zero-sum game for b is $v(b) = 0.25$. For a, we can say that this argument belongs to the set of stable (and so admissible) extension because it has only defense branches. Moreover, the Proposition 5 [MATT & TONI 2008] says that if an argument belong to a stable extension (which is unique here then its strength is greater or equal to $\frac{1}{2}$ so $v(a) \geq \frac{1}{2}$. Consequently we have $v(b) = 0.25 < 0.5 \leq v(a)$ and so $a \succ b$.

Counter-examples

(AE) Incompatible with SC which is satisfied.

(⊕DB) Incompatible with VP which is satisfied.

(DP) To show that the ranking-based semantics M&T does not satisfy the property Defense Precedence (DP), consider the argumentation framework AF from Figure B.23.



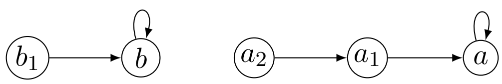
$s(a_1) = s(a_2) = s(b_2) = 1$
$s(b_1) = 0.25$
$s(a) = s(b) \simeq 0.16$

$$a_1 \simeq^{\text{MT}} a_2 \simeq^{\text{MT}} b_2 \succ^{\text{MT}} b_1 \succ^{\text{MT}} a \simeq^{\text{MT}} b$$

Figure B.23 – The ranking-based semantics M&T falsifies the property DP

The property says that b should be strictly more acceptable than a because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$ but $|\mathcal{R}_2(a)| = 0 < 1 = |\mathcal{R}_2(b)|$. But in using the semantics, we can see that a and b are equally acceptable, contradicting the property.

(QP) To show that the ranking-based semantics M&T does not satisfy the property Quality Precedence (QP), consider the argumentation framework AF from Figure B.24. The property says that a should be



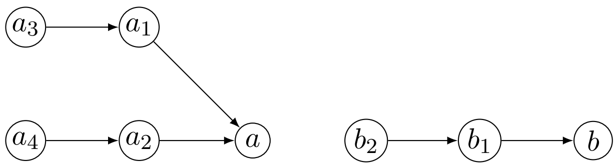
$s(a_2) = s(b_1) = 1$
$s(a_1) = 0.25$
$s(a) = s(b) = 0$

$$a_2 \simeq^{\text{MT}} b_1 \succ^{\text{MT}} a_1 \succ^{\text{MT}} a \simeq^{\text{MT}} b$$

Figure B.24 – The ranking-based semantics M&T falsifies the property QP

strictly more acceptable than b because $b_1 \succ^{\text{MT}} a_1$ and $b_1 \succ^{\text{MT}} a$. But the semantics considers that a and b are equally acceptable, contradicting the property.

(CP) To show that the ranking-based semantics M&T does not satisfy the property Cardinality Precedence (CP), consider the argumentation framework AF from Figure B.25.



$s(a_3) = s(a_4) = s(b_1) = 1$
$s(a_1) = s(a_2) = 0.25$
$s(a) = s(b) = 0.5$

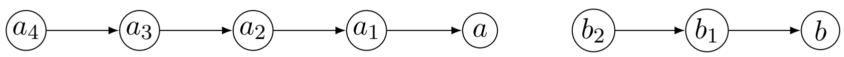
$$a_3 \simeq^{\text{MT}} a_4 \simeq^{\text{MT}} b_2 \succ^{\text{MT}} a \simeq^{\text{MT}} b \succ^{\text{MT}} a_1 \simeq^{\text{MT}} a_2 \simeq^{\text{MT}} b_1$$

Figure B.25 – The ranking-based semantics M&T falsifies the properties CP and +DB

The property says that b should be strictly more acceptable than a because $|\mathcal{R}_1(a)| = 2 > 1 = |\mathcal{R}_1(b)|$. But in using the semantics, we can see that a and b are equally acceptable, contradicting the property.

(+DB) To show that the ranking-based semantics M&T does not satisfy the property Addition of Defense Branch (+DB), consider the argumentation framework AF from Figure B.25.
The property says that a should be strictly more acceptable than b because a has one more defense branch that b. But in using the semantics, we can see that a and b are equally acceptable, contradicting the property.

(↑DB) To show that the ranking-based semantics M&T does not satisfy the property Increase of Defense branch (↑DB), consider the argumentation framework AF from Figure B.26.



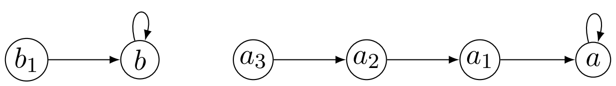
$$s(a_4) = s(b_2) = 1,\ s(a_1) \simeq 0.386,\ s(a_3) = s(b_1) = 0.25 \text{ and } s(a) = s(b) = s(a_2) = 0.5$$

$$a_3 \simeq^{\text{MT}} b_1 \succ^{\text{MT}} a \simeq^{\text{MT}} b \simeq^{\text{MT}} a_2 \succ^{\text{MT}} a_1 \succ^{\text{MT}} a_3 \simeq^{\text{MT}} b_1$$

Figure B.26 – The ranking-based semantics M&T falsifies the property ↑DB

The property says that b should be strictly more acceptable than a because a has one defense branch longer than the defense branch of b. But in using the semantics, we can see that a and b are equally acceptable, contradicting the property.

(↑AB) To show that the ranking-based semantics M&T does not satisfy the property Increase of Attack branch (↑AB), consider the argumentation framework AF from Figure B.27.



$s(b_1) = s(a_3) = 1$
$s(a_2) = 0.25$
$s(a_1) = 0.5$
$s(a) = s(b) = 0$

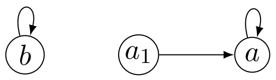
$$b_1 \simeq^{\text{MT}} a_3 \succ^{\text{MT}} a_1 \succ^{\text{MT}} a_2 \succ^{\text{MT}} a \simeq^{\text{MT}} b$$

Figure B.27 – The ranking-based semantics M&T falsifies the property ↑AB

The property says that a should be strictly more acceptable than b because a has one attack branch longer than the attack branch of b. But in using the semantics, we can see that a and b are equally acceptable, contradicting the property.

(+AB) To show that the ranking-based semantics M&T does not satisfy the property Addition of Attack Branch (+AB), consider the argumentation framework AF from Figure B.28.

The property says that b should be strictly more acceptable than a because a has one more attack branch that b. But in using the semantics, we can see that a and b are equally acceptable, contradicting



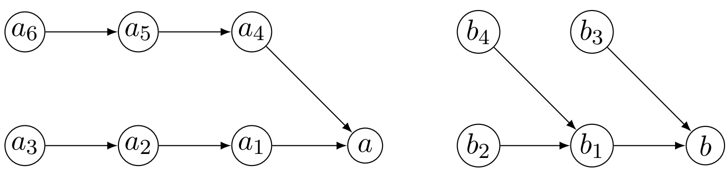
$$s(a_1) = 1$$
$$s(a) = s(b) = 0$$

$$a_1 \succ^{\text{MT}} a \simeq^{\text{MT}} b$$

Figure B.28 – The ranking-based semantics M&T falsifies the property +AB

the property.

(DDP) To show that the ranking-based semantics M&T does not satisfy the property Distributed Defense Precedence (DDP), consider the argumentation framework AF from Figure B.29.



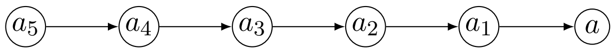
$$s(a_3) = s(a_6) = 1$$
$$s(b_2) = s(b_3) = s(b_4) = 1$$
$$s(a_2) = s(a_5) = s(b) = 0.25$$
$$s(a_1) = s(a_4) = 0.5$$
$$s(b_1) = s(a) = 0.1667$$

$$a_3 \simeq^{\text{MT}} a_6 \simeq^{\text{MT}} b_2 \simeq^{\text{MT}} b_3 \simeq^{\text{MT}} b_4 \succ^{\text{MT}} a_1 \simeq^{\text{MT}} a_4 \succ^{\text{MT}} b \simeq^{\text{MT}} a_2 \simeq^{\text{MT}} a_5 \succ^{\text{MT}} a \simeq^{\text{MT}} b_1$$

Figure B.29 – The ranking-based semantics M&T falsifies the property Distributed-Defense Precedence

The definition says that a should be strictly more acceptable than b because they have the same number of direct attackers ($|\mathcal{R}_1^S(a)| = |\mathcal{R}_1^S(b)| = 2$) and the same number of direct defenders ($|\mathcal{R}_2^S(a)| = |\mathcal{R}_2^S(b)| = 2$) but the defense of a is simple and distributed whereas the defense of b is simple and not distributed. But in using the semantics, b is strictly more acceptable than a, contradicting the property.

(OE) To show that the ranking-based semantics M&T does not satisfy the property Ordinal Equivalence (OE), consider the argumentation framework AF from Figure B.30.



$$s(a_5) = 1$$
$$s(a_4) = 0.25$$
$$s(a_3) = s(a_1) = 0.5$$
$$s(a_2) \simeq 0.3863$$
$$s(a) = 0.425$$

$$a_5 \succ^{\text{MT}} a_1 \simeq^{\text{MT}} a_3 \succ^{\text{MT}} a \succ^{\text{MT}} a_2 \succ^{\text{MT}} a_4$$

Figure B.30 – The ranking-based semantics M&T falsifies the property OE

The property says that a and a_2 should be equally acceptable because there exists a bijective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(a_2)$ such that $\forall b \in \mathcal{R}_1(a)$, $f(b) \simeq^{\text{MT}} b$ ($a_3 \simeq^{\text{MT}} a_1$). But in using the semantics, a

is strictly more acceptable than a_2, contradicting the property.

(CT) To show that the ranking-based semantics M&T does not satisfy the property Counter-Transitivity (CT), consider the argumentation framework AF from Figure B.30.
The property says that a_2 should be at least as acceptable as a because there exists an injective function f from $\mathcal{R}_1(a_2)$ to $\mathcal{R}_1(a)$ such that $\forall b \in \mathcal{R}_1(a_2)$, $f(b) \succeq b$ ($a_1 \simeq^{\text{MT}} a_3$ which implies $a_1 \succeq^{\text{MT}} a_3$) so $\mathcal{R}_1(a) \geq_S^{\text{MT}} \mathcal{R}_1(a_2)$. But the semantics considers that a is strictly more acceptable than a_2, contradicting the property.

(SCT) To show that the ranking-based semantics M&T does not satisfy the property Strict Counter-Transitivity (SCT), consider the argumentation framework AF from Figure B.30.
The property says that a_3 should be strictly more acceptable than a_1 because it exists an injective function f from $\mathcal{R}_1(a_3)$ to $\mathcal{R}_1(a_1)$ such that $\forall b \in \mathcal{R}_1(a_3)$, $f(b) \succeq b$ ($a_2 \succeq^{\text{MT}} a_4$) and especially $a_2 \succ^{\text{MT}} a_4$ (so $\mathcal{R}_1(a_1) >_S^{\text{MT}} \mathcal{R}_1(a_3)$). But the semantics considers that a and b are equally acceptable, contradicting the property.

Iterated Graded Defense semantics

The results concerning the property Total (Tot) can be found in [GROSSI & MODGIL 2015].

Properties satisfied

(Abs) The nature of an argument is not used in the computation of its score. Only the attack relation is needed.

(In) According to the definition 2.3.16, the graded defense is computed from the direct attackers and the direct defenders of arguments. So if there exists no path between two arguments then it cannot influence the acceptability to the other.

(VP) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x, y \in \mathcal{A}$ such that x is not attacked and y is attacked. As x is not attacked, then $\forall i \in \mathbb{N}^*$ and $\forall \mathcal{X} \in \mathcal{A}$, $x \in \mathfrak{N}_i(\mathcal{X})$. So $\forall m, n \in \mathbb{N}^*$, $x \in d^*_{\substack{m\\n}}(\mathcal{X})$. Let us now find two values of m and n such that y does appears in $d^*_{\substack{m\\n}}(\mathcal{X})$. If $m' = 1$ and $n' = |\mathcal{A}| + 1$ then for any $\mathcal{X} \in \mathcal{A}$, $d_{\substack{m'\\n'}}(\mathcal{X})$ selects the arguments such that none of their direct attackers are directly attacked at most $n - 1$ times. It is clear that only the non-attacked arguments respect this condition so $x \in d^*_{\substack{m'\\n'}}(\mathcal{X})$ and $y \notin d^*_{\substack{m'\\n'}}(\mathcal{X})$ because it is attacked, so $x \succ^{\text{IGD}} y$.

(NaE) NaE are obviously satisfied because for any non-attacked argument x, then $\forall i \in \mathbb{N}^*$ and $\forall \mathcal{X} \in \mathcal{A}$, $x \in \mathfrak{N}_i(\mathcal{X})$. So $\forall m, n \in \mathbb{N}^*$, $x \in d^*_{\substack{m\\n}}(\mathcal{X})$. Consequently, the non-attacked arguments always belong to the same graded defense which means that, according to the definition of the IGD semantics, they are equally acceptable.

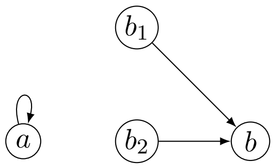
(+AB) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that an isomorphism γ exists such that $AF = \gamma(AF')$. Let $a \in \mathcal{A}$ and its image $\gamma(a) \in \mathcal{A}'$ be two arguments. As the semantics satisfies Argument Equivalence, a and $\gamma(a)$ are equally acceptable, so $\forall m, n \in \mathbb{N}^*$ and $\forall \mathcal{X} \in \mathcal{A} \cup \mathcal{A}'$, $a \in d^*_{\substack{m\\n}}(\mathcal{X})$ if and only if $\gamma(a) \in d^*_{\substack{m\\n}}(\mathcal{X})$.
Let m' be the minimum value such that $a \in d^*_{\substack{m'\\2}}(\mathcal{X})$ and $\nexists m < m'$, $a \in d^*_{\substack{m\\2}}(\mathcal{X})$ (idem for $\gamma(a)$). This

result contains the arguments such that at most $m' - 1$ arguments are not attacked or are attacked once. Let us now add an attack branch to $\gamma(a)$ which implies that it has now one additional direct attacker which can be attacked (if the length of the branch is greater than 1) or not (if the length of the branch is 1). In both cases, $\gamma(a) \notin d^*_{\frac{m'}{2}}(\mathcal{X})$ because m' is the minimum value while $a \in d^*_{\frac{m'}{2}}(\mathcal{X})$. Moreover, the initial isomorphism between a and $\gamma(a)$ guarantees that $\forall \mathcal{X} \in Arg(AF^*)$, if $\gamma(a) \in d^*_{\frac{m}{n}}(\mathcal{X})$ then $a \in d^*_{\frac{m}{n}}(\mathcal{X})$ too. So according the definition, $a \succ^{\text{IGD}}_{\text{AF}^*} \gamma(a)$.

Counter-examples

($\oplus$DB) Incompatible with VP which is satisfied.

(SC) To show that the iterated graded defense semantics does not satisfy the property Self-Contradiction (SC), consider the argumentation framework AF from Figure B.31.

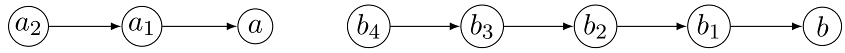


$b_1 \simeq^{\text{IGD}} b_2 \succ^{\text{IGD}} a \succ^{\text{IGD}} b$

Figure B.31 – The iterated graded defense semantics falsifies the property SC

The property says that b should be strictly more acceptable than a because a attacks itself while b does not attack itself. But, one can remark that a is strictly more acceptable than b, contradicting the property.

($\uparrow$DB) To show that the iterated graded defense semantics does not satisfy the property Increase of Defense branch ($\uparrow$DB), consider the argumentation framework AF from Figure B.32.



$a_2 \simeq^{\text{IGD}} b_4 \succ^{\text{IGD}} b_2 \simeq^{\text{IGD}} a \simeq^{\text{IGD}} b \succ^{\text{IGD}} a_1 \simeq^{\text{IGD}} b_1 \simeq^{\text{IGD}} b_3$

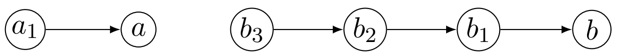
Figure B.32 – The iterated graded defense semantics falsifies the property $\uparrow$DB

The property says that a should be strictly more acceptable than b because the length of the defense branch of b is greater than the length of the defense branch of a. But in using the semantics, we can see that a and b are equally acceptable, contradicting the property.

($\uparrow$AB) To show that the iterated graded defense semantics does not satisfy the property Increase of Attack branch ($\uparrow$AB), consider the argumentation framework AF from Figure B.33.

The property says that b should be strictly more acceptable than a because the length of the attack branch of b is greater than the length of the attack branch of a. But in using the semantics, we can see that a and b are equally acceptable, contradicting the property.

(DP) To show that the iterated graded defense semantics does not satisfy the property Defense Precedence (DP), consider the argumentation framework AF from Figure B.33.



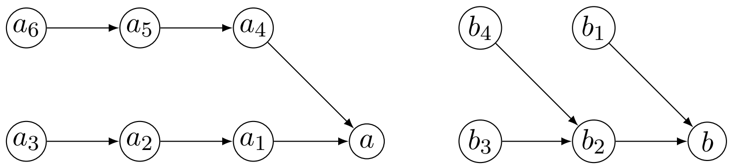
$a_1 \simeq^{\text{IGD}} b_3 \succ^{\text{IGD}} b_1 \succ^{\text{IGD}} b_2 \simeq^{\text{IGD}} a \simeq^{\text{IGD}} b$

Figure B.33 – The iterated graded defense semantics falsifies the properties ↑AB, DP and SCT

Defense Precedence says that b should be strictly more acceptable than a because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 1$ and $|\mathcal{R}_2(a)| = 0 < 1 = |\mathcal{R}_2(b)|$. But the semantics considers that a and b are equally acceptable, contradicting the property.

