

# Flipping time versus Satisfying in Local Search for SAT

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Given a CNF formula  $\mathcal{F}$  and an assignment  $A$ , the objective function that local search for SAT attempts to minimize is usually the total number of unsatisfied clauses in  $\mathcal{F}$  under  $A$ . The score of a variable  $x$  with respect to  $A$  is the decrease of the objective function when it is flipped. The flipping time (`flip_time`) of a variable usually refers to the last step where the variable has changed the value. The previous flipping time (`previous_flip_time`) of a variable is the second last step where the variable is flipped. The satisfying time (`sat_time`) of a variable related to a clause  $c$  is the last step where the variable is flipped to satisfy  $c$ ,  $c$  being false before this step.

The *Novelty* [4] family algorithms use the score and the flipping time of the variables in a falsified clause to choose a variable to flip in this clause as follows:

*Novelty*( $p, c$ ): Sort the variables in clause  $c$  by their scores, breaking ties in favor of the least recently flipped variable. Consider the best and second best variables from the sorted variables. If the best variable is not the most recently flipped one in  $c$ , then pick it. Otherwise, with probability  $p$ , pick the second best variable, and with probability  $1-p$ , pick the best variable.

The *Novelty* family algorithms are very efficient for random  $kSAT$  instances, but have some difficulties to solve structured SAT instances, as showed in recent SAT competitions. We propose a new local search solver called *Sattime* which is *Novelty* but uses the satisfying time of the variables related to the falsified clause  $c$  to select the next variable to flip. Algorithm *Sattime* is sketched as follows.

*Sattime*( $p, wp, c$ ):

1. If there are promising decreasing variables, pick the oldest one;
2. Otherwise, with probability  $wp$ , randomly pick a variable from  $c$ ;
3. With probability  $1-wp$ , sort the variables in clause  $c$  by their scores, breaking ties in favor of the least recent satisfying variable of  $c$ . Consider the best and second best variables from the sorted variables. If the best variable is not the most recent satisfying variable of  $c$ , then pick it. Otherwise, with probability  $p$ , pick the second best variable, and with probability  $1-p$ , pick the best variable.

The notion of promising decreasing variable was defined in [3]. Random walk probability  $wp$  was defined in [1], and adaptive noise mechanism was defined in [2].

The intuition behind *Sattime* is that if the last time flipping  $x$  to satisfy  $c$  does not prevent  $c$  from being falsified later during search (since  $c$  is now falsified), then one should be more careful to flip  $x$  to satisfy  $c$  again. So, if  $x$  is the best variable,  $x$  is not flipped with probability  $p$ .

Unfortunately, the satisfying time of a variable related to a clause is time consuming to memorize, because the satisfying time of the variable related to a different clause may be different. So, we relax the constraint related to the clause  $c$ , so that when a variable is flipped to satisfy  $c$ ,  $s$  may or may not be false. In this case, given a falsified clause  $c$ , the previous flipping time of a variable in  $c$  is the last step the variable satisfies  $c$ ,  $c$  may or may not have been satisfied by other variables in  $c$ , since the current value of the variable falsifies the corresponding literal in  $c$ , but its previous value satisfied this literal. We then propose a new local search solver *Sattime+* in which `sat.time` is replaced with `previous_flip_time`. In other words, the variables of the falsified clause  $c$  are sorted by their scores and ties are broken using `previous_flip_time` instead of `sat.time`.

Note that the previous flipping time of a variable is independent of any clause. However, the satisfying time (`sat.time`) of a variable should be recorded for every clause in which the variable occurs.

## References

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