(SCT) To show that the iterated graded defense semantics does not satisfy the property Strict Counter-Transitivity (SCT), consider the argumentation framework AF from Figure B.33.
The property says that b should be strictly more acceptable than a because it exists an injective function f from $\mathcal{R}_1(b)$ to $\mathcal{R}_1(a)$ such that $\forall b' \in \mathcal{R}_1(b)$, $f(b') \succeq^{\text{IGD}} b'$ ($a_1 \succeq^{\text{IGD}} b_1$ because $a_1 \succ^{\text{IGD}} b_1$) so $\mathcal{R}_1(a) \geq_S^{\text{IGD}} \mathcal{R}_1(b)$. But $a_1 \succ^{\text{IGD}} b_1$ so $\mathcal{R}_1(a) >_S^{\text{IGD}} \mathcal{R}_1(b)$. But the semantics considers that a and b are equally acceptable, contradicting the property.

(DDP) To show that the iterated graded defense semantics does not satisfy the property Distributed-Defense Precedence (DDP), consider the argumentation framework AF from Figure B.34.



$b_1 \simeq^{\text{IGD}} b_3 \simeq^{\text{IGD}} b_4 \simeq^{\text{IGD}} a_3 \simeq^{\text{IGD}} a_6 \succ^{\text{IGD}} a_1 \simeq^{\text{IGD}} a_4 \succ^{\text{IGD}} a_1 \simeq^{\text{IGD}} a_5 \succ^{\text{IGD}} a \succ^{\text{IGD}} b \succ^{\text{IGD}} b_2$

Figure B.34 – The iterated graded defense semantics falsifies the property DDP

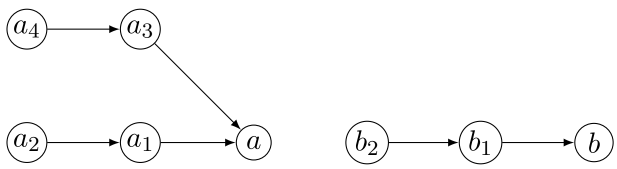
The property says that b should be strictly more acceptable than a because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$, $|\mathcal{R}_2(a)| = |\mathcal{R}_2(b)| = 2$ and the defense of b is simple and distributed while the defense of a is simple but not distributed. But the semantics considers that a is strictly more acceptable than b.

(QP) To show that the iterated graded defense semantics does not satisfy the property Quality Precedence (QP), consider the argumentation framework AF from Figure B.34.
The property says that b should be strictly more acceptable than a because $a_1 \succ^{\text{IGD}} b_1$ and $a_1 \succ^{\text{IGD}} b_4$. But the semantics considers that a is strictly more acceptable than b which contradicts the property.

(+DB) To show that the iterated graded defense semantics does not satisfy the property Addition of Defense Branch (+DB), consider the argumentation framework AF from Figure B.35.

The property says that a should be strictly more acceptable than b because a has two defense branches while b has only one defense branch. But the semantics considers that b is strictly more acceptable than b which contradicts the property.

(CP) To show that the iterated graded defense semantics does not satisfy the property Cardinality Precedence (CP), consider the argumentation framework AF from Figure B.36.



$$a_2 \simeq^{\text{IGD}} a_4 \simeq^{\text{IGD}} b_2 \succ^{\text{IGD}} a \succ^{\text{IGD}} b \succ^{\text{IGD}} a_1 \simeq^{\text{IGD}} a_3 \simeq^{\text{IGD}} b_1$$

Figure B.35 – The iterated graded defense semantics falsifies the property +DB.

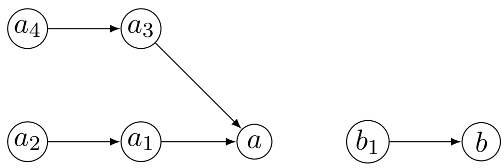


Figure B.36 – The iterated graded defense semantics falsifies the properties CP and AvsFD

The property says that b should be strictly more acceptable than a because $|\mathcal{R}_1(b)| = 1 < 2 = |\mathcal{R}_1(a)|$. But the semantics considers that a and b are incomparable because $a \in d^*_{\substack{1\\1}}(\emptyset)$ but $a \notin d^*_{\substack{2\\2}}(\emptyset)$ while $b \notin d^*_{\substack{1\\1}}(\emptyset)$ but $b \in d^*_{\substack{2\\2}}(\emptyset)$, which contradicts the property.

(AvsFD) To show that the iterated graded defense semantics does not satisfy the property Attack vs Full Defense (AvsFD), consider the argumentation framework AF from Figure B.36.
The property says that a should be strictly more acceptable than b because a has only defense branches while b has exactly one direct attacker and no defense branch. But the semantics considers that a and b are incomparable because $a \in d^*_{\substack{1\\1}}(\emptyset)$ but $a \notin d^*_{\substack{2\\2}}(\emptyset)$ while $b \notin d^*_{\substack{1\\1}}(\emptyset)$ but $b \in d^*_{\substack{2\\2}}(\emptyset)$, which contradicts the property.

(CT) To show that the iterated graded defense semantics does not satisfy the property Counter-Transitivity (CT), consider the argumentation framework AF from Figure B.36.
The property says that a should be at least as acceptable than b because it exists an injective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(b)$ such that $\forall a' \in \mathcal{R}_1(a)$, $f(a') \succeq a'$ ($b_1 \succ^{\text{IGD}} a_1$ and so $b_1 \succeq^{\text{IGD}} a_1$) so $\mathcal{R}_1(b) \geq_S^{\text{IGD}} \mathcal{R}_1(a)$. But the semantics considers that a and b are incomparable because $a \in d^*_{\substack{1\\1}}(\emptyset)$ but $a \notin d^*_{\substack{2\\2}}(\emptyset)$ while $b \notin d^*_{\substack{1\\1}}(\emptyset)$ but $b \in d^*_{\substack{2\\2}}(\emptyset)$, which contradicts the property.

□

Proposition 17. Let $\oplus \in \{M, S\}$ and $\epsilon \in]0, 1]$.

- The ranking-based semantics $Propa_{\epsilon}^{\epsilon,\oplus}$ satisfies Abs, In, VP, DP, ↑AB, ↑DB, +AB, NaE, Tot and AE.
- The ranking-based semantics $Propa_{1+\epsilon}^{\epsilon,\oplus}$ satisfies Abs, In, VP, DP, DDP, ↑AB, ↑DB, +AB, Tot, NaE, AE and AvsFD.
- The ranking-based semantics $Propa_{1\to\epsilon}^{\epsilon,\oplus}$ satisfies Abs, In, VP, DP, DDP, +DB, ↑AB, ↑DB, +AB, Tot, NaE, AE and AvsFD.

Proof.

Propa$_\epsilon$

Properties satisfied

(Abs) The nature of an argument is not used in the computation of its score. Only the attack relation is needed (see the definition of $Propa_\epsilon$ and the definition of the propagation principle).

(In) An argument a only receives the scores from its attackers and defenders. Thus, an argument b, such that there exists no path between a and b, cannot propagate its initial value to a and then cannot influence the propagation number of a.

(VP) Let $\oplus \in \{M, S\}$ and $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $a, b \in \mathcal{A}$ such that a is not attacked ($\mathcal{R}_1^{\oplus}(a) = \emptyset$) and b is attacked ($\mathcal{R}_1^{\oplus}(b) \neq \emptyset$).
<u>If $\epsilon < 1$:</u> During the first step ($i = 0$), the non-attacked arguments begin with a score of 1, in particular $P_0^{\epsilon,\oplus}(a) = 1$, and all the others begin with a score of ϵ, in particular $P_0^{\epsilon,\oplus}(b) = \epsilon$. At the end of the first step, $P_0^{\epsilon,\oplus}(a) > P_0^{\epsilon,\oplus}(b)$ so $a \succ^{P}_{\text{AF}} b$, in agreement with the property.
<u>If $\epsilon = 1$:</u> During the first step ($i = 0$), we have $P_0^{\epsilon,\oplus}(a) = P_0^{\epsilon,\oplus}(b) = 1$. So, a second step is needed to distinguish them. Indeed, a keeps its score of 1 because it is not attacked while the score of b decreases because it is directly attacked by at least one argument (we note n the number of direct attackers of b with $n > 0$). Consequently, $P_1^{\epsilon,\oplus}(b) = 1 - n < 1 = P_1^{\epsilon,\oplus}(a)$ implies that $a \succ^{P}_{\text{AF}} b$, in agreement with the property.

(DP) Let $\oplus \in \{M, S\}$ and $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $a, b \in \mathcal{A}$ such that a is not defended ($\mathcal{R}_2^{\oplus}(a) = \emptyset$), b is defended ($\mathcal{R}_2^{\oplus}(b) \neq \emptyset$) and $|\mathcal{R}_1^{\oplus}(a)| = |\mathcal{R}_1^{\oplus}(b)| = n > 0$.
During the step $i = 0$, both arguments have exactly the same score because they are attacked by at least one argument, so $P_0^{\epsilon,\oplus}(a) = P_0^{\epsilon,\oplus}(b) = \epsilon$.
<u>If $\epsilon < 1$:</u> During the step $i = 1$, a is directly attacked by n non-attacked arguments because it is not defended so $P_1^{\epsilon,\oplus}(a) = \epsilon - n$ while b is attacked by m non-attacked arguments and $n - m$ attacked arguments so $P_1^{\epsilon,\oplus}(b) = \epsilon - (m + (n - m)\epsilon)$. Consequently, as $\epsilon < 1$, $P_1^{\epsilon,\oplus}(a) = \epsilon - n < \epsilon - (m + (n - m)\epsilon) = P_1^{\epsilon,\oplus}(b)$ so $b \succ^{P}_{\text{AF}} a$, in agreement with the property.
<u>If $\epsilon = 1$:</u> During the step $i = 1$, $P_1^{\epsilon,\oplus}(a) = P_1^{\epsilon,\oplus}(b) = \epsilon - n$. So, a third step is needed to distinguish them. Indeed, when $i = 2$, b is defended by at least one argument so its score increases while the score of a stays the same because it has no defender, so $P_2^{\epsilon,\oplus}(b) > P_2^{\epsilon,\oplus}(a)$. Consequently, we obtain $b \succ^{P}_{AF} a$, in agreement with the property.

(AE) Obvious because two arguments with the same ancestors' graph receive exactly the same values from its attackers and defenders.

(NaE) NaE is implied by AE which is satisfied.

(Tot) It is clear that the computation of the propagation vectors is always guaranteed for each argument. And the lexicographical order always returns a result between two vectors which guarantees the comparison between two arguments.

(+AB,↑AB and ↑DB) Let $\oplus \in \{M, S\}$, $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that it exists an isomorphism γ with $AF = \gamma(AF')$. Let $a \in \mathcal{A}$ and its image $\gamma(a) \in \mathcal{A}'$ be two arguments. As $Propa_\epsilon$ satisfies the properties Argument Equivalence (AE), we can say that a and $\gamma(a)$, with the same ancestors' graph, have the same propagation vector: $\forall \epsilon \in]0,1], P^{\epsilon,\oplus}(a) = P^{\epsilon,\oplus}(\gamma(a))$.

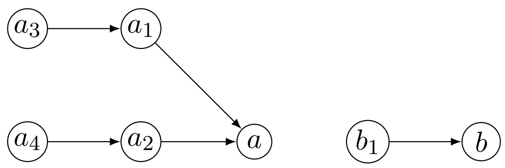
+AB Let us add an attack branch $P^-(\gamma(a))$ to $\gamma(a)$ with a length of $n \in 2\mathbb{N}+1$. If a is not attacked, then a becomes more acceptable than $\gamma(a)$ which is now attacked (see the property Void Precedence). Otherwise, when a is attacked by at least one argument, a and $\gamma(a)$ keep the same score during the first step : $P_0^{\epsilon,\oplus}(a) = P_0^{\epsilon,\oplus}(\gamma(a)) = \epsilon$. But during the second step, $\gamma(a)$ has now one more direct attackers than a, so the score of $\gamma(a)$ becomes lower than the score of a ($P_1^{\epsilon,\oplus}(\gamma(a)) = p_1 - 1$ if $n = 1$ or $P_1^{\epsilon,\oplus}(\gamma(a)) = p_1 - \epsilon$ if $n > 1$). Consequently, we obtain $P_1^{\epsilon,\oplus}(a) > P_1^{\epsilon,\oplus}(\gamma(a)) \Rightarrow a \succ^P_{\text{AF}^*} \gamma(a)$, in agreement with the property.

↑AB Let us suppose $\exists b \in \mathcal{B}_-(a)$, $b \notin \mathcal{B}_+(a)$ such that the length of the path from b to a is n. Let us now add a defense branch to the non-attacked argument $\gamma(b)$. Until the step $n-1$, there is no change in the values received by a and $\gamma(a)$ so they keep the same score: $\forall i \leq n-1, P_i^{\epsilon,\oplus}(a) = P_i^{\epsilon,\oplus}(\gamma(a))$. But during the step n, $\gamma(a)$ receives a negative value of ϵ from $\gamma(b)$ which is now attacked while a receives a negative value of 1 from b which is not attacked. So if $\epsilon < 1$ then $P_n^{\epsilon,\oplus}(a) < P_n^{\epsilon,\oplus}(\gamma(a)) \Rightarrow \gamma(a) \succ^P_{\text{AF}^*} a$, in agreement with the property. However, if $\epsilon = 1$ then a supplementary step is necessary to obtain a difference between a and $\gamma(a)$. Indeed, $\gamma(a)$ receives an additional positive value coming from the new direct attacker of $\gamma(b)$ so $P_{n+1}^{\epsilon,\oplus}(a) < P_{n+1}^{\epsilon,\oplus}(\gamma(a)) \Rightarrow \gamma(a) \succ^P_{\text{AF}^*} a$, in agreement with the property.

↑DB The proof is similar to ↑AB except that b is a defense root. So during the step n, $\gamma(a)$ receives a positive value of ϵ from $\gamma(b)$ which is now attacked while a receives a positive value of 1 from b which is not attacked. So, when $\epsilon < 1$, $P_n^{\epsilon,\oplus}(a) > P_n^{\epsilon,\oplus}(\gamma(a)) \Rightarrow a \succ^P_{\text{AF}^*} \gamma(a)$ and when $\epsilon = 1$, $P_{n+1}^{\epsilon,\oplus}(a) > P_{n+1}^{\epsilon,\oplus}(\gamma(a)) \Rightarrow a \succ^P_{\text{AF}^*} \gamma(a)$, in agreement with the property.

Counter-examples

(CP) To show that $Propa_\epsilon$ does not satisfy the property Cardinality Precedence (CP), consider the argumentation framework AF from Figure B.37.



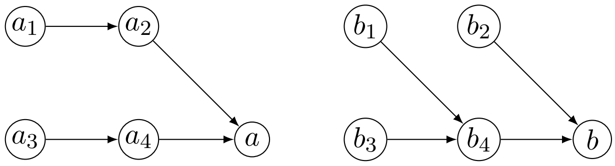
$P_i^{0.1,\oplus}$	a_3, a_4, b_1	a_1, a_2	a	b
0	1	0.1	0.1	0.1
1	1	-0.9	-0.1	-0.9
2	1	-0.9	1.9	-0.9

$$a_3 \simeq^P a_4 \simeq^P b_1 \succ^P a \succ^P a_1 \simeq^P a_2 \simeq^P b$$

Figure B.37 – $Propa_\epsilon$ falsifies the property CP

The property says that b should be strictly more acceptable than a because $|\mathcal{R}_1^\oplus(a)| = 2 > 1 = |\mathcal{R}_1^\oplus(b)|$. But, with the semantics $Propa_\epsilon^{\oplus,0.1}$, we can see that a is strictly more acceptable than b, contradicting the property.

(QP) To show that $Propa_\epsilon$ does not satisfy the property Quality Precedence (QP), consider the argumentation framework AF from Figure B.38.



$P_i^{1,\oplus}$	a_1, a_3, b_1, b_2, b_3	a_2, a_4	b_4	a	b
0	1	1	1	1	1
1	1	0	-1	-1	-1
2	1	0	-1	1	1

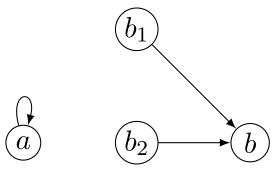
$$a_1 \simeq^P a_3 \simeq^P b_1 \simeq^P b_2 \simeq^P b_3 \succ^P a_2 \simeq^P a_4 \succ^P a \simeq^P b \succ^P b_4$$

Figure B.38 – $Propa_\epsilon$ falsifies the properties QP and DDP

The property says that a should be strictly more acceptable than b because $b_2 \succ^P a_2$ and $b_2 \succ^P a_4$. But, with the semantics $Propa_\epsilon^{\oplus,1}$, we can see that a and b are equally acceptable, contradicting the property.

(DDP) To show that $Propa_\epsilon$ does not satisfy the property Distributed-Defense Precedence (QP), consider the argumentation framework AF from Figure B.38.
The property says that a should be ranked strictly higher than b because the defense of a is simple and distributed while the defense of b is simple but not distributed. But, with the semantics $Propa_\epsilon$, we can see that a and b are equally acceptable, contradicting the property.

(SC) To show that $Propa_\epsilon$ does not satisfy the property Self-Contradiction (SC), consider the argumentation framework AF from Figure B.39.



$P_i^{0.1,\oplus}$	b_1, b_2	a	b
0	1	0.1	0.1
1	1	0	-1.9
2	1	0.1	-1.9

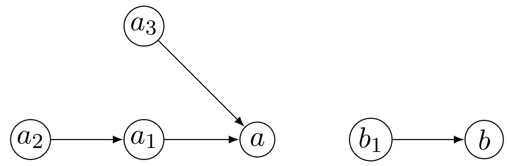
$$b_1 \simeq^P b_2 \succ^P a \succ^P b$$

Figure B.39 – $Propa_\epsilon$ falsifies the property SC

The property says that b should be ranked strictly higher than a because a attacks itself while b does not. But, with the semantics $Propa_\epsilon^{\oplus,0.1}$, we can see that a is ranked strictly higher than b, contradicting the property.

($\oplus$DB and +DB) To show that $Propa_\epsilon$ does not satisfy the property Addition of Defense Branch (+DB) and the property Strict addition of Defense Branch ($\oplus$DB), consider the argumentation framework AF from Figure B.40.

Both properties say that a should be ranked strictly higher than b because a has one defense branch while b has no defense branch. But, with $Propa_\epsilon^{\oplus,0.1}$, we can see that b is ranked strictly



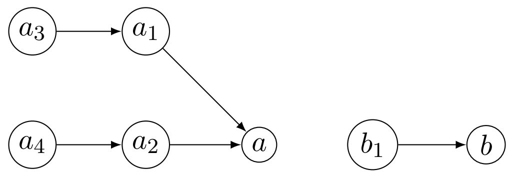
$P_i^{0.1,\oplus}$	a_2, a_3, b_1	a	a_1, b
0	1	0.1	0.1
1	1	-1	-0.9
2	1	0	-0.9

$$a_2 \simeq^P a_3 \simeq^P b_1 \succ^P a_1 \simeq^P b \succ^P a$$

Figure B.40 – $Propa_\epsilon$ falsifies the properties $\oplus$DB and +DB

higher than a, contradicting $\oplus$DB and +DB.

(AvsFD) To show that $Propa_\epsilon$ does not satisfy the property Attack vs Full Defense (AvsFD), consider the argumentation framework AF from Figure B.41.



$P_i^{0.7,\oplus}$	a_3, a_4, b_1	a_1, a_2	a	b
0	1	0.7	0.7	0.7
1	1	-0.3	-0.7	-0.3
2	1	-0.3	1.3	-0.3

$$a_3 \simeq^P a_4 \simeq^P b_1 \succ^P a_1 \simeq^P a_2 \simeq^P b \succ^P a$$

Figure B.41 – $Propa_\epsilon$ falsifies the property AvsFD

The property says that a should be ranked strictly higher than b because a has only defense branches while b has not. But, with $Propa_\epsilon^{\oplus,0.7}$, we can see that b is ranked strictly higher than a, contradicting the property.

Propa$_{1+\epsilon}$

Properties satisfied

(Abs) The nature of an argument is not used in the computation of its score. Only the attack relation is needed (see the definition of $Propa_{1+\epsilon}$ and the definition of the propagation principle).

(In) An argument a only receives the scores from its attackers and defenders. Thus, an argument b, such that there exists no path between a and b, cannot propagate its initial value to a and then cannot influence the propagation number of a.

(VP) Let $\oplus \in \{M, S\}$ and $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $a, b \in \mathcal{A}$ such that a is not attacked ($\mathcal{R}_1^\oplus(a) = \emptyset$) and b is attacked ($\mathcal{R}_1^\oplus(b) \neq \emptyset$). During the step $i = 0$, when $\epsilon = 0$, the non-attacked arguments begin with a score equal to 1, in particular $P_0^{0,\oplus}(a) = 1$, and the attacked arguments begin with a score equal to 0, in particular so $P_0^{0,\oplus}(b) = 0$. So, we have

$P_0^{0,\oplus}(b) = 0 < 1 = P_0^{0,\oplus}(a)$ which implies that $a \succ_{\text{AF}}^{\widehat{P}} b$, in agreement with the property.

(DP) Let $\oplus \in \{M, S\}$ and $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $a, b \in \mathcal{A}$ such that a is not defended ($\mathcal{R}_2^{\oplus}(a) = \emptyset$), b is defended ($\mathcal{R}_2^{\oplus}(b) \neq \emptyset$) and $|\mathcal{R}_1^{\oplus}(a)| = |\mathcal{R}_1^{\oplus}(b)| = n > 0$. During the first step where $\epsilon = 0$, both arguments have exactly the same score because they are attacked by at least one argument, so $P_0^{0,\oplus}(a) = P_0^{0,\oplus}(b) = 0$. During the first step where $\epsilon \neq 0$, for the same reason both arguments have exactly the same score : $P_0^{\epsilon,\oplus}(a) = P_0^{\epsilon,\oplus}(b) = \epsilon$. But during the second step where $\epsilon = 0$, a is attacked by n non-attacked arguments so $P_1^{0,\oplus}(a) = -n$ while b is attacked by m non-attacked arguments (with $m < n$) so $P_1^{0,\oplus}(b) = -m$. Consequently, $P_1^{0,\oplus}(a) < P_1^{0,\oplus}(b)$ which implies that $b \succ_{\text{AF}}^{\widehat{P}} a$, in agreement with the property.

(AE) Obvious because two arguments with the same ancestors' graph receive exactly the same values from its attackers and defenders whatever the value of ϵ.

(NaE) NaE is implied by AE which is satisfied.

(+AB,↑AB and ↑DB) Let $\oplus \in \{M, S\}$, $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that it exists an isomorphism γ with $AF = \gamma(AF')$. Let $a \in \mathcal{A}$ and its image $\gamma(a) \in \mathcal{A}'$ be two arguments. As $Propa_\epsilon$ satisfies the properties Argument Equivalence (AE), we can say that a and $\gamma(a)$, with the same ancestors' graph, have the same propagation vector: $\forall \epsilon \in]0, 1], P^{\epsilon,\oplus}(a) = P^{\epsilon,\oplus}(\gamma(a))$.

+AB Let us add an attack branch $P^-(\gamma(a))$ to $\gamma(a)$ with a length of $n \in 2\mathbb{N} + 1$. If a is not attacked, then a becomes more acceptable than $\gamma(a)$ which is now attacked (see the property Void Precedence). Otherwise, when a is attacked by at least one argument, a and $\gamma(a)$ have the same score during the first step when $\epsilon = 0$ and $\epsilon \neq 0$: $P_0^{0,\oplus}(a) = P_0^{0,\oplus}(\gamma(a)) = 0$ and $P_0^{\epsilon,\oplus}(a) = P_0^{\epsilon,\oplus}(\gamma(a)) = \epsilon$.
For the second step, there is two possible ways to say that a becomes better than $\gamma(a)$ according to the length of the new attack branch:

- If $n = 1$ then $\gamma(a)$ is attacked by one more non-attacked argument than a. In this case, the score of $\gamma(a)$ decreases and we obtain $P_1^{0,\oplus}(a) > P_1^{0,\oplus}(\gamma(a))$. Consequently, a is ranked strictly higher $\gamma(a)$, in agreement with the property.
- If $n \neq 1$ then a and $\gamma(a)$ keep the same score during the second step where $\epsilon = 0$ because they have the same number of direct attacker which are non-attacked, so $P_1^{0,M}(a) = P_1^{0,M}(\gamma(a))$. However, during the second step where $\epsilon \neq 0$, $\gamma(a)$, with one more direct attacker, receives one more negative value than a, so $P_1^{\epsilon,\oplus}(a) > P_1^{\epsilon,\oplus}(\gamma(a))$. Consequently, a is ranked strictly higher $\gamma(a)$, in agreement with the property.

↑AB Let us suppose $\exists b \in \mathcal{B}_-(a), b \notin \mathcal{B}_+(a)$ such that the length of the path from b to a is n. Let us now add a defense branch to the non-attacked argument $\gamma(b)$. Until the step $n-1$, the score of a and $\gamma(a)$ stay similar : $\forall i < n, P_i^{0,\oplus}(a) = P_i^{0,\oplus}(\gamma(a))$ and $P_i^{\epsilon,\oplus}(a) = P_i^{\epsilon,\oplus}(\gamma(a))$.
But during the n^{th} step where $\epsilon = 0$, a receives one more value come from a non-attacked argument than $\gamma(a)$ because $\gamma(b)$ is now attacked. So $P_n^{0,\oplus}(a) < P_n^{0,\oplus}(\gamma(a))$ which means that $\gamma(a)$ becomes strictly more acceptable than a, in agreement with the property.

↑DB We have almost the same proof that Increase of Attack Branch to the difference that this time b is situated at the beginning of a defense branch. So the score of $\gamma(a)$ decreases during the n^{th} step because it loses one defense branch with a length equal of n. Indeed, we obtain $P_n^{0,\oplus}(a) = P_n^{0,\oplus}(\gamma(a)) + 1$, so $P_n^{0,\oplus}(a) > P_n^{0,\oplus}(\gamma(a)) \Rightarrow a \succ^{\widehat{P}\oplus} \gamma(a)$, in agreement with the property.

(Tot) It is clear that the computation of the propagation vectors is always guaranteed for each argument. And the lexicographical order always returns a result between two vectors which guarantees the comparison between two arguments.

(DDP) Let $\oplus \in \{M, S\}$ and $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $a, b \in \mathcal{A}$ such that $|\mathcal{R}_1^{\oplus}(a)| = |\mathcal{R}_1^{\oplus}(b)| \neq 0$ and $|\mathcal{R}_2^{\oplus}(a)| = |\mathcal{R}_2^{\oplus}(b)| \neq 0$ and the defense of a is simple and distributed whereas the defense of b is simple but not distributed. During the step $i = 0$, when $\epsilon = 0$, we have $P_0^{0,\oplus}(a) = P_0^{0,\oplus}(b) = 0$ and when $\epsilon \neq 0$, we have $P_0^{\epsilon,\oplus}(a) = P_0^{\epsilon,\oplus}(b) = \epsilon$ because they are both attacked by at least one argument. The fact that the defense of a is distributed means that all the direct attackers of a are attacked by at most one argument, so there exists no direct attacker of a which is non-attacked so $P_1^{0,\oplus}(a) = 0$. It is not the case of b because its defense is not distributed but only simple. Consequently it exists at least one of its direct attacker which is not attacked, so $P_1^{0,\oplus}(b) \leq -1$. So $P_1^{0,\oplus}(a) = 0 > -1 \geq P_1^{0,M}(b)$ which means that $a \succ_{\text{AF}}^{\widehat{P}} b$, in agreement with the property.

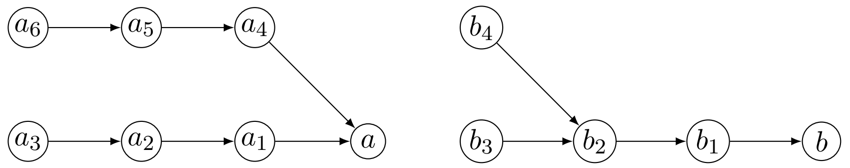
(AvsFD) Let $\oplus \in \{M, S\}$ and $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework where $a \in \mathcal{A}$ is attacked by only one non-attacked argument and $b \in \mathcal{A}$ is only defended (i.e. without attack branch). It is obvious that if b is a non-attacked argument, it is better than a (see Void Precedence which is satisfied). Otherwise, during the step $i = 0$, when $\epsilon = 0$, we have $P_0^{0,\oplus}(a) = P_0^{0,\oplus}(b) = 0$ and when $\epsilon \neq 0$, we have $P_0^{\epsilon,\oplus}(a) = P_0^{\epsilon,\oplus}(b) = \epsilon$ because they are both attacked. But during the second step with $\epsilon = 0$, a, which is directly attacked by a non-attacked argument, receives a negative value from this argument so $P_1^{0,\oplus}(a) = -1$ whereas b do not receive any value so $P_1^{0,\oplus}(b) = 0$. Consequently, we have $P_1^{0,\oplus}(a) = -1 < 0 = P_1^{0,\oplus}(b)$ which implies that $b \succ_{\text{AF}}^{\widehat{P}} a$, in agreement with the property.

Counter-examples

(CP) To show that $Propa_{1+\epsilon}$ does not satisfy the property Cardinality Precedence (CP), consider the argumentation framework AF from Figure B.42. The property says that a_2 (or a_5) should be strictly more acceptable than a because $|\mathcal{R}_1^{\oplus}(a)|$ = 2 > 1 = $|\mathcal{R}_1^{\oplus}(a_2)|$. But, with the semantics $Propa_{1+\epsilon}$, we can see that a is strictly more acceptable than b, contradicting the property.

(QP) To show that $Propa_{1+\epsilon}$ does not satisfy the property Quality Precedence (QP), consider the argumentation framework AF from Figure B.42. The property says that a should strictly more acceptable than b because b_1 is strictly more acceptable than all the direct attackers of a: $b_1 \succ^{\widehat{P}} a_4$ and $b_1 \succ^{\widehat{P}} a_1$. But, with the semantics $Propa_{1+\epsilon}$, we can see that b is strictly more acceptable than a, contradicting the property.

(SC) To show that $Propa_{1+\epsilon}$ does not satisfy the property Self-Contradiction (SC), consider the argumentation framework AF from Figure B.43.

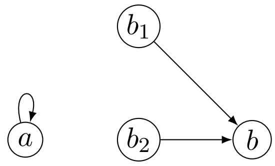


$P_i^{0,\oplus}$	a_3, a_6, b_3, b_4	a_2, a_5	a_1, a_4	b_2	b_1	a	b
0	1	0	0	0	0	0	0
1	1	-1	0	-2	0	0	0
2	1	-1	1	-2	2	0	0
3	1	-1	1	-2	2	-2	-2

$P_i^{0.1,\oplus}$	a_3, a_6, b_3, b_4	a_2, a_5	a_1, a_4	b_2	b_1	a	b
0	1	0.1	0.1	0.1	0.1	0.1	0.1
1	1	-0.9	0	-1.9	0	-0.1	0
2	1	-0.9	1	-1.9	2	0.1	0.1
3	1	-0.9	1	-1.9	2	-1.9	-1.9

$$a_3 \simeq^{\widehat{P}} a_6 \simeq^{\widehat{P}} b_3 \simeq^{\widehat{P}} b_5 \succ^{\widehat{P}} b_1 \succ^{\widehat{P}} a_1 \simeq^{\widehat{P}} a_4 \succ^{\widehat{P}} b \succ^{\widehat{P}} a \succ^{\widehat{P}} a_2 \simeq^{\widehat{P}} a_5 \succ^{\widehat{P}} b_2$$

Figure B.42 – $Propa_{1+\epsilon}$ falsifies the properties CP and QP



$$b_1 \simeq^{\widehat{P}} b_2 \succ^{\widehat{P}} a \succ^{\widehat{P}} b$$

$P_i^{0,\oplus}$	b_1, b_2	a	b
0	1	0	0
1	1	0	-2
2	1	0	-2

$P_i^{0.1,\oplus}$	b_1, b_2	a	b
0	1	0.1	0.1
1	1	0	-1.9
2	1	0.1	-1.9

Figure B.43 – $Propa_{1+\epsilon}$ falsifies the property SC

The definition says that b should be ranked higher than a because a attacks itself while b does not. But, with the semantics $Propa_{1+\epsilon}$, we can see that a is ranked strictly higher than b, contradicting the property.

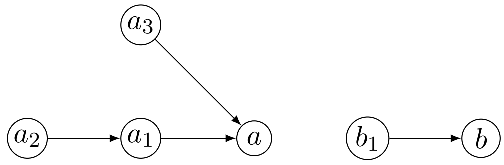
($\oplus$DB and +DB) To show that $Propa_{1+\epsilon}$ does not satisfy the property Addition of Defense Branch (+DB) and the property Strict addition of Defense Branch ($\oplus$DB), consider the argumentation framework AF from Figure B.40.

Both properties say that a should be ranked strictly higher than b because a has one defense branch while b has no defense branch. But, with $Propa_{\epsilon}^{\oplus,0.1}$, we can see that b is ranked strictly higher than a, contradicting $\oplus$DB and +DB.

Propa$_{1\to\epsilon}$

Properties satisfied

(Abs) The nature of an argument is not used in the computation of its score. Only the attack relation is needed (see the definition of $Propa_{1\to\epsilon}$ and the definition of the propagation princi-



$P_i^{0,\oplus}$	a_2, a_3, b_1	a	a_1, b
0	1	0	0
1	1	-1	-1
2	1	0	-1

$P_i^{0.1,\oplus}$	a_2, a_3, b_1	a	a_1, b
0	1	0.1	0.1
1	1	-1	-0.9
2	1	0	-0.9

$$a_2 \simeq^{\widehat{P}} a_3 \simeq^{\widehat{P}} b_1 \succ^{\widehat{P}} a_1 \simeq^{\widehat{P}} b \succ^{\widehat{P}} a$$

Figure B.44 – $Propa_{1+\epsilon}$ falsifies the properties $\oplus$DB and +DB

ple).

(In) An argument a only receives the scores from its attackers and defenders. Thus, an argument b, such that there exists no path between a and b, cannot propagate its initial value to a and then cannot influence the propagation number of a.

(VP) Let $\oplus \in \{M, S\}$ and $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $a, b \in \mathcal{A}$ such that a is not attacked ($\mathcal{R}_1^{\oplus}(a) = \emptyset$) and b is attacked ($\mathcal{R}_1^{\oplus}(b) \neq \emptyset$). During the step $i = 0$, when $\epsilon = 0$, the non-attacked arguments begin with a score equal to 1, in particular $P_0^{0,\oplus}(a) = 1$, and the attacked arguments begin with a score equal to 0, in particular so $P_0^{0,\oplus}(b) = 0$. So, we have $P_0^{0,\oplus}(b) = 0 < 1 = P_0^{0,\oplus}(a)$ which implies that $a \succ_{AF}^{\overline{P}} b$, in agreement with the property.

(DP) Let $\oplus \in \{M, S\}$ and $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $a, b \in \mathcal{A}$ such that a is not defended ($\mathcal{R}_2^{\oplus}(a) = \emptyset$), b is defended ($\mathcal{R}_2^{\oplus}(b) \neq \emptyset$) and $|\mathcal{R}_1^{\oplus}(a)| = |\mathcal{R}_1^{\oplus}(b)| = n > 0$. When $\epsilon = 0$, during the first step, both arguments have exactly the same score because they are attacked by at least one argument, so $P_0^{0,\oplus}(a) = P_0^{0,\oplus}(b) = 0$. But during the second step, a is attacked by n non-attacked arguments so $P_1^{0,\oplus}(a) = -n$ while b is attacked by m non-attacked arguments (with $m < n$) so $P_1^{0,\oplus}(b) = -m$. Consequently, $P_1^{0,\oplus}(a) < P_1^{0,\oplus}(b)$ which implies that $a \succ_{\text{AF}}^{\overline{P}} b$, in agreement with the property.

(DDP) Let $\oplus \in \{M, S\}$ and $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $a, b \in \mathcal{A}$ such that $|\mathcal{R}_1^{\oplus}(a)| = |\mathcal{R}_1^{\oplus}(b)| \neq 0$ and $|\mathcal{R}_2^{\oplus}(a)| = |\mathcal{R}_2^{\oplus}(b)| \neq 0$ and the defense of a is simple and distributed whereas the defense of b is simple but not distributed. According the definition of $Propa_{1\rightarrow\epsilon}$, let us first consider the case $\epsilon = 0$. During the step $i = 0$, it is clear that $P_0^{0,\oplus}(a) = P_0^{0,\oplus}(b) = 0$ because they are both attacked by at least one argument. The fact that the defense of a is distributed means that all the direct attackers of a are attacked by at most one argument, so there exists no direct attacker of a which is non-attacked so $P_1^{0,\oplus}(a) = 0$. It is not the case of b because its defense is not distributed but only simple.

Consequently it exists at least one of its direct attacker which is not attacked, so $P_1^{0,\oplus}(b) \leq -1$. So $P_1^{0,\oplus}(a) = 0 > -1 \geq P_1^{0,M}(b)$ which means that $a \succ_{\mathrm{AF}}^{\overline{P}} b$, in agreement with the property.

(AE) Obvious because two arguments with the same ancestors' graph receive exactly the same values from its attackers and defenders whatever the value of ϵ.

(NaE) NaE is implied by AE which is satisfied.

(+AB, +DB, ↑AB and ↑DB) Let $\oplus \in \{M, S\}$, $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that it exists an isomorphism γ with $AF = \gamma(AF')$. Let $a \in \mathcal{A}$ and its image $\gamma(a) \in \mathcal{A}'$ be two arguments. As $Propa_\epsilon$ satisfies the properties Argument Equivalence (AE), we can say that a and $\gamma(a)$, with the same ancestors' graph, have the same propagation vector: $\forall \epsilon, P^{\epsilon,\oplus}(a) = P^{\epsilon,\oplus}(\gamma(a))$.

+AB Let us add an attack branch $P^-(\gamma(a))$ to $\gamma(a)$ with a length of $n \in 2\mathbb{N}+1$. Let us note b the new attack root of $\gamma(a)$. If a is not attacked, then a becomes more acceptable than $\gamma(a)$ which is now attacked (see the property Void Precedence). Otherwise, according the definition of $Propa_{1\to\epsilon}$, let us first consider the case $\epsilon = 0$. So a is attacked by at least one argument which means that a and $\gamma(a)$ have the same score during the first step: $P_0^{0,\oplus}(a) = P_0^{0,\oplus}(\gamma(a)) = 0$. For the second step, there is two possible ways to say that a becomes better than $\gamma(a)$ according to the length of the new attack branch:

- If $n = 1$ then $\gamma(a)$ is attacked by one more non-attacked argument than a. In this case, the score of $\gamma(a)$ decreases and we obtain $P_1^{0,\oplus}(a) > P_1^{0,\oplus}(\gamma(a))$. Consequently, a is ranked strictly higher $\gamma(a)$, in agreement with the property.
- If $n \neq 1$ then a and $\gamma(a)$ keep the same score during the $n - 1$ first step : $\forall i < n, P_i^{0,\oplus}(a) = P_i^{0,\oplus}(\gamma(a))$. But during the n^{th} step, $\gamma(a)$ receives the negative score of b, so its score decreases : $P_i^{0,\oplus}(a) - 1 = P_i^{0,\oplus}(\gamma(a))$, so $P_i^{0,\oplus}(a) > P_i^{0,\oplus}(\gamma(a)) \Rightarrow a \succ^{\overline{P}} \gamma(a)$, in agreement with the property.

+DB Let us add a defense branch $P^+(\gamma(a))$ to $\gamma(a)$ with a length of $n \in 2\mathbb{N}$. Let us note b the new defense root of $\gamma(a)$. According the definition of $Propa_{1\to\epsilon}$, let us first consider the case $\epsilon = 0$. So, $\forall i < n, P_i^{0,\oplus}(a) = P_i^{0,\oplus}(\gamma(a))$. But during the n^{th} step, $\gamma(a)$ receives the positive score of b, so its score increases and become better than a: $P_i^{0,\oplus}(a) + 1 = P_i^{0,\oplus}(\gamma(a))$, so $P_i^{0,\oplus}(a) < P_i^{0,\oplus}(\gamma(a)) \Rightarrow \gamma(a) \succ^{\overline{P}} a$, in agreement with the property.

↑AB Let us suppose $\exists b \in \mathcal{B}_-(a), b \notin \mathcal{B}_+(a)$ such that the length of the path from b to a is n. Let us now add a defense branch to the non-attacked argument $\gamma(b)$. According the definition of $Propa_{1\to\epsilon}$, let us first consider the case $\epsilon = 0$. Until the step $n - 1$, the score of a and $\gamma(a)$ stay similar : $\forall i < n, P_i^{0,\oplus}(a) = P_i^{0,\oplus}(\gamma(a))$.
But during the n^{th} step, a receives one more value come from a non-attacked argument than $\gamma(a)$ because $\gamma(b)$ is now attacked. So $P_n^{0,\oplus}(a) < P_n^{0,\oplus}(\gamma(a))$ which means that $\gamma(a)$ becomes strictly more acceptable than a, in agreement with the property.

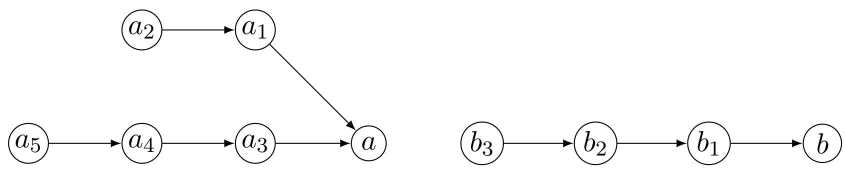
↑DB We have almost the same proof that ↑AB to the difference that this time b is situated at the beginning of a defense branch. So the score of $\gamma(a)$ decreases during the n^{th} step because it loses one defense branch with a length equal of n. Indeed, we obtain $P_n^{0,\oplus}(a) = P_n^{0,\oplus}(\gamma(a))+1$, so $P_n^{0,\oplus}(a) > P_n^{0,\oplus}(\gamma(a)) \Rightarrow a \succ^{\overline{P}} \gamma(a)$, in agreement with the property.

(Tot) It is clear that the computation of the propagation vectors is always guaranteed for each argument. And the lexicographical order always returns a result between two vectors which guarantees the comparison between two arguments.

(AvsFD) Let $\oplus \in \{M, S\}$ and $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework where $a \in \mathcal{A}$ is attacked by only one non-attacked argument and $b \in \mathcal{A}$ is only defended (i.e. without attack branch). It is obvious that if b is a non-attacked argument, it is better than a (see Void Precedence which is satisfied). Otherwise, according the definition of $Propa_{1\to\epsilon}$, let us first consider the case $\epsilon = 0$. During the step $i = 0$, we have $P_0^{0,\oplus}(a) = P_0^{0,\oplus}(b) = 0$ because they are both attacked. But during the second step, a, which is directly attacked by a non-attacked argument, receives a negative value from this argument so $P_1^{0,\oplus}(a) = -1$ whereas b do not receive any value so $P_1^{0,\oplus}(b) = 0$. Consequently, we have $P_1^{0,\oplus}(a) = -1 < 0 = P_1^{0,\oplus}(b)$ which implies that $b \succ_{\text{AF}}^{\overline{P}} a$, in agreement with the property.

Counter-examples

(CT) To show that $Propa_{1\to\epsilon}$ does not satisfy the property Counter-Transitivity (CT), consider the argumentation framework AF from Figure B.45.



$P_i^{0,\oplus}$	a_2, a_5, b_3	a_1, a_4, b_2	a_3, b_1	a	b
0	1	0	0	0	0
1	1	-1	0	0	0
2	1	-1	1	1	0
3	1	-1	1	0	-1

$P_i^{0.1,\oplus}$	a_2, a_5, b_3	a_1, a_4, b_2	a_3, b_1	a	b
0	1	0.1	0.1	0.1	0.1
1	1	-0.9	0	-0.1	0
2	1	-0.9	1	1	0.1
3	1	-0.9	1	0	-0.9

$$a_2 \simeq^{\overline{P}} a_5 \simeq^{\overline{P}} b_3 \succ^{\overline{P}} a_3 \simeq^{\overline{P}} b_1 \succ^{\overline{P}} a \succ^{\overline{P}} b \succ^{\overline{P}} a_1 \simeq^{\overline{P}} a_4 \simeq^{\overline{P}} b_2$$

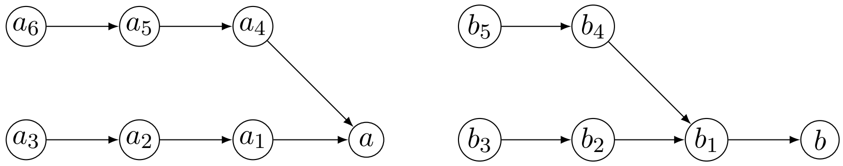
Figure B.45 – $Propa_{1\to\epsilon}$ falsifies the properties CT and SCT

The property says that b should be at least as acceptable as a ($b \succeq^{\overline{P}} a$) because it exists an injective function f from $\mathcal{R}_1^{\oplus}(b)$ to $\mathcal{R}_1^{\oplus}(a)$ such that $\forall b' \in \mathcal{R}_1^{\oplus}(b), f(b') \succeq^{\overline{P}} b'$. Indeed, we have $\mathcal{R}_1^{\oplus}(b) = \{b_1\}$ and $\mathcal{R}_1^{\oplus}(a) = \{a_1, a_3\}$ where $a_3 \succeq^{\overline{P}} b_1$. But, with the semantics $Propa_{1\to\epsilon}$, we can see that a is strictly more acceptable than b, contradicting the property.

(SCT) To show that $Propa_{1\to\epsilon}$ does not satisfy the property Strict Counter-Transitivity (SCT), consider the argumentation framework AF from Figure B.45.
The property says that b should be strictly more acceptable than a ($b \succ^{\overline{P}} a$) because it exists an injective function f from $\mathcal{R}_1^{\oplus}(b)$ to $\mathcal{R}_1^{\oplus}(a)$ such that $\forall b' \in \mathcal{R}_1^{\oplus}(b), f(b') \succeq^{\overline{P}} b'$. Indeed, we have $\mathcal{R}_1^{\oplus}(b) = \{b_1\}$ and $\mathcal{R}_1^{\oplus}(a) = \{a_1, a_3\}$ where $a_3 \succeq^{\overline{P}} b_1$ and $|\mathcal{R}_1^{\oplus}(b)| = 1 < 2 = \mathcal{R}_1^{\oplus}(a)$.

But, with the semantics $Propa_{1\to\epsilon}$, we can see that a is ranked higher than b, contradicting the property.

(CP) To show that $Propa_{1\to\epsilon}$ does not satisfy the property Cardinality Precedence (CP), consider the argumentation framework AF from Figure B.46.



$P_i^{0,\oplus}$	a_3, a_6, b_3, b_5	a_2, a_5, b_2, b_4	a_1, a_4	b_1	a	b
0	1	0	0	0	0	0
1	1	-1	0	0	0	0
2	1	-1	1	2	0	0
3	1	-1	1	2	-2	-2

$P_i^{0.1,\oplus}$	a_3, a_6, b_3, b_5	a_2, a_5, b_2, b_4	a_1, a_4	b_1	a	b
0	1	0.1	0.1	0.1	0.1	0.1
1	1	-0.9	0	-0.1	-0.1	0
2	1	-0.9	1	1.9	0.1	0.2
3	1	-0.9	1	1.9	-1.9	-1.8

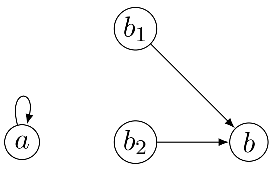
$$a_3 \simeq^{\overline{P}} a_6 \simeq^{\overline{P}} b_3 \simeq^{\overline{P}} b_5 \succ^{\overline{P}} b_1 \succ^{\overline{P}} a_1 \simeq^{\overline{P}} a_4 \succ^{\overline{P}} b \succ^{\overline{P}} a \succ^{\overline{P}} a_2 \simeq^{\overline{P}} a_5 \simeq^{\overline{P}} b_2 \simeq^{\overline{P}} b_4$$

Figure B.46 – $Propa_{1\to\epsilon}$ falsifies the properties CP and QP

The property says that b should be strictly more acceptable than a because $|\mathcal{R}_1^{\oplus}(a)| = 2 > 1 = |\mathcal{R}_1^{\oplus}(b)|$. But, with the semantics $Propa_{1\to\epsilon}$, we can see that a is ranked strictly more acceptable than b, contradicting the property.

(QP) To show that $Propa_{1\to\epsilon}$ does not satisfy the property Quality Precedence (QP), consider the argumentation framework AF from Figure B.46.
The property says that a should be strictly more acceptable than b because $b_1 \succ^{\overline{P}} a_4$ and $b_1 \succ^{\overline{P}} a_1$. But, with the semantics $Propa_{1\to\epsilon}$, we can see that b is strictly more acceptable than a, contradicting the property.

(SC) To show that $Propa_{1\to\epsilon}$ does not satisfy the property Self-Contradiction (SC), consider the argumentation framework AF from Figure B.47.



$$b_1 \simeq^{\overline{P}} b_2 \succ^{\overline{P}} a \succ^{\overline{P}} b$$

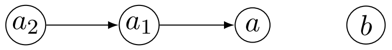
$P_i^{0,\oplus}$	b_1, b_2	a	b
0	1	0	0
1	1	0	-2
2	1	0	-2

$P_i^{0.1,\oplus}$	b_1, b_2	a	b
0	1	0.1	0.1
1	1	0	-1.9
2	1	0.1	-1.9

Figure B.47 – $Propa_{1\to\epsilon}$ falsifies the property SC

The definition says that b should be strictly more acceptable than a because a attacks itself while b does not. But, with the semantics $Propa_{1\to\epsilon}$, we can see that a is strictly more acceptable than b, contradicting the property.

(⊕DB) To show that $Propa_{1\to\epsilon}$ does not satisfy the property Strict addition of Defense Branch (⊕DB), consider the argumentation framework AF from Figure B.48.



$a_2 \simeq^{\overline{P}} b \succ^{\overline{P}} a \succ^{\overline{P}} a_1$

$P_i^{0,\oplus}$	a_2	a_1	a	b
0	1	0	0	1
1	1	-1	0	1
2	1	-1	2	1

$P_i^{0.1,\oplus}$	a_2	a_1	a	b
0	1	0.1	0.1	1
1	1	-0.9	0	1
2	1	-0.9	1	1

Figure B.48 – $Propa_{1\to\epsilon}$ falsifies the property ⊕DB

The property says that a should be strictly more acceptable than b because a has one defense branch while that b has not. But, with the semantics $Propa_{1\to\epsilon}$, we can see that b is strictly more acceptable than a, contradicting the property.

□

Proposition 18. Let $\epsilon \in]0,1]$.

- The ranking-based semantics $Propa_{\epsilon}^{\epsilon,M}$ satisfies CT, SCT and OE.
- The ranking-based semantics $Propa_{1+\epsilon}^{\epsilon,M}$ satisfies CT, SCT and OE.
- The ranking-based semantics $Propa_{1\to\epsilon}^{\epsilon,M}$ satisfies OE.

It is not the case when $\oplus = S$.

Proof. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. It is possible to compute, only when the multiset is used, the vector propagation of an argument on the basis of the vector propagation of its direct attackers with the following formula:

$$P_n^{\epsilon,M}(x) = \begin{cases} v_\epsilon(x) & \text{if } n = 0 \\ P_0^{\epsilon,M}(x) - \sum\limits_{y \in \mathcal{R}_1(x)} P_0^{\epsilon,M}(y) & \text{if } n = 1 \\ P_{n-1}^{\epsilon,M}(x) + (-1)^n \sum\limits_{y \in \mathcal{R}_1(x)} |P_{n-1}^{\epsilon,M}(y) - P_{n-2}^{\epsilon,M}(y)| & \text{if } n \geq 2 \end{cases} \quad \text{(B.1)}$$

Indeed, the values received by the direct attackers of an argument, will be received by this argument to the next step.

Propa$_\epsilon^M$

(OE) Let $x_1, x_2 \in \mathcal{A}$. If x_1 and x_2 are not attacked then $x_1 \simeq^P x_2$ because NaE is satisfied. Otherwise, let us suppose that there exists a bijective function f from $\mathcal{R}_1(x_1)$ to $\mathcal{R}_1(x_2)$ such that $\forall y \in \mathcal{R}_1(x_1), y \simeq^P_{\text{AF}} f(y)$. According to the definition of $Propa_\epsilon$, $y \simeq^P_{\text{AF}} f(y)$ if and only if $P^{\epsilon,M}(y) \simeq_{lex} P^{\epsilon,M}(f(y))$, *i.e.* they have the same propagation vector ($\forall i \geq 0$, $P_i^{\epsilon,M}(y) = P_i^{\epsilon,M}(f(y))$). Let us use the equation B.1 to show that, in this case, x_1 and x_2 are equally acceptable too.

n=0: x_1 and x_2 are both directly attacked so $P_0^{\epsilon,M}(x_1) = P_0^{\epsilon,M}(x_2) = \epsilon$;

n=1: Thanks to the bijective function between the direct attacker of x_1 and x_2, we have:

$$P_1^{\epsilon,M}(x_1) = P_0^{\epsilon,M}(x_1) - \sum_{y \in \mathcal{R}_1(x_1)} P_0^{\epsilon,M}(y) \quad = P_0^{\epsilon,M}(x_2) - \sum_{f(y) \in \mathcal{R}_1(x_2)} P_0^{\epsilon,M}(f(y)) = P_1^{\epsilon,M}(x_2)$$

n≥2: For the same reason, we have:

$$\begin{aligned}
P_n^{\epsilon,M}(x_1) &= P_{n-1}^{\epsilon,M}(x_1) + (-1)^n \sum_{y \in \mathcal{R}_1(x_1)} |P_{n-1}^{\epsilon,M}(y) - P_{n-2}^{\epsilon,M}(y)| \\
&= P_{n-1}^{\epsilon,M}(x_2) + (-1)^n \sum_{f(y) \in \mathcal{R}_1(x_2)} |P_{n-1}^{\epsilon,M}(f(y)) - P_{n-2}^{\epsilon,M}(f(y))| \\
&= P_n^{\epsilon,M}(x_2)
\end{aligned}$$

So $\forall i \geq 0$, $P_i^{\epsilon,M}(x_1) = P_i^{\epsilon,M}(x_2)$ so $P^{\epsilon,M}(x_1) \simeq_{lex} P^{\epsilon,M}(x_2)$. Consequently, according to the definition of $Propa_\epsilon$, $x_1 \simeq^P_{\text{AF}} x_2$, in agreement with the property.

(SCT) Let $x_1, x_2 \in \mathcal{A}$. Let us suppose that there exists an injective function f from $\mathcal{R}_1(x_1)$ to $\mathcal{R}_1(x_2)$ such that $\forall y \in \mathcal{R}_1(x_1), f(y) \succeq^P_{\text{AF}} y$.

- Let us suppose that $\mathcal{R}_1(x_2) > \mathcal{R}_1(x_1)$. If $\mathcal{R}_1(x_1) = \emptyset$ then $x_1 \succ^P_{\text{AF}} x_2$ because VP is satisfied. Otherwise, $P_0^{\epsilon,M}(x_1) = P_0^{\epsilon,M}(x_2) = \epsilon$. But during the step $i = 1$, x_2 has now more direct attackers than x_1 so, with the injective function, we have:

$$\begin{gathered}
\sum_{y \in \mathcal{R}_1(x_1)} P_0^{\epsilon,M}(y) < \sum_{f(y) \in \mathcal{R}_1(x_2)} P_0^{\epsilon,M}(f(y)) \\
- \sum_{y \in \mathcal{R}_1(x_1)} P_0^{\epsilon,M}(y) > - \sum_{f(y) \in \mathcal{R}_1(x_2)} P_0^{\epsilon,M}(f(y)) \\
P_0^{\epsilon,M}(x_1) - \sum_{y \in \mathcal{R}_1(x_1)} P_0^{\epsilon,M}(y) > P_0^{\epsilon,M}(x_2) - \sum_{f(y) \in \mathcal{R}_1(x_2)} P_0^{\epsilon,M}(f(y)) \\
P_1^{\epsilon,M}(x_1) > P_1^{\epsilon,M}(x_2)
\end{gathered}$$

  Consequently, we have $x_1 \succ^P_{\text{AF}} x_2$, in agreement with the property.

- Now, let us suppose that $\exists y' \in \mathcal{R}_1(x_1), f(y') \succ^{P}_{\text{AF}} y'$ and that $\mathcal{R}_1(x_2) = \mathcal{R}_1(x_1)$ (otherwise see the previous case where $\mathcal{R}_1(x_2) > \mathcal{R}_1(x_1)$).
  During the initial step, we have $P_0^{\epsilon,M}(x_1) = P_0^{\epsilon,M}(x_2) = \epsilon$.
  If y' is not attacked then $P_0^{\epsilon,M}(f(y')) = 1 > \epsilon = P_0^{\epsilon,M}(y')$. So, during the step $i = 1$, thanks to the injective function, we have:

$$P_1^{\epsilon,M}(x_1) = P_0^{\epsilon,M}(x_1) - \sum_{y \in \mathcal{R}_1(x_1)} P_0^{\epsilon,M}(y) \quad > P_0^{\epsilon,M}(x_2) - \sum_{f(y) \in \mathcal{R}_1(x_2)} P_0^{\epsilon,M}(f(y)) = P_1^{\epsilon,M}(x_2)$$

  This result implies that $x_1 \succ^{P}_{\text{AF}} x_2$, in agreement with the property.
  But if y' is attacked then $\exists i > 1$ such that $P_i^{\epsilon,M}(f(y')) > P_i^{\epsilon,M}(y')$ because $f(y') \succ^{P}_{\text{AF}} y'$. So, during the step $i+1$, with the same reasoning, we have $P_{i+1}^{\epsilon,M}(x_1) > P_{i+1}^{\epsilon,M}(x_2)$. Consequently, we have $x_1 \succ^{P}_{\text{AF}} x_2$, in agreement with the property.

(CT) CT is implied by SCT and OE which are satisfied.

Propa$^M_{1+\epsilon}$

(OE) The proof is similar to the one done to prove that OE is satisfied by Propa$^M_\epsilon$. Indeed, given two arguments $x_1, x_2 \in \mathcal{A}$, we showed that when there exists a bijective function f from $\mathcal{R}_1(x_1)$ to $\mathcal{R}_1(x_2)$ such that $\forall y \in \mathcal{R}_1(x_1), y \simeq^{P}_{\text{AF}} f(y)$ then $P^{\epsilon,M}(x_1) \simeq_{lex} P^{\epsilon,M}(x_2)$. This is also the case when $\epsilon = 0$ so $P^{0,M}(x_1) \simeq_{lex} P^{0,M}(x_2)$. So if we combine the two results with the shuffle $\cup_s$, we obviously obtain that $P^{0,M}(x_1) \cup_s P^{\epsilon,M}(x_1) \simeq_{lex} P^{0,M}(x_2) \cup_s P^{\epsilon,M}(x_2)$ which implies that $x_1 \simeq^{\widehat{P}} x_2$.
(SCT) The proof is globally the same to the one done to prove that SCT is satisfied by Propa$^M_\epsilon$. We just need to add an additional check when $\epsilon = 0$ which gives the same result.

(CT) CT is implied by SCT and OE which are satisfied.

Propa$_{1\to\epsilon}$

(OE) The proof is similar to the ones done to prove that OE is satisfied by Propa$^M_\epsilon$ and Propa$^M_{1+\epsilon}$. Indeed, given two arguments $x_1, x_2 \in \mathcal{A}$, we showed that when there exists a bijective function f from $\mathcal{R}_1(x_1)$ to $\mathcal{R}_1(x_2)$ such that $\forall y \in \mathcal{R}_1(x_1), y \simeq^{P}_{\text{AF}} f(y)$ then $P^{\epsilon,M}(x_1) \simeq_{lex} P^{\epsilon,M}(x_2)$. This is also the case when $\epsilon = 0$ so $P^{0,M}(x_1) \simeq_{lex} P^{0,M}(x_2)$. According to the definition of Propa$_{1\to\epsilon}$, then $x_1 \simeq^{\overline{P}} x_2$.

□

Proposition 19. The grounded semantics (Gr) satisfies Abs, In, Tot, NaE, AE and AvsFD. The other properties are not satisfied.

Proof.

Properties satisfied

(Abs) Obvious because we work with abstract arguments.

(In) Obvious (see definition of the grounded semantics).

(Tot) All the arguments are either accepted or not accepted so a comparison between two arguments are always possible.

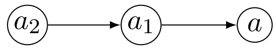
(NaE) All the non-attacked argument are accepted, so they are all equally acceptable.

(AE) This is directly connected with the property Directionality [BARONI *et al.* 2011] which says that if an argument a attacks an argument b then a affects b and not vice versa. So the only arguments which are a direct or indirect impact on an argument are the argument belonging to its ancestors' graph. So two arguments with the "same" ancestors' graph are equally acceptable.

(AvsFD) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an acyclic argumentation framework with $a, b \in \mathcal{A}$ be two arguments where b is attacked by a non-attacked argument and a only have defense branches and no attack branch. It is clear that b is not accepted because it is directly attacked by a non-attacked argument which is accepted. If a is not attacked then it is directly accepted. And if it is attacked then it is accepted too by the grounded semantics because AF is acyclic (so there exists at least one non-attacked argument) and as a has no attack branch so all its direct attackers are not accepted (see the algorithm detailed in section 3.1). So a which is accepted is more acceptable than b which is rejected ($a \succ^{gr} b$) in agreement with the property.

Counter-examples

(VP) To show that the grounded semantics does not satisfy the property Void Precedence (VP), consider the argumentation framework AF from Figure B.49.



$$\mathcal{E}_{gr}(AF) = \{a_2, a\}$$

$$a_2 \simeq^{gr} a \succ^{gr} a_1$$

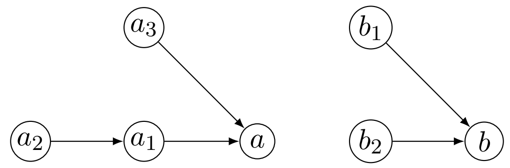
Figure B.49 – The grounded semantics falsifies the property VP

Void Precedence says that a_2 should be strictly more acceptable than a ($a_2 \succ^{gr} a$) because a_2 is a non-attacked argument while a is attacked by a_1. But the grounded semantics considers that a_2 and a are equally acceptable (a_2 and a are both accepted), contradicting the property.

(DP) To show that the grounded semantics does not satisfy the property Defense Precedence (DP), consider the argumentation framework AF from Figure B.50.

Defense Precedence says that a should be strictly more acceptable than b ($a \succ^{gr} b$) because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$ and $|\mathcal{R}_2(a)| = 1 > 0 = |\mathcal{R}_2(b)|$. But the grounded semantics considers that a and b are equally acceptable, contradicting the property.

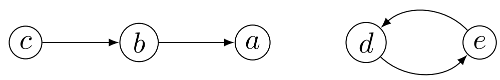
(CT) To show that the grounded semantics does not satisfy the property Counter Transitivity (CT), consider the argumentation framework AF from Figure B.51.



$\mathcal{E}_{gr}(AF) = \{a_2, a_3, b_1, b_2\}$

$a_2 \simeq^{gr} a_3 \simeq^{gr} b_1 \simeq^{gr} b_2 \succ^{gr} a_1 \simeq^{gr} a \simeq^{gr} b$

Figure B.50 – The grounded semantics falsifies the properties DP, SCT and CP



$\mathcal{E}_{gr}(AF) = \{a, c\}$

$a \simeq^{gr} c \succ^{gr} b \simeq^{gr} d \simeq^{gr} e$

Figure B.51 – The grounded semantics falsifies the properties CT, QP and OE

The property says that e should be at least as acceptable than a ($e \succeq^{gr} a$) because it exists an injective function f from $\mathcal{R}_1(e)$ to $\mathcal{R}_1(a)$ such that $\forall e' \in \mathcal{R}_1(e)$, $f(e') \succeq e'$ ($b \simeq^{gr} d$ which implies $b \succeq^{gr} d$) so $\mathcal{R}_1(a) \geq_S^{gr} \mathcal{R}_1(e)$. But the grounded semantics considers that a is strictly more acceptable than e, contradicting the property.

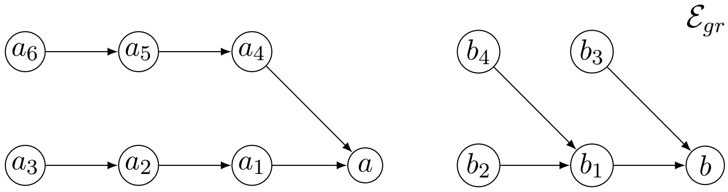
(SCT) To show that the grounded semantics does not satisfy the property Strict-Counter Transitivity (SCT), consider the argumentation framework AF from Figure B.50.
The property says that a should be strictly more acceptable than b ($a \succ^{gr} b$) because it exists an injective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(b)$ such that $\forall a' \in \mathcal{R}_1(a)$, $f(a') \succeq a'$ ($b_1 \succeq^{gr} a_3$ and $b_2 \succeq^{gr} a_1$) and especially $b_2 \succ^{gr} a_1$ (so $\mathcal{R}_1(b) >_S^{gr} \mathcal{R}_1(a)$). But the grounded semantics considers that a and b are equally acceptable, contradicting the property.

(CP) To show that the grounded semantics does not satisfy the property Cardinality Precedence (CP), consider the argumentation framework AF from Figure B.50.
The property says that a_1 should be strictly more acceptable than b because $|\mathcal{R}_1(b)|$ = 2 > 1 = $|\mathcal{R}_1(a_1)|$. But the grounded semantics considers that a_1 and b are equally acceptable, contradicting the property.

(QP) To show that the grounded semantics does not satisfy the property Quality Precedence (QP), consider the argumentation framework AF from Figure B.51.
The property says that e should be strictly more acceptable than b ($e \succ^{gr} b$) because $c \succ^{gr} d$. But the grounded semantics considers that e and b are equally acceptable, contradicting the property.

(DDP) To show that the grounded semantics does not satisfy the property Distributed-Defense Precedence (DDP), consider the argumentation framework AF from Figure B.52.

The property says that a should be strictly more acceptable than b ($a \succ^{gr} b$) because $|\mathcal{R}_1(a)| = |\mathcal{R}_1(b)| = 2$, $|\mathcal{R}_2(a)| = |\mathcal{R}_2(b)| = 2$ and the defense of a is simple and distributed while the defense of b is simple but not distributed. But the grounded semantics considers that a and b are equally acceptable, contradicting the property.

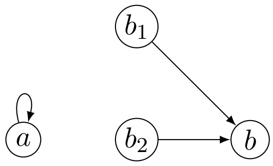


$$\mathcal{E}_{gr}(AF) = \{a_3, a_6, b_2, b_3, b_4, a_1, a_4\}$$

$$\boxed{a_3 \simeq^{gr} a_6 \simeq^{gr} b_2 \simeq^{gr} b_3 \simeq^{gr} b_4 \simeq^{gr} a_1 \simeq^{gr} a_4 \succ^{gr} a_2 \simeq^{gr} a_5 \simeq^{gr} b \simeq^{gr} a \simeq^{gr} b_1}$$

Figure B.52 – The grounded semantics falsifies the property DDP

(SC) To show that the grounded semantics does not satisfy the property Self-Contradiction (SC), consider the argumentation framework AF from Figure B.53.



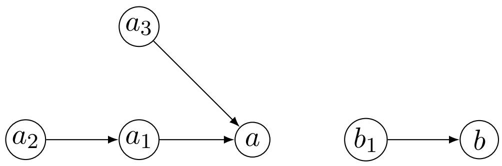
$$\mathcal{E}_{gr}(AF) = \{b_1, b_2\}$$

$$\boxed{b_1 \simeq^{gr} b_2 \succ^{gr} a \simeq^{gr} b}$$

Figure B.53 – The grounded semantics falsifies the property SC

The definition says that b should be strictly more acceptable than a ($b \succ^{gr} a$) because a attacks itself while b does not attack itself. But the grounded semantics considers that a and b are equally acceptable, contradicting the property.

(+DB and $\oplus$DB) To show that the grounded semantics does not satisfy the property Addition of Defense Branch (+DB) and the property Strict addition of Defense Branch ($\oplus$DB), consider the argumentation framework AF from Figure B.54.



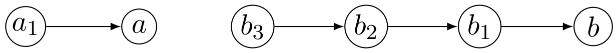
$$\mathcal{E}_{gr}(AF) = \{a_2, a_3, b_1\}$$

$$\boxed{a_2 \simeq^{gr} a_3 \simeq^{gr} b_1 \succ^{gr} a_1 \simeq^{gr} a \simeq^{gr} b}$$

Figure B.54 – The grounded semantics falsifies the properties +DB and $\oplus$DB

Both properties say that a should be strictly more acceptable than b ($a \succ^{Gr} b$) because a has one defense branch while b has no defense branch. But in using the grounded semantics, a and b are equally acceptable, contradicting both properties.

($\uparrow$AB) To show that the grounded semantics does not satisfy the property Increase of Attack branch ($\uparrow$AB), consider the argumentation framework AF from Figure B.55.



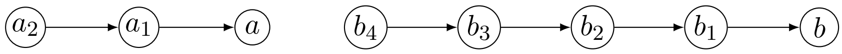
$\mathcal{E}_{gr}(AF) = \{a_1, b_3, b_1\}$

$a_1 \simeq^{gr} b_3 \simeq^{gr} b_1 \succ^{gr} b_2 \simeq^{gr} a \simeq^{gr} b$

Figure B.55 – The grounded semantics falsifies the property ↑AB

The property says that b should be strictly more acceptable than a ($b \succ^{gr} a$) because the length of the attack branch of b is greater than the length of the attack branch of a. But in using the grounded semantics, we can see that a and b are equally acceptable, contradicting the property.

(↑DB) To show that the grounded semantics does not satisfy the property Increase of Defense branch (↑DB), consider the argumentation framework AF from Figure B.56.



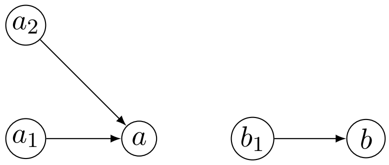
$\mathcal{E}_{gr}(AF) = \{a_2, a, b_4, b_2, b\}$

$a_2 \simeq^{gr} a \simeq^{gr} b_4 \simeq^{gr} b_2 \simeq^{gr} b \succ^{gr} a_1 \simeq^{gr} b_3 \simeq^{gr} b_1$

Figure B.56 – The grounded semantics falsifies the property ↑DB

The property says that a should be strictly more acceptable than b ($a \succ^{gr} b$) because the length of the defense branch of b is greater than the length of the defense branch of a. But in using the grounded semantics, we can see that a and b are equally acceptable, contradicting the property.

(+AB) To show that the grounded semantics does not satisfy the property Addition of Attack Branch (+AB), consider the argumentation framework AF from Figure B.57.



$\mathcal{E}_{gr}(AF) = \{a_1, a_2, b_1\}$

$a_1 \simeq^{gr} a_2 \simeq^{gr} b_1 \succ^{gr} a \simeq^{gr} b$

Figure B.57 – The grounded semantics falsifies the property +AB

The property says that b should be strictly more acceptable than a ($b \succ^{Gr} a$) because a has one attack branch while b has two attack branches. But in using the grounded semantics, a and b are equally acceptable, contradicting the property.

(OE) To show that the grounded semantics does not satisfy the property Ordinal Equivalence (OE), consider the argumentation framework AF from Figure B.51.
The property says that a and e should be equally acceptable ($a \simeq^{gr} e$) because there exists a bijective function f from $\mathcal{R}_1(a)$ to $\mathcal{R}_1(e)$ such that $\forall a' \in \mathcal{R}_1(a)$, $f(a') \simeq^{gr} a'$ (b and d are both

rejected). But in using the grounded semantics, a is accepted whereas e is rejected ($a \succ^{gr} e$), contradicting the property. $\square$

Appendix C

Proofs of the Results from Chapter 5

Proposition 20. Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $\delta \in \,]0,1[$ and $\epsilon \in \,]0,1]$. For all $x \in \mathcal{A}$, the sequence $\{P_i^{\epsilon,\delta}(x)\}_{i=0}^{+\infty}$ converges.

Proof. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $a \in \mathcal{A}$ and $\delta \in]0,1[$.
To simplify the proof (and the formula), we focus on the "worst" case where each argument receives the maximal value of 1 from all others arguments at each step i (according to the parity of i). In this way, if the method converges in this case, the method will be converge too when there are less attacks or with a smaller ϵ. Please note that the set of arguments is finite, so to each step the set of attackers/defenders is finite too. So we have $\forall i > 0, \sum_{b \in \mathcal{R}_i^S(a)} v_\epsilon(b) = |\mathcal{A}| = k$.
Let $PP(a)$ (resp. $NP(a)$) be the positive (resp. negative) propagation that only focus on the score received by the attackers (resp. defenders) of a:

$$PP_n^{\epsilon,\delta}(a) = v_\epsilon(a) + \sum_{i=1}^{n} k\delta^i \ with\ k \geq 0\ and\ \delta \in]0,1[$$

$$NP_n^{\epsilon,\delta}(a) = \sum_{i=1}^{n} k\delta^i \ with\ k \geq 0\ and\ \delta \in]0,1[$$

$$P_n^{\epsilon,\delta}(a) = PP_n^{\epsilon,\delta}(a) + (-NP_n^{\epsilon,\delta}(a))$$

We can observe that NP and PP (if we remove the initial value $v_\epsilon(a)$) correspond to a geometric series with a common ratio $\delta \in]0,1[$. As we know, when n goes to infinity, a geometric series always converges when the common ration (here δ) is strictly smaller than 1. Thus, PP and NP (and more precisely -NP) converge. In using these results combining with the fact that the addition of two convergent function converge too, we can conclude that $P_n^{\epsilon,\delta}(a)$ converge. □

Proposition 21. Let $\delta \in \,]0,1[$ and $\epsilon, \epsilon' \in \,]0,1]$. For any argumentation framework AF,

$$\mathsf{vdp}^{\epsilon,\delta}(AF) = \mathsf{vdp}^{\epsilon',\delta}(AF)$$

Proof. The goal of this proof consists in showing that for all values of $\epsilon \in \,]0,1]$, the ranking obtained stays the same. In other words, the value of ϵ has no influence on the result.

During the step where $\epsilon = 0$, it is obvious that the obtained pre-order is identical because the value of ϵ does not interfere in the computation of the propagation number.
Let us show, that during the step $\epsilon \neq 0$, it is not necessary to know the value of ϵ. Indeed, this step aims to distinguish arguments which have the same propagation number when $\epsilon = 0$. Thus, only attacked arguments can influence the ranking between two arguments. Let $a, b \in \mathcal{A}$ be both of these arguments: $\mathcal{R}_1^S(a) \neq \emptyset$, $\mathcal{R}_1^S(b) \neq \emptyset$ and $P^{0,\delta}(a) = P^{0,\delta}(b)$.
We can split the way to compute the propagation number when $\epsilon \neq 0$ in three parts: the initial value (ϵ) + the scores received by the non-attacked arguments ($P^{0,\delta}(a)$) + the scores received by the attacked arguments ($AP^{\epsilon,\delta}(a)$):

$$P^{\epsilon,\delta}(a) = \epsilon + P^{0,\delta}(a) + AP^{\epsilon,\delta}(a)$$

However, we know that $\epsilon + P^{0,\delta}(a) = \epsilon + P^{0,\delta}(b)$ because a and b are attacked so they begin with the same ϵ and that $P^{0,\delta}(a) = P^{0,\delta}(b)$. So if $a \simeq b$ (the reasoning is the same with $\succ$ and $\prec$) it is because $AP^{\epsilon,\delta}(a) = AP^{\epsilon,\delta}(b)$.
We denote by $k_i(x) = \mathcal{R}_i^S(x) \backslash \mathcal{B}_i^S(x)$ the number of attacked arguments at the end of a path with a length of i to x. According the previous definition,

$$\begin{aligned}
a \simeq b &\Rightarrow AP^{\epsilon,\delta}(a) = AP^{\epsilon,\delta}(b) \\
&\Rightarrow -(k_1(a)\epsilon)\delta + (k_2(a)\epsilon)\delta^2 - (k_3(a)\epsilon)\delta^3 + \dots = -(k_1(b)\epsilon)\delta + (k_2(b)\epsilon)\delta^2 - (k_3(b)\epsilon)\delta^3 + \dots \\
&\Rightarrow \epsilon \times (-k_1(a)\delta + k_2(a)\delta^2 - k_3(a)\delta^3 + \dots) = \epsilon \times (-k_1(b)\delta + k_2(b)\delta^2 - k_3(b)\delta^3 + \dots) \\
&\Rightarrow -k_1(a)\delta + k_2(a)\delta^2 - k_3(a)\delta^3 + \dots = -k_1(b)\delta + k_2(b)\delta^2 - k_3(b)\delta^3 + \dots
\end{aligned}$$

The fact that ϵ vanished from the formula means that the value of ϵ has no impact on the ranking between a and b. □

Proposition 22. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework, $i \in \mathbb{N} \backslash \{0\}$ be the maximal depth and μ be the precision threshold. If $\delta < \sqrt[i]{\frac{\mu}{\max_{a \in \mathcal{A}}(|\mathcal{R}_i^S(a)|)}}$ then, for all $a \in \mathcal{A}$, the sequence $\{P_i^{\epsilon,\delta}(a)\}_{i=0}^{+\infty}$ converges before step $i + 1$.

Proof. The process is stopped when, between two steps, the difference with the previous step for all the valuations P is smaller than a fixed precision threshold μ, i.e. $\forall a \in \mathcal{A}$,

$$\begin{aligned}
|P_i^{\epsilon,\delta}(a) - P_{i\text{-}1}^{\epsilon,\delta}(a)| &< \mu \\
|P_{i\text{-}1}^{\epsilon,\delta}(a) + (-1)^i \delta^i \sum_{b \in \mathcal{R}_i a} v_\epsilon(b) - P_{i\text{-}1}^{\epsilon,\delta}(a)| &< \mu \\
\delta^i \sum_{b \in \mathcal{R}_i a} v_\epsilon(b) &< \mu
\end{aligned}$$

It is clear that $\forall a \in \mathcal{A}$, $\sum_{b \in \mathcal{R}_i^S(a)} v_\epsilon(b) \leq |\mathcal{R}_i^S(a)| \leq \max_{a \in \mathcal{A}}\big(|\mathcal{R}_i^S(a)|\big)$. Use the maximum allows to be sure to that the difference between two steps is small enough w.r.t μ for all the arguments. So, if the method converges with $\max_{a \in \mathcal{A}}\big(|\mathcal{R}_i^S(a)|\big)$ then it also converges with the smallest values:

$$\delta^i \max_{a\in\mathcal{A}} \left(|\mathcal{R}_i^S(a)|\right) < \mu \Rightarrow \delta^i < \frac{\mu}{\max\limits_{a\in\mathcal{A}}\left(|\mathcal{R}_i^S(a)|\right)} \Rightarrow \delta < \sqrt[i]{\frac{\mu}{\max\limits_{a\in\mathcal{A}}\left(|\mathcal{R}_i^S(a)|\right)}}$$

□

Proposition 23. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $x, y \in \mathcal{A}$ such that $\mathcal{R}_1^S(x) = \emptyset$ and $\mathcal{R}_1^S(y) \neq \emptyset$.

If $\delta < \delta^M$ such that $\delta^M = \sqrt{\frac{1}{\max_{z\in\mathcal{A}}(|\mathcal{R}_2^S(z)|)}}$ then $P^{0,\delta}(x) > P^{0,\delta}(y)$

Proof. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework with $y \in \mathcal{A}$ the argument with the biggest number of direct defenders ($m = |\mathcal{R}_2^S(y)| = \max_{a\in\mathcal{A}}(|\mathcal{R}_2^S(a)|)$) and $x \in \mathcal{A}$ a non-attacked argument.
Let us determine the case where an attacked argument y can obtain a maximal score. According to the formal definition of the propagation principle, in order to not receive any negative value, y must not have any attack branch. Inversely, it must have many defense branches to receive a maximum of positive values. In addition, because of the attenuation factor, it is better to receive positive values from direct defenders. So the best score of y is when it receives m positive values from its m direct attackers: $P^{0,\delta}(y) = m\delta^2$. Recall that the condition is that $\delta < \sqrt{\frac{1}{m}}$, so:

$$\delta < \sqrt{\frac{1}{m}} \Rightarrow \delta^2 < \frac{1}{m} \Rightarrow m\delta^2 < 1 \Rightarrow P^{0,\delta}(y) < 1 \Rightarrow P^{0,\delta}(y) < P^{0,\delta}(x)$$

□

Proposition 24. Let $PP = \langle \mathcal{A}, \mathcal{R} \rangle$ be a persuasion pitch with $x \in \mathcal{A}$ as the targeted argument and $y \in \mathcal{A}$ be a non-attacked argument. Then,

(i) if $|\mathcal{B}_+(x)| < 2$ then $y \succ_{PP}^{\text{vdp}} x$;

(ii) if $|\mathcal{B}_+(x)| \geq 2$ and $\delta > \sqrt[m]{\frac{1}{|\mathcal{B}_+(x)|}}$ with m the length of the longest defense branch of x then $x \succ_{PP}^{\text{vdp}} y$.

Proof. (i) Let y be the only defense root of x ($\mathcal{B}_+(x) = \{y\}$ and $\mathcal{B}_-(x) = \emptyset$). The length of the path from y to x is n with $n \in 2\mathbb{N}$. According the definition of the propagation principle, when $\epsilon = 0$, x only receives the score from y which is attenuated by δ^n. So, $P^{0,\delta}(x) = \delta^n$ but $\delta \in]0, 1[$ so $\forall n, \delta^n < 1 = P^{0,\delta}(y)$ which implies that $y \succ^{\text{vdp}} x$, in agreement with the property Void Precedence.

(ii) Let y be a non-attacked argument ($P^{0,\delta}(y) = 1$) and x an argument with only defense branches with a length of m ($P^{0,\delta}(x) = |\mathcal{B}_+(x)|\delta^m$).

$$\delta > \sqrt[m]{\frac{1}{|\mathcal{B}_+(x)|}} \Rightarrow \delta^m > \frac{1}{|\mathcal{B}_+(x)|} \Rightarrow |\mathcal{B}_+(x)|\delta^m > 1 \Rightarrow P^{0,\delta}(x) > P^{0,\delta}(y) \Rightarrow x \succ^{\text{vdp}} y$$

The result will be similar with shorter defense branches. □

Proposition 25. If vdp^δ satisfies VP then it satisfies DP.

Proof. Let a and b be two arguments with the same number of direct attackers ($|\mathcal{R}_1^S(a)| = |\mathcal{R}_1^S(b)| = n$ with $n \in \mathbb{N}^*$) but b is defended whereas a is not. DP states that b should be strictly more acceptable than a.
As VP is satisfied, all the direct attackers of b which are attacked have a smaller propagation number than any direct attacker of a which are non-attacked : $\exists x \in \mathcal{R}_1^S(b)$ such that $\forall y \in \mathcal{R}_1^S(a)$, $y \succ^{\text{vdp}} x$. That means that $P^{0,\delta}(y) > P^{0,\delta}(x)$ or $P^{0,\delta}(y) = P^{0,\delta}(x)$ and $P^{\epsilon,\delta}(y) > P^{\epsilon,\delta}(x)$. Let us show that whatever the value of $\epsilon \in [0,1]$, we obtain the same result:

$$\begin{aligned} y \succ^{\text{vdp}} x &\Rightarrow P^{\epsilon,\delta}(y) > P^{\epsilon,\delta}(x) \\ &\Rightarrow 1 > P^{\epsilon,\delta}(x) \text{ because y is not attacked} \\ &\Rightarrow \delta > P^{\epsilon,\delta}(x)\delta \text{ with } \delta \in]0,1[\\ &\Rightarrow -\delta < -P^{\epsilon,\delta}(x)\delta \end{aligned}$$

a is only attacked by non-attacked arguments so $P^{\epsilon,\delta}(a) = \epsilon - n\delta = \epsilon - (n-1)\delta - \delta$. Suppose that b has only one direct attacker which is attacked (the same reasoning holds with more attacked attackers) then $P^{\epsilon,\delta}(b) = \epsilon - (n-1)\delta - P^{\epsilon,\delta}(x)\delta$. Consequently, according to the previous result ($-\delta < -P^{\epsilon,\delta}(x)\delta$), we can say that

$$\epsilon - (n-1)\delta - \delta < \epsilon - (n-1)\delta - P^{\epsilon,\delta}(x)\delta \Rightarrow P^{\epsilon,\delta}(a) < P^{\epsilon,\delta}(b) \Rightarrow a \prec^{\text{vdp}} b$$

□

Proposition 26. Let $\delta \in]0,1[$. vdp^δ satisfies Abs, In, Tot, NaE, +AB, AE and AvsFD. The other properties are not satisfied.

Proof.

Properties satisfied

(Abs) The nature of an argument is not used in the computation of its score. Only the attack relation is needed.

(In) An argument a only receives the scores from its attackers and defenders. Thus, an argument b, such that there exists no path between a and b, cannot propagate its initial value to a and then cannot influence the propagation number of a.

(Tot) Each argument receives a real number as propagation number. As all the real numbers can be compared, it is possible to compare all the arguments.

(AE) Obvious because two arguments with the same ancestor graph receive exactly the same value from their attackers and defenders whatever the values of ϵ and δ. Thus, they have the same propagation number which implies that they are equally acceptable.

(NaE) NaE is implied by AE which is satisfied.

(+AB) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that it exists an isomorphism γ with $AF = \gamma(AF')$. Let $a \in \mathcal{A}$ and its image $\gamma(a) \in \mathcal{A}'$ be two arguments, $\epsilon \in]0, 1]$ and $\delta \in]0, 1[$. As the semantics satisfies Argument Equivalence (AE), we can say that a and $\gamma(a)$, which have the same ancestor's graph, have always the same propagation number ($P^{\epsilon,\delta}(a) = P^{\epsilon,\delta}(\gamma(a))$).
Let us now add an attack branch from b to $\gamma(a)$ with a length $n \in 2\mathbb{N} + 1$.
There exists two possibilities with respect to δ:

- If the maximal depth m is greater than n, then during the first phase where $\epsilon = 0$, $\gamma(a)$ receives the negative value from b attenuated by δ^n: $P^{0,\delta}(a) - \delta^n = P^{0,\delta}(\gamma(a)) \Rightarrow P^{0,\delta}(a) > P^{0,\delta}(\gamma(a)) \Rightarrow a \succ^{\text{vdp}} \gamma(a)$.
- If the maximal depth m is smaller than n, then during the first step, no distinction is done between a and $\gamma(a)$ ($P^{0,\delta}(a) = P^{0,\delta}(\gamma(a))$) because the method converges before that $\gamma(a)$ receives the value from b. So, we restart with $\epsilon \neq 0$, and $\gamma(a)$ receives several additional values from its new attackers and defenders in the added attack branch. Thus, we have $P^{\epsilon,\delta}(a) + \sum_{i=1}^{m}(-1)^i \delta^i \epsilon = P^{\epsilon,\delta}(\gamma(a))$ but as $\sum_{i=1}^{m}(-1)^i \delta^i \epsilon < 0$, then $P^{\epsilon,\delta}(a) > P^{\epsilon,\delta}(\gamma(a)) \Rightarrow a \succ^{\text{vdp}} \gamma(a)$.

Then, in both cases, when an attack branch is added to an argument, its acceptability decreases, in agreement with the property.

(AvsFD) Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework where $a \in \mathcal{A}$ is attacked by only one non-attacked argument and $b \in \mathcal{A}$ is only defended (i.e. it has no attack branch). The property says that b should be more acceptable than a.
If b is a not attacked, then it is better than a: $P^{0,\delta}(b) = 1 > -\delta = P^{0,\delta}(a) \Rightarrow b \succ^{\text{vdp}} a$.
When b is attacked, we need to distinguish two cases with respect to δ :

- If the length of all the defense branches of b are greater than the maximal depth, then b does not receive any value from its defense roots. But a always receives the score from its non-attacked direct attacker so: $P^{0,\delta}(b) = 0 > -\delta = P^{0,\delta}(a) \Rightarrow b \succ^{\text{vdp}} a$.

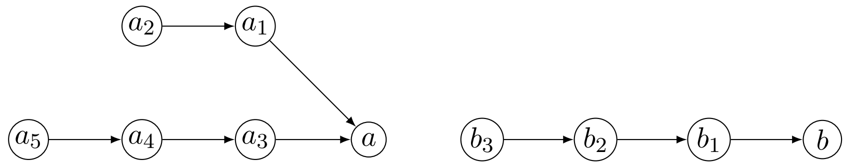
- Otherwise, b receives only positive values from its defense roots, so $P^{0,\delta}(b) > 0 > -\delta = P^{0,\delta}(a) \Rightarrow b \succ^{\text{vdp}} a$.

In summary, for all values of δ, b is more acceptable than a in agreement with the property.

Counter-examples

(CT) Considering the argumentation framework depicted in Figure C.1, let us show that CT is not satisfied.

The property says that b should be at least as acceptable as a because it exists an injective function f from $\mathcal{R}_1^S(b)$ to $\mathcal{R}_1^S(a)$ such that $\forall b' \in \mathcal{R}_1^S(b)$, $f(b') \succeq b'$. Indeed, we have $\mathcal{R}_1^S(b) = \{b_1\}$ and $\mathcal{R}_1^S(a) = \{a_1, a_3\}$ where $a_3 \succeq b_1$. However, vdp considers that, when $\delta = 0.5$, a is strictly more acceptable than b, contradicting the property.



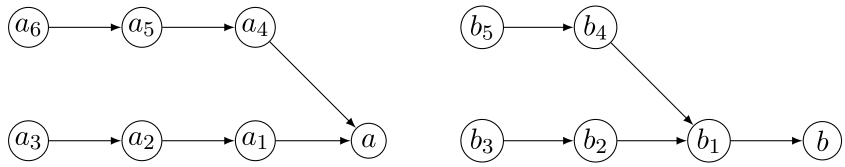
$\delta = 0.5 \quad a_2 \simeq a_5 \simeq b_3 \succ a_3 \simeq b_1 \succ a \succ b \succ a_1 \simeq a_4 \simeq b_2$

Figure C.1 – vdp falsifies the properties (S)CT and CP

(SCT) Considering the argumentation framework depicted in Figure C.1, let us show that SCT is not satisfied.
The property says that b should be at least as acceptable as a because it exists an injective function f from $\mathcal{R}_1^S(b)$ to $\mathcal{R}_1^S(a)$ such that $\forall b' \in \mathcal{R}_1^S(b)$, $f(b') \succeq b'$ and $|\mathcal{R}_1^S(a)| > |\mathcal{R}_1^S(b)|$. Indeed, we have $\mathcal{R}_1^S(b) = \{b_1\}$ and $\mathcal{R}_1^S(a) = \{a_1, a_3\}$ (so $|\mathcal{R}_1^S(a)| = 2 > 1 = |\mathcal{R}_1^S(b)|$) where $a_3 \succeq b_1$. However, vdp considers that, when $\delta = 0.5$, a is strictly more acceptable than b, contradicting the property.

(CP) Considering the argumentation framework depicted in Figure C.1, let us show that CP is not satisfied.
The property considers that b should be strictly more acceptable than a because $|\mathcal{R}_1^S(a)| = 2 > 1 = |\mathcal{R}_1^S(b)|$. However, vdp considers that, when $\delta = 0.5$, a is strictly more acceptable than b, contradicting the property.

(QP) Considering the argumentation framework depicted in Figure C.2, let us show that QP is not satisfied.



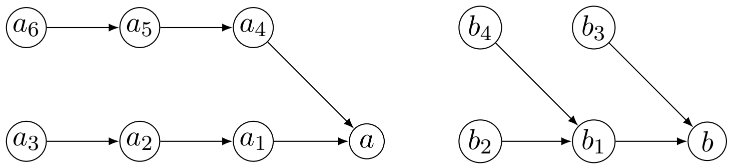
$\delta = 0.5 \quad a_3 \simeq a_6 \simeq b_3 \simeq b_5 \succ b_1 \succ a_1 \simeq a_4 \succ b \succ a \succ a_2 \simeq a_5 \simeq b_2 \simeq b_4$

Figure C.2 – vdp falsifies the property QP

The property says that a should be strictly more acceptable than b because $b_1 \succ a_4$ and $b_1 \succ a_1$. However, vdp considers that, when $\delta = 0.5$, b is strictly more acceptable than a, contradicting the property.

(DDP) Considering the argumentation framework depicted in Figure C.3, let us show that DDP is not satisfied.

The definition says that a should be strictly more acceptable than b because they have the same number of direct attackers ($|\mathcal{R}_1^S(a)| = |\mathcal{R}_1^S(b)| = 2$) and the same number of direct defenders ($|\mathcal{R}_2^S(a)| = |\mathcal{R}_2^S(b)| = 2$) but the defense of a is simple and distributed whereas the

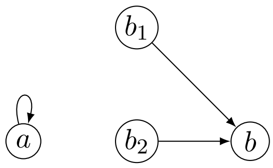


$$\delta = 0.5 \quad a_3 \simeq a_6 \simeq b_2 \simeq b_3 \simeq b_4 \succ a_1 \simeq a_4 \succ b \succ a \succ a_2 \simeq a_5 \succ b_1$$

Figure C.3 – vdp falsifies the property DDP

defense of b is simple and not distributed. However, vdp considers that, when $\delta = 0.5$, b is strictly more acceptable than a, contradicting the property.

(SC) Considering the argumentation framework depicted in Figure C.4, let us show that SC is not satisfied.

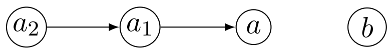


$$\delta = 0.5 \quad b_1 \simeq b_2 \succ a \succ b$$

Figure C.4 – vdp falsifies the property SC

The definition says that b should be strictly more acceptable than a because a attacks itself while b does not. However, vdp considers that, when $\delta = 0.5$, a is strictly more acceptable than b, contradicting the property.

($\oplus$DB) Considering the argumentation framework depicted in Figure C.5, let us show that $\oplus$DB is not satisfied.

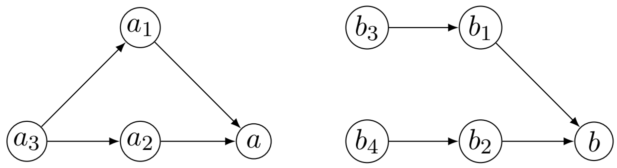


$$\delta = 0.5 \quad a_2 \simeq b \succ a \succ a_1$$

Figure C.5 – vdp falsifies the property $\oplus$DB

The property says that a should be strictly more acceptable than b because a has a defense branch while that b has not. However, vdp considers that, when $\delta = 0.5$, b is strictly more acceptable than a, contradicting the property.

(OE) Considering the argumentation framework depicted in Figure C.6, let us show that OE is not satisfied. The property considers that a and b should be equally acceptable because there



$\delta = 0.2 \quad a_3 \simeq b_3 \simeq b_4 \succ b \succ a \succ a_1 \simeq a_2 \simeq b_1 \simeq b_2$

Figure C.6 – vdp falsifies the property OE

exists a bijective function f from $\mathcal{R}_1^S(a)$ to $\mathcal{R}_1^S(b)$ such that $\forall c \in \mathcal{R}_1^S(a), c \simeq f(c)$. Indeed, one can remark that $a_1 \simeq b_1$ and $a_2 \simeq b_2$. However, vdp considers that, when $\delta = 0.2$, b is strictly more acceptable than a, contradicting the property.

□

Proposition 27. Let μ be a precision threshold and i the expected maximal length.

If $\delta \in]\delta^m, 1[$ such that $\delta^m = \sqrt[i]{\mu}$ then vdp^δ satisfies also +DB$_i$, ↑DB$_i$ and ↑AB$_i$

Proof. The proposition 22 assures that if $\delta > \delta^m$ then the argument with an additional branch (+DB) or a extended branch (↑DB and ↑AB) will receive all the values from the arguments belonging to this branch.
Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ be two argumentation frameworks such that it exists an isomorphism γ with $AF = \gamma(AF')$. Let $a \in \mathcal{A}$ and its image $\gamma(a) \in \mathcal{A}'$ be two arguments, $\epsilon \in]0, 1]$ and $\delta \in]0, 1[$. As the semantics satisfies the properties Argument Equivalence (AE), we can say that a and $\gamma(a)$, with the same ancestor's graph, have always the same propagation number $(P^{\epsilon,\delta}(a) = P^{\epsilon,\delta}(\gamma(a)))$.

(+DB) Let us now add a defense branch from b to $\gamma(a)$ (so $\mathcal{R}_1^S(b) = \emptyset$) with a length of $i \in 2\mathbb{N}$. Recall that the first step of this semantics (when $\epsilon = 0$) consists in checking only the impact of non-attacked arguments. So now $\gamma(a)$ receives one additional positive value from b ($v_\epsilon(b) = 1$), so $P^{\epsilon,\delta}(a) + \delta^i = P^{\epsilon,\delta}(\gamma(a)) \Rightarrow P^{\epsilon,\delta}(a) < P^{\epsilon,\delta}(\gamma(a)) \Rightarrow \gamma(a) \succ^{\text{vdp}} a$, in agreement with the property.

(↑AB) We suppose $\exists n \in 2\mathbb{N}+1$ such that b is an argument situated at the beginning of an attack branch to a with a length of n. Thus a receives a score of $-\delta^n$ from b during the first phase where $\epsilon = 0$. Now, we add a new defense branch to the non-attacked argument $\gamma(b)$. We denoted by b' the argument at the beginning of this new branch which has a length of m. It is clear than $\gamma(b)$ is now attacked so, during the step where $\epsilon = 0$, $\gamma(b)$ does not send its negative value anymore to $\gamma(a)$ but it receives the negative score of b' (-δ^{n+m}) : $P^{0,\delta}(\gamma(a)) = P^{0,\delta}(a) + \delta^n - \delta^{n+m}$. Consequently, $\delta^n > \delta^{m+n}$ (because $m + n > n$) and $P^{0,\delta}(\gamma(a)) > P^{0,\delta}(a)$ implies that $\gamma(a) \succ^{\text{vdp}} a$, in agreement with the property.

(↑DB) The proof is very similar to the one of ↑AB. The difference is that b is situated at the beginning of a defense branch. So $P^{0,\delta}(\gamma(a)) = P^{0,\delta}(a) - \delta^n + \delta^{m+n}$ with $\delta^n > \delta^{m+n}$, which implies that $P^{0,\delta}(a) > P^{0,\delta}(\gamma(a)) \Rightarrow a \succ^{\text{vdp}} \gamma(a)$ in agreement with the property.

□

Appendix D

List of properties for ranking-based semantics

Abstraction **(page 63)**

The ranking on the set of arguments should be defined only on the basis of the attacks between arguments.

Property 1 (Abstraction (Abs))**.** [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Abstraction if and only if for any $AF, AF' \in \mathbb{AF}$, for every isomorphism γ such that $AF' = \gamma(AF)$, we have $x \succeq^{\sigma}_{\text{AF}} y$ if and only if $\gamma(x) \succeq^{\sigma}_{\text{AF}'} \gamma(y)$.

Independence **(page 64)**

The ranking between two arguments x and y should be independent of any argument that is neither connected to x nor to y.

Property 2 (Independence (In))**.** [MATT & TONI 2008, AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Independence if and only if for any argumentation framework AF such that $\forall AF' \in cc(AF), \forall x, y \in Arg(AF')$, $x \succeq^{\sigma}_{\text{AF}'} y$ if and only if $x \succeq^{\sigma}_{\text{AF}} y$.

Void Precedence **(page 64)**

A non-attacked argument should be strictly more acceptable than an attacked argument.

Property 3 (Void Precedence (VP))**.** [MATT & TONI 2008, AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Void Precedence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $\mathcal{R}_1(x) = \emptyset$ and $\mathcal{R}_1(y) \neq \emptyset$ then $x \succ^{\sigma}_{\text{AF}} y$.

Self-Contradiction **(page 64)**

An argument that attacks itself should be strictly less acceptable than an argument that does not.

Property 4 (Self-Contradiction (SC))**.** [MATT & TONI 2008]
A ranking-based semantics σ satisfies Self-Contradiction if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $(x, x) \notin \mathcal{R}$ and $(y, y) \in \mathcal{R}$ then $x \succ^{\sigma}_{\text{AF}} y$.

Cardinality Precedence **(page 65)**

If x has strictly more direct attackers than y, then y should be strictly more acceptable than x.

Property 5 (Cardinality Precedence (CP)). [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Cardinality Precedence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $|\mathcal{R}_1(x)| < |\mathcal{R}_1(y)|$ then $x \succ^{\sigma}_{\text{AF}} y$.

Quality Precedence **(page 66)**

If x has a direct attacker strictly more acceptable than any direct attacker of y, then x should be strictly more acceptable than y.

Property 6 (Quality Precedence (QP)). [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Quality Precedence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $\exists y' \in \mathcal{R}_1(y)$ such that $\forall x' \in \mathcal{R}_1(x)$, $y' \succ^{\sigma}_{\text{AF}} x'$ then $x \succ^{\sigma}_{\text{AF}} y$.

Counter-Transitivity **(page 66)**

If the direct attackers of y are at least as numerous and acceptable as those of x, then x should be at least as acceptable as y.

Property 7 (Counter-Transitivity (CT)). [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Counter-Transitivity if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $\mathcal{R}_1(y) \geq^{\sigma}_{S} \mathcal{R}_1(x)$ then $x \succeq^{\sigma}_{\text{AF}} y$.

Strict Counter-Transitivity **(page 67)**

If CT is satisfied and if the direct attackers of y are either strictly more numerous, or strictly more acceptable than those of x, then x should be strictly more acceptable than y.

Property 8 (Strict Counter-Transitivity (SCT)). [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Strict Counter-Transitivity if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $\mathcal{R}_1(y) >^{\sigma}_{S} \mathcal{R}_1(x)$ then $x \succ^{\sigma} y$.

Defense Precedence **(page 67)**

If arguments x and y have the same number of direct attackers, and if x is defended at least once whereas y is not, x should be ranked higher than y.

Property 9 (Defense Precedence (DP)). [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Defense Precedence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$ such that $|\mathcal{R}_1(x)| = |\mathcal{R}_1(y)|$, if $\mathcal{R}_2(x) \neq \emptyset$ and $\mathcal{R}_2(y) = \emptyset$ then $x \succ^{\sigma}_{\text{AF}} y$.

Distributed-Defense Precedence **(page 67)**

A defense where each defender attacks a distinct attacker is better than any other.

Property 10 (Distributed-Defense Precedence (DDP))**.** [AMGOUD & BEN-NAIM 2013]
A ranking-based semantics σ satisfies Distributed-Defense Precedence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$ such that $|\mathcal{R}_1(x)| = |\mathcal{R}_1(y)|$ and $|\mathcal{R}_2(x)| = |\mathcal{R}_2(y)|$, if the defense of x is simple and distributed and the defense of y is simple but not distributed, then $x \succ^{\sigma}_{\text{AF}} y$.

Addition of an Attack Branch **(page 97)**

Adding an attack branch to any argument decreases its level of acceptability.

Property 11 (Addition of an Attack Branch (+AB))**.** [BONZON *et al.* 2016a]
A ranking-based semantics σ satisfies Addition of an Attack Branch if and only if for any $AF, AF' \in \mathbb{AF}$ and $x \in Arg(AF)$, for every isomorphism γ such that $AF' = \gamma(AF)$, if $AF^{\star} = AF \cup AF' \cup P^{-}(\gamma(x))$, then $x \succ^{\sigma}_{\text{AF}^{\star}} \gamma(x)$.

Strict Addition of an Defense Branch **(page 97)**

Adding a defense branch to any argument increases its level of acceptability.

Property 12 (Strict Addition of a Defense Branch ($\oplus$DB))**.** [BONZON *et al.* 2016a]
A ranking-based semantics σ satisfies Strict Addition of a Defense Branch if and only if for any $AF, AF' \in \mathbb{AF}$ and $x \in Arg(AF)$, for every isomorphism γ such that $AF' = \gamma(AF)$, if $AF^{\star} = AF \cup AF' \cup P^{+}(\gamma(x))$, then $\gamma(x) \succ^{\sigma}_{\text{AF}^{\star}} x$.

Addition of an Defense Branch **(page 98)**

Adding a defense branch to any attacked argument increases its level of acceptability.

Property 13 (Addition of a Defense Branch (+DB))**.** [BONZON *et al.* 2016a]
A ranking-based semantics σ satisfies Addition of a Defense Branch if and only if for any $AF, AF' \in \mathbb{AF}$ and $x \in Arg(AF)$, for every isomorphism γ such that $AF' = \gamma(AF)$, if $AF^{\star} = AF \cup AF' \cup P^{+}(\gamma(x))$ and $\mathcal{R}_1(x) \neq \emptyset$, then $\gamma(x) \succ^{\sigma}_{\text{AF}^{\star}} x$.

Increase of an Attack Branch **(page 98)**

Increasing the length of an attack branch of an argument increases its level of acceptability.

Property 14 (Increase of an Attack Branch ($\uparrow$AB))**.** [BONZON *et al.* 2016a]
A ranking-based semantics σ satisfies Increase of an Attack Branch if and only if for any $AF, AF' \in \mathbb{AF}$ and $x \in Arg(AF)$, for every isomorphism γ such that $AF' = \gamma(AF)$, if $\exists y \in \mathcal{B}_{-}(x)$, $y \notin \mathcal{B}_{+}(x)$ and $AF^{\star} = AF \cup AF' \cup P^{+}(\gamma(y))$, then $\gamma(x) \succ^{\sigma}_{\text{AF}^{\star}} x$.

Increase of a Defense Branch **(page 98)**

Increasing the length of a defense branch of an argument decreases its level of acceptability.

Property 15 (Increase of a Defense Branch (↑DB))**.** [BONZON *et al.* 2016a]
A ranking-based semantics σ satisfies Increase of a Defense Branch if and only if for any $AF, AF' \in \mathbb{AF}$ and $x \in Arg(AF)$, for every isomorphism γ such that $AF' = \gamma(AF)$, if $\exists y \in \mathcal{B}_+(x), y \notin \mathcal{B}_-(x)$ and $AF^\star = AF \cup AF' \cup P^+(\gamma(y))$, then $x \succ^\sigma_{\text{AF}^\star} \gamma(x)$.

Total **(page 99)**

All arguments can be compared.

Property 16 (Total (Tot))**.** [BONZON *et al.* 2016a]
A ranking-based semantics σ satisfies total if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, $x \succeq^\sigma_{\text{AF}} y$ or $y \succeq^\sigma_{\text{AF}} x$.

Argument Equivalence **(page 99)**

If there exists an isomorphism between the ancestors' graph of two arguments, then they are equally acceptable.

Property 17 (Argument Equivalence (AE))**.** [BONZON *et al.* 2016b]
A ranking-based semantics σ satisfies argument equivalence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, for every isomorphism γ such that $Anc_{\text{AF}}(x) = \gamma(Anc_{\text{AF}}(y))$ then $x \simeq^\sigma_{\text{AF}} y$.

Non-attacked Equivalence **(page 100)**

All non-attacked arguments should be equally acceptable.

Property 18 (Non-attacked Equivalence (NaE))**.** [BONZON *et al.* 2016a]
A ranking-based semantics σ satisfies non-attacked equivalence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, $\mathcal{R}_1(x) = \emptyset$ and $\mathcal{R}_1(y) = \emptyset$ then $x \simeq^\sigma_{\text{AF}} y$.

Ordinal Equivalence **(page 100)**

Suppose that two arguments, x and y, have the same number of direct attackers. If, for each direct attacker of x, there exists a direct attacker of y such that the two attackers are equally acceptable, then x and y are equally acceptable too.

Property 19 (Ordinal Equivalence (OE))**.**
A ranking-based semantics σ satisfies ordinal equivalence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if there exists a bijective function f from $\mathcal{R}_1(x)$ to $\mathcal{R}_1(y)$ such that $\forall z \in \mathcal{R}_1(x), z \simeq^\sigma_{\text{AF}} f(z)$ then $x \simeq^\sigma_{\text{AF}} y$.

Attack vs Full Defense **(page 100)**

A fully defended argument (without any attack branch) should be strictly more acceptable than an argument attacked once by a non-attacked argument.

Property 20 (Attack vs Full Defense (AvsFD))**.** [BONZON *et al.* 2016a]
A ranking-based semantics σ satisfies attack vs full defense if and only if for any acyclic $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $|\mathcal{B}_{-}(x)| = 0$, $|\mathcal{R}_1(y)| = 1$ and $|\mathcal{R}_2(y)| = 0$ then $x \succ^{\sigma}_{\text{AF}} y$.

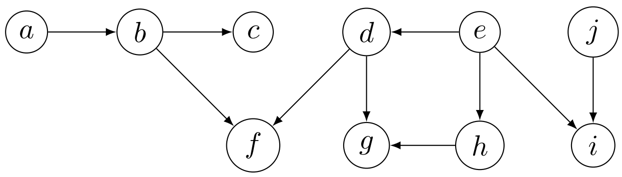
Weak Void Precedence **(page 201)**

A non-attacked argument should be at least as acceptable as an attacked argument.

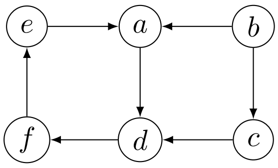
Property 21 (Weak Void Precedence (WVP))**.** [THIMM & KERN-ISBERNER 2014]
A ranking-based semantics σ satisfies Weak Void Precedence if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall x, y \in \mathcal{A}$, if $\mathcal{R}_1(x) = \emptyset$ and $\mathcal{R}_1(y) \neq \emptyset$ then $x \succeq^{\sigma}_{\text{AF}} y$.

Appendix E

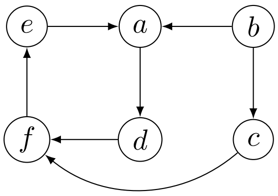
Examples



Semantics	*Ranking*
M&T	$a \simeq e \simeq j \succ c \simeq f \simeq g \succ b \simeq d \simeq h \simeq i$
FL	$a \simeq e \simeq j \simeq c \simeq f \simeq g \succ b \simeq d \simeq h \simeq i$
Cat	$a \simeq e \simeq j \succ c \succ b \simeq d \simeq f \simeq g \simeq h \succ i$
1-Bbs	
Dbs	$a \simeq e \simeq j \succ c \succ b \simeq d \simeq h \succ f \simeq g \succ i$
Bbs	
0.5-Bbs	
CS	
Propa$_\epsilon^{0.75,M}$	
Propa$_\epsilon^{0.75,S}$	$a \simeq e \simeq j \succ c \succ b \simeq d \simeq h \succ f \succ g \succ i$
5-Bbs	$a \simeq e \simeq j \succ c \succ f \simeq g \succ b \simeq d \simeq h \succ i$
IGD	
Propa$_\epsilon^{0.3,M}$	
Propa$_{1+\epsilon}^{\epsilon,M}$	
Propa$_\epsilon^{0.3,S}$	$a \simeq e \simeq j \succ c \succ f \succ g \succ b \simeq d \simeq h \succ i$
Propa$_{1+\epsilon}^{\epsilon,S}$	
Propa$_{1\to\epsilon}^{\epsilon,S}$	$a \simeq e \simeq j \succ f \succ c \succ g \succ b \simeq d \simeq h \succ i$
Tuples	$a \simeq e \simeq j \succ f \simeq g \succ c \succ b \simeq d \simeq h \succ i$
Propa$_{1\to\epsilon}^{\epsilon,M}$	



<table>
<tr><th>Semantics</th><th>Ranking</th></tr>
<tr><td>FL</td><td>$b \succ d \succ e \succ f \succ a \simeq c$</td></tr>
<tr><td>$\mathsf{vdp}^{0.3}$</td><td rowspan="3">$b \succ d \succ e \succ f \succ c \succ a$</td></tr>
<tr><td>5-Bbs</td></tr>
<tr><td>$Propa^{\epsilon,\oplus}_{1\to\epsilon}$</td></tr>
<tr><td>Dbs/Bbs</td><td rowspan="4">$b \succ f \succ e \succ c \succ d \succ a$</td></tr>
<tr><td>CS</td></tr>
<tr><td>0.5-Bbs</td></tr>
<tr><td>$Propa^{0.8,\oplus}_{\epsilon}$</td></tr>
<tr><td>Cat</td><td rowspan="3">$b \succ f \succ e \succ d \succ c \succ a$</td></tr>
<tr><td>$Propa^{0.3,\oplus}_{\epsilon}$</td></tr>
<tr><td>$Propa^{\epsilon,\oplus}_{1+\epsilon}$</td></tr>
<tr><td>$\mathsf{vdp}^{0.8}$</td><td>$d \succ b \succ e \succ c \succ f \succ a$</td></tr>
</table>



<table>
<tr><th>Semantics</th><th>Ranking</th></tr>
<tr><td>FL</td><td>$b \simeq d \simeq e \succ f \simeq a \simeq c$</td></tr>
<tr><td>Cat</td><td rowspan="6">$b \succ d \succ e \succ c \succ f \succ a$</td></tr>
<tr><td>Bbs/Dbs</td></tr>
<tr><td>0.3-Bbs</td></tr>
<tr><td>1-Bbs</td></tr>
<tr><td>CS</td></tr>
<tr><td>$Propa^{0.8,\oplus}_{\epsilon}$</td></tr>
<tr><td>10-Bbs</td><td rowspan="3">$b \succ d \succ e \succ f \succ c \succ a$</td></tr>
<tr><td>$Propa^{0.3,\oplus}_{\epsilon}$</td></tr>
<tr><td>$Propa^{\epsilon,\oplus}_{1+\epsilon}$</td></tr>
<tr><td>$Propa^{\epsilon,\oplus}_{1\to\epsilon}$</td><td rowspan="3">$b \succ d \succ f \succ e \succ a \succ c$</td></tr>
<tr><td>$\mathsf{vdp}^{0.3}$</td></tr>
<tr><td>$\mathsf{vdp}^{0.8}$</td></tr>
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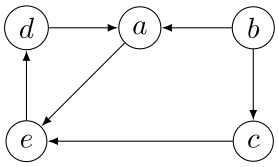

<table>
<tr><th>Semantics</th><th>Ranking</th></tr>
<tr><td>FL</td><td>$b \simeq e \succ a \simeq c \simeq d$</td></tr>
<tr><td>Cat</td><td rowspan="5">$b \succ d \succ e \succ c \succ a$</td></tr>
<tr><td>1-Bbs</td></tr>
<tr><td>$Propa^{0.3,\oplus}_{\epsilon}$</td></tr>
<tr><td>$Propa^{\epsilon,\oplus}_{1+\epsilon}$</td></tr>
<tr><td>CS</td></tr>
<tr><td>Bbs</td><td rowspan="4">$b \succ d \succ c \succ e \succ a$</td></tr>
<tr><td>Dbs</td></tr>
<tr><td>0.3-Bbs</td></tr>
<tr><td>$Propa^{0.8,\oplus}_{\epsilon}$</td></tr>
<tr><td>10-Bbs</td><td rowspan="2">$b \succ e \succ d \succ c \succ a$</td></tr>
<tr><td>$Propa^{\epsilon,\oplus}_{1\to\epsilon}$</td></tr>
<tr><td>$\mathsf{vdp}^{0.3}$</td><td rowspan="2">$b \succ e \succ d \succ c \succ a$</td></tr>
<tr><td>$\mathsf{vdp}^{0.8}$</td></tr>
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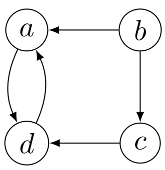
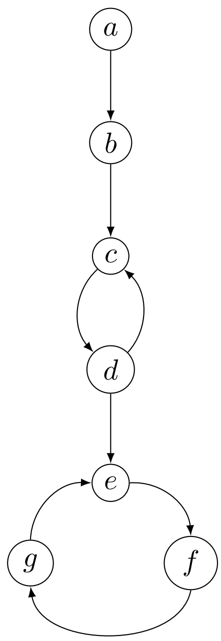


Semantics	*Ranking*
FL	$b \simeq d \succ a \simeq c$
Cat	$b \succ d \succ c \succ a$
1-Bbs	
10-Bbs	
CS	
$Propa_\epsilon^{0.3,\oplus}$	
$Propa_{1+\epsilon}^{\epsilon,\oplus}$	
$Propa_{1\to\epsilon}^{\epsilon,\oplus}$	
$\mathsf{vdp}^{0.3}$	
$\mathsf{vdp}^{0.8}$	
Bbs	$b \succ c \succ d \succ a$
Dbs	
0.3-Bbs	
$Propa_\epsilon^{0.8,\oplus}$	



Semantics	*Ranking*
FL	$a \succ c \simeq d \simeq e \simeq f \simeq g \succ b$
Cat	$a \succ f \succ d \succ g \succ b \succ c \succ e$
Bbs	
Dbs	
0.3-Bbs	
1-Bbs	
CS	
$Propa_\epsilon^{0.8,\oplus}$	
10-Bbs	$a \succ f \succ d \succ g \succ c \succ e \succ b$
$Propa_\epsilon^{0.8,\oplus}$	
$Propa_{1+\epsilon}^{\epsilon,\oplus}$	
$Propa_{1\to\epsilon}^{\epsilon,\oplus}$	$a \succ f \succ d \succ g \succ c \succ e \succ b$
$\mathsf{vdp}^{0.3}$	
$\mathsf{vdp}^{0.8}$	

List of Figures

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Abstract

Dung's theory of abstract argumentation is a formalism that represents conflicting information using an argumentation framework. Extension-based semantics have been introduced to determine, given an argumentation framework, the justifiable points of view on the acceptability of the arguments. However, these semantics are not appropriate for some applications. So alternative semantics, called ranking-based semantics, have recently been evolved. Such semantics produces, for a given argumentation framework, a ranking on its arguments from the most acceptable to the least one(s). The overall aim of this thesis is to propose and study ranking-based semantics in the context of abstract argumentation.

We first define a new family of ranking-based semantics based on a propagation principle which allow us to control the influence of non-attacked arguments on the acceptability of arguments. We investigate the properties of these semantics, the relationships between them but also with other existing semantics.

Then, we provide a thorough analysis of ranking-based semantics in two different ways. The first one is an empirical comparison on randomly generated argumentation frameworks which reveals insights into similarities and differences between ranking-based semantics. The second one is an axiomatic comparison of all these semantics with respect to the proposed properties aiming to better understand the behavior of each semantics.

At last, we question the ability of the existing ranking-based semantics to capture persuasion settings and introduce a new parametrized ranking-based semantics which is more appropriate in this context.

Keywords: Abstract argumentation, ranking-based semantics.

Résumé

La théorie de l'argumentation abstraite de Dung est un formalisme permettant d'utiliser un système d'argumentation afin de représenter des informations conflictuelles. Des sémantiques à base d'extensions ont d'abord été introduites dans le but de déterminer quels arguments peuvent être conjointement acceptés. Cependant, ces sémantiques ne sont pas appropriées pour certaines applications, c'est pourquoi des sémantiques à base de classement, permettant de classer les arguments du plus acceptable au moins acceptable, ont été introduites. Le but de cette thèse est donc de proposer et d'étudier ces sémantiques à base de classement dans le contexte de l'argumentation abstraite.

Nous définissons d'abord une nouvelle famille de sémantiques à base de classement basées sur un principe de propagation permettant de contrôler l'influence des arguments non-attaqués sur l'acceptabilité des arguments. Nous étudions les propriétés de ces sémantiques, les relations entre elles ainsi qu'avec d'autres sémantiques existantes.

Nous proposons ensuite deux méthodes pour comparer les sémantiques à base de classement. La première est une comparaison empirique sur des systèmes d'argumentation générés aléatoirement donnant un aperçu des similitudes et des différences entre ces sémantiques. La seconde est une comparaison axiomatique de toutes ces sémantiques à la lumière des propriétés proposées visant à mieux comprendre le comportement de chaque sémantique.

Enfin, nous remettons en question la capacité des sémantiques existantes à capturer certains principes de persuasion et introduisons une nouvelle sémantique paramétrée à base de classement plus appropriée pour ce contexte précis.

Mots-clés: Argumentation abstraite, Sémantique à base de classement